July 2014

Improving Mathematics: An Examination of the Effects of Specific Cognitive Abilities on College-age Students' Mathematics Achievement

Gordon E. Taub
University of Central Florida, gtaub@ucf.edu

Nicholas Benson
University of South Dakota, Nicholas.Benson@usd.edu

Judit Szente
University of Cental Florida, jszente@ucf.edu

Improving Mathematics: An Examination of the Effects of Specific Cognitive Abilities on College-age Students' Mathematics Achievement

Abstract

This study investigated the effects of general intelligence and seven specific cognitive abilities on college-age students' mathematics achievement. The present investigation went beyond previous research by employing structural equation modeling. It also represents the first study to examine the direct and indirect effects of general and specific cognitive abilities, simultaneously, on the mathematics achievement of college-age students. A model developed using the Cattell-Horn-Carroll theory of intelligence was the theoretical model used in all analyses. Data from 1,054 college-age students who participated in the standardization of the Woodcock–Johnson III (Woodcock, McGrew, & Mather, 2001) were divided into a calibration sample set and validation sample set. The calibration data set was used for model testing and modification and the independent validation sample data set was used for model validation. The specific areas of intelligence demonstrating direct effects on the mathematics achievement dependent variable were Crystallized Intelligence and Fluid Reasoning. The effects of general intelligence were found to be *indirect* in the college-age sample. Implications for instruction and intervention to improve college student's mathematics achievement are provided.

Improving Mathematics: A Examination of the Effects of Specific Cognitive Abilities on College-age Students' Mathematics Achievement

Research investigating quantitative reasoning within the scholarship of teaching and learning (SoTL) community indicates that the cognitive abilities important for mathematics success are also important for students' general academic success. The academic skills associated with these cognitive abilities include numeracy, problem solving, and quantitative reasoning (e.g., Bok, 2006; Grawe, 2011; Richardson & McCallum, 2004). Much of this research encourages faculty to foster students' academic development across curricular domains and disciplines. Many of these studies are process oriented, meaning their focus is on students' output or the application of quantitative reasoning knowledge and/or skills in our classrooms. Another area of research in the relationship between cognitive abilities and mathematics achievement focuses on the specific underlying cognitive abilities important in the acquisition of quantitative reasoning skills.

Knowledge of the specific cognitive abilities important for mathematics success has the potential to impact faculty instruction and curriculum development. It is impractical for faculty to assess and provide feedback and instruction relative to each student's problem solving and reasoning skill-level. However, each cognitive ability is composed of specific narrow areas or what might be thought of as specific sub-skills. For example, two sub-skills important in reasoning activities include deductive and inductive reasoning. Knowing the specific narrow areas or sub-skills underlying each cognitive ability that is important for quantitative reasoning skill acquisition may assist faculty in curriculum development, instruction, and improve students' learning outcomes.

Prior research investigating the role of cognitive abilities and mathematics achievement focused on general intelligence. For the purposes of the present paper, we are defining general intelligence as being inferred from the common variance shared by multiple tasks used to measure intellectual functioning. Although this phenomenon is not directly observable, it is a strong predictor of academic and occupational success. Research suggests that general intelligence is the single best predictor of academic achievement across all academic areas including mathematics (Cronbach & Snow, 1977; Hunter & Hunter, 1984; Jensen, 1984, 1998). Although general intelligence accounts for about .50% of the variance in the prediction of academic achievement, knowledge of an individual's general intelligence or IQ score does little to assist faculty during instruction or curriculum development; except when working with individuals at the extreme ends of the distribution.

More recently, researchers have identified specific cognitive abilities important for success across many academic domains. This research identifies the specific cognitive abilities important for mathematics success– beyond general intelligence. Knowledge of the specific cognitive abilities important for mathematics success may be used by faculty to guide instruction and curriculum development. To date, the majority of this research focused on school-age populations（e.g., Floyd, Keith, Taub, McGrew, 2007; McGrew, Flanagan, Keith, & Vanderwood, 1997; Taub, Floyd, Keith, & McGrew, 2008）. This study represents the second investigation of this nature involving college-age participants（Taub & Benson, 2013）.

The development of this body of research is driven by two factors. The first is advances in the areas of intellectual assessment, research methods, and intellectual theory. This is most evident in the Cattell-Horn-Carroll（CHC）theory of intelligence（McGrew & Flanagan, 1998）, which serves as the theoretical foundation for the research identifying the specific cognitive abilities important for academic success. The second factor driving this research is the technological advancement provided through structural equation modeling（SEM）, which allows for the inclusion of composite scores or general intelligence and the scores from the tests measuring the specific cognitive factors contributing to general intelligence to be analyzed simultaneously. This is not possible with multiple regression（Thorndike, Hagen, & Sattler, 1986）.

Cattell-Horn-Carroll Theory of Intelligence

The CHC theory of intelligence is the most comprehensive research-based model of intelligence（McGrew 2005; 2009）and serves as the primary theoretical foundation for most contemporary tests of intelligence（Keith & Reynolds, 2010）. Within CHC theory, there are seven specific cognitive abilities which may be measured. These include Auditory Processing, Crystalized Intelligence, Fluid Reasoning, Long-Term Retrieval, Processing Speed, Short-Term Memory, and Visual-Spatial Thinking（see Taub & Benson, 2013; Taub & McGrew, 2004 for a detailed explanation of each cognitive ability）. Recent CHC-based research employing SEM identified the specific cognitive abilities Crystalized Intelligence, Fluid Reasoning, and Processing Speed as strong predictors of mathematics achievement（McGrew, 1997; Keith 2009）. This research was supported by Taub, Floyd, Keith, & McGrew's（2007）study which identified these three specific cognitive factors, in various combinations, as important for mathematics achievement across four age differentiated school-age samples.

Purpose of the Study

Investigating the cognitive abilities important for mathematics achievement to guide faculty instruction and improve student achievement is valued within the scholarship of teaching and learning community（Dewar & Benett, 2010; Hutchings,

Huber, & Ciccone, 2011; Mitchell, Anderson, Sensibaugh, & Osgood, 2011). The first purpose of the study is to answer the question, which specific CHC cognitive abilities are most important for college students' quantitative reasoning skill acquisition, beyond general intelligence? This will expand discipline-based knowledge through the development of curriculum and instructional strategies to assist students as they acquire, apply, and master mathematics algorithms and skills. The second purpose of this research is to identify how students process information to improve academic learning and more specifically, mathematics achievement (Cerbin, 2013). A final purpose of this study is to provide the SoTL community with knowledge of the development of this body of research.

<div align="center">Method</div>

Participants

The participants were derived from a portion of the WJ III standardization sample (McGrew & Woodcock, 2001). The WJ III was standardized on individuals ranging from 2 years of age to over 90 years old. The subsection of the standardization sample used in the present study consisted of the portion of the standardization sample that ranged between 20 years of age and 39 years of age (n = 1,041).

Instruments

The seven CHC-based specific cognitive (or broad) abilities used in this study were derived from correlations matrixes obtained from participant scores on 27 tests and one composite. The indicators included 4 tests from the WJ III Tests of Achievement (ACH; Mather & Woodcock, 2001), 5 tests and 1 special composite (a combination of Number Series and Number Matrices tests) from the WJ III Diagnostic Supplement (Woodcock, McGrew, Mather, & Schrank, 2003), and 18 tests from the WJ III Tests of Cognitive Abilities. More information about the tests and abilities they measure may be found in the instrument's Technical Manual or in Taub, Floyd, Keith, & McGrew (2008).

Scores from the Applied Problems and Calculation tests from the WJ III ACH served as the dependent variable, Quantitative Reasoning. These tests require participants to identify relevant from irrelevant information, perform calculations ranging from simple subtraction and addition to calculus, and comprehend the nature of a mathematical problem for successful problem completion.

<div align="center">Analysis</div>

All analyses were conducted using covariance matrixes derived from the correlations and standard deviations of participant scores. Following the recommendations of MacCallum, Roznowski, Mar, and Reith (1994), the sample was randomly divided into two subsamples. One sample served as the *calibration sample*

and the other was the *validation sample*. Randomly dividing participant data into both a calibration and validation sample allowed for model cross-validation. The AMOS (Arbuckle, 2007) statistical program was used to conduct all analyses with SEM. This means scores from the participants were divided into two independent samples or datasets. One data set was used for model testing (i.e., calibration) and the second sample or dataset was used for model validation.

Models

The first two models provide an overview of the development of the body of research into the relationship between intelligence, specific cognitive abilities, and academic achievement. They are included for informational purposes and are not estimated (results are not provided for these models). The first model, Figure 1, presents the Traditional model to measure intelligence. The Traditional model in Figure 1 is presented sideways and is hierarchical. The hierarchical nature of the model may be seen as the tests on the left side of the figure (rectangles) provide scores that contribute to the calculation of each of the seven specific CHC-based cognitive abilities (ovals). In CHC nomenclature, the seven specific cognitive abilities are called *broad abilities*. The term *broad abilities* will be used for consistency when referring to these specific cognitive abilities. The scores from the broad abilities then contribute to the calculation of general intelligence (g) at the apex of the hierarchy. It is important to note that using scores from 28 different tests permits the inclusion of at least three indicators for each of the seven broad abilities. Additionally, each of these 28 tests measures a different aspect of each broad ability. In CHC nomenclature, each test contributes scores from (or measures) a different narrow area, thus each specific broad ability is a combination of scores from more than one narrow area or subskill.

The lines with single-headed arrows represent the impact of one variable on another. The lines with single-headed arrows connecting the tests to the seven broad abilities as well as the lines connecting the broad abilities to general intelligence (g) are referred to as *paths* in SEM nomenclature. AMOS provides estimates for these paths and other sources of variance within the model. The estimates for the paths are referred to as *path coefficients*. This model is not estimated in the present study, it is presented for informational purposes.

The second model, Figure 2, presents a Broad model identifying the effect of each of the seven specific CHC-based broad abilities on Quantitative Reasoning. It too is a hierarchical model similar to the Traditional model in Figure 1. The Broad model provides an estimate of the relative contribution of each of the seven CHC-based broad abilities, in combination, on Quantitative Reasoning (general intelligence is not included in this model).

The third model is an integration of the models in Figures 1 and 2. This integrated model is referred to as the *baseline* model and is presented in Figure 3. The baseline model integrates the tests contributing the seven CHC-based broad abilities, general intelligence, and Quantitative Reasoning. The baseline model identifies the specific broad abilities, beyond general intelligence, having the strongest effect on the acquisition of Quantitative Reasoning skills. It is important to note that general intelligence is identified as g in the integrated baseline model. This change makes labeling within the baseline model consistent with prior research. The SEM measurement model presented in Figure 3 was used in previous research and has empirical support (e.g., McGrew & Woodcock, 2001; Taub & Benson, 2013; Taub, Floyd, Keith, & McGrew, 2007; Taub & McGrew, 2004).

Analysis

The first two models, the Broad and Traditional models displayed in Figures 1 and 2 are not analyzed in this study. They are presented to provide a background to explain the baseline model displayed in Figure 3. Analyses were conducted in two phases. The purpose of the first phase, the Calibration phase, was to identify the specific or combination of specific CHC-based broad abilities that were statistically significant predictors of the Quantitative Reasoning dependent variable. Thus, the first phase only used calibration data and was complete when the best fitting model, using the calibration data, was identified. Model estimations were obtained via the AMOS program which provided estimations for each of the structural paths. The initial model tested in the Calibration phase was the baseline model presented in Figure 3. Next, the single structural path with the highest negative value was removed from the baseline model. The model was then re-estimated. This process of model examination, deletion, and reiteration was carried out until all structural paths with negative values and paths with critical values below 1.96 ($p. \geq .05$) were removed. This resulted in a final model that contained only positive and statistically significant values. Once identified, this model served as the final calibration model and signaled the end of the first phase of the study.

In the second phase of the study, the Validation phase, the final calibration model was re-estimated using data from the validation sample. It is important to note that the *final model* derived from analyses using the calibration data in the first phase of the study was then validated using an independent dataset (validation data) in the second phase of the study, the Validation phase. The *final model* derived during the Calibration phase is presented in Figure 4 and will now be referred to as the Validation model. One benefit of using separate calibration/validation samples is the results are more stable (e.g., higher likelihood of obtaining similar results) upon replication (MacCallum et al., 1994).

Results

The Traditional and Broad models presented in Figures 1 and 2 were not estimated; they are included for informational purposes. The third model presented as Figure 3 is the baseline model used in the Calibration phase. The Calibration phase employed a process of model generation, path deletion, and reiteration using data from the calibration sample. The *final model* from theses analyses was identified as the best fitting model and is presented in Figure 4. Once identified, the model in Figure 4 was referred to as the *Validation model* and was tested using the independent validation data set in the second phase of the study, the Validation phase.

All structural paths in the final model, the Validation model displayed in Figure 4, were statistically significant. The standardized direct effects of the positive and statistically significant structural paths associated with the Validation model are presented in Figure 4.

An examination of fit indices associated with the Validation model, presented in Figure 4, provided evidence of the goodness of fit of the model to the data. These fit indices included the Akaike Information Criterion (AIC; 2445.019), comparative fit index (CFI; .721), root mean square error of approximation (RMSEA; .092), and the Tucker-Lewis index (TLI; .693). For reference, lower values indicate a better fit for the AIC and RMSEA and higher values on the CFI and TLI indicate that the model fit the data better, with a 1.0 indicative of a perfect fit. The RMSEA of .092 is above the threshold of .08 to consider the model to have adequate fit, but is less than the .10 cutoff indicating poor fit (Hu & Bentler. 1999). The model χ^2 is 2,299.019 with 423 degrees of freedom.

Overall, the results indicate that general intelligence had an *indirect* effect on the dependent Quantitative Reasoning variable. Meaning general intelligence had a direct effect on the specific abilities, which in turn had a direct effect on the mathematics achievement dependent variable. The specific broad abilities factors Crystallized Intelligence and Fluid Reasoning had *direct* effects on Quantitative Reasoning. The standardized path coefficient for Crystallized Intelligence → Quantitative Reasoning was .26 and Fluid Reasoning → Quantitative Reasoning was .54. The total indirect effect of *g* on Quantitative Reasoning can be calculated by first multiplying the path coefficient g → Gf by the path coefficient Gf → Gq (.95 x .54 = .513) as presented in Figure 4. The next step is to multiply the path coefficient g → Gc by the path coefficient Gc →Gq (.83 x .26 = .215). The sum of these two products is the total indirect effect of general intelligence on Quantitative Reasoning (.513 + .215 = .728) or .73.

Discussion

Recent advances in intellectual theory and statistical software（e.g., SEM）allow researchers to simultaneously analyze the relative contribution of specific broad abilities and general intelligence on mathematics achievement. The purpose of the present study was to go beyond earlier investigations of SEM and mathematics achievement to identify the relative contribution of seven specific broad abilities, based on the CHC theoretical model of intelligence, and general intelligence on college-age students' mathematics achievement.

The model presented in Figure 1, the Traditional model, provides a visual figure of the measurement of general intelligence. Initial research investigating the relationship between broad abilities and achievement focused specifically on the relationship between general intelligence and achievement.

The Broad model presented in Figure 2 examines the direct relationship between specific broad abilities and Quantitative Reasoning. One inherent problem with the Broad model is that it does not account for the variance associated with strong and consistent relationship between general intelligence and academic achievement. One purpose of this study was to identify the effects of the specific CHC-based broad abilities beyond the strong effect of general intelligence. In other words, if we take general intelligence out of the equation（or account for all variance associated with general intelligence）, which CHC broad ability or combination of broad abilities are most important to Quantitative Reasoning skill acquisition？

To answer this question, the models presented in Figure 1 and 2 were combined into one model, the baseline model, as displayed in Figure 3. This integrated model accounts for the direct effect of seven specific CHC-based broad abilities on general intelligence and the portion of remaining variance associated Quantitative Reasoning. This model was tested in two phases: a Calibration phase and a Validation phase. In the first phase, calibration data were used as input data and all negative structural paths were deleted from the model one at a time, after the deletion of each individual path the model was retested. This iteration continued until all structural paths with negative or non-statistically significant structural paths were removed from the model in Figure 3. This was then referred to as the *final model*. Next, the final model served as the Validation model presented in Figure 4, was tested using an independent validation data set.

The structural paths remaining in Figure 4 linking Crystalized Intelligence and Fluid Reasoning with Quantitative Reasoning indicate that these are the only specific broad abilities having direct effect on Quantitative Reasoning beyond general intelligence for this sample. The specific broad abilities Crystallized Intelligence and Fluid Reasoning had statistically significant direct effects on Quantitative Reasoning. In previous studies, Crystallized Intelligence also demonstrated a consistent relationship

with mathematics, specifically from 6 years of age through 19 years of age, with the exception of 7-8 years of age (e.g., Floyd, et al., 2003; Hale, Fiorello, Kavanagh, Hoeppner, & Gaither, 2001; Keith, 1999; McGrew, 1997; McGrew & Hessler, 1995; Taub, et al., 2008); as did Fluid Reasoning across all age groups. Thus, it was not surprising to find that Crystallized Intelligence and Fluid Reasoning were statistically related to mathematics achievement in the present college-age sample.

The specific CHC broad ability Crystalized Intelligence accounts for the depth and breadth of an individuals' acquired knowledge. This includes information that is overlearned and automatized across several domains including academic, cultural, and linguistic domains. It also accounts for the communication of this knowledge and for reasoning using previously acquired experiences and procedural skills.

The Fluid Reasoning broad ability accounts for one's ability to solve problems using both inductive and deductive reasoning skills. It also accounts for the ability to form concepts and solve problems using novel or less familiar information and procedures. This is in contrast to using overlearned and automatized problem solving skills associated with Crystalized Intelligence. The results from the present study support previous results indicating that Crystalized Intelligence is important across all areas of the curriculum and Fluid Reasoning is as important across all age ranges in the acquisition of mathematics skills. These results support prior SoTL research which identified the cognitive processes associated with mathematics as important across all areas of the curriculum in higher education.

Limitations

The findings from this study are limited by the data set used in the analyses. Specifically, all data used in this research came from a single battery of tests. Second, the participants were 20 years of age or older, thus college-age students between the ages of 18 and 19 were not included. Third, the mathematics achievement variable was general in nature and may not represent all mathematics skills.

Despite such limitations, several strengths in the present study are worth noting. The study used separate calibration and validation data for all analyses. This permitted the development of a model based on one sample (data set) which was validated on a different data set. This should result in more stable findings when compared to using a single data set for exploratory and confirmatory analyses. Finally, the specific broad abilities tested in this study and the general factors of intelligence were derived from an instrument that is well standardized and validated on a nationally represented sample.

Implications

The first implication of the study was general intelligence only had an indirect effect on the mathematics achievement of college-age students. This finding was important because previous research indicated that general intelligence had a direct effect between the ages of 14 to 19 years of age. It is also noteworthy that the stable effect of general intelligence for college-age students, 20 years of age and above, was indirect and only observed through its direct effect on the specific broad ability Crystallized Intelligence and Fluid Reasoning, which in turn had a direct effect on the mathematics achievement dependent variable.

Possibly a more noteworthy implication was the important role of Crystallized Intelligence and Fluid Reasoning on mathematics achievement of college-age students. Within the classroom, these findings may indicate that students experiencing difficulty with mathematics may benefit from explicit strategies targeting these two constructs. Specifically, in regards to Crystallized Intelligence, students who have not over learned mathematical algorithms may benefit from repeated instruction, instruction in the development of memorization strategies, or compensatory strategies (e.g., note cards) to assist in problem solving. The appearance of Fluid Reasoning as an important broad ability in the completion of mathematics may indicate that conceptual knowledge of the relationship between numbers and mathematics algorithms may be foundational for mathematics achievement (Hecht, Close, & Santisi, 2003). Unlike calculation, problem solving involves linguistic information for students to construct a problem model. Thus, instructional strategies assisting students with problem representation may prove beneficial in the classroom (Fuchs, Fuchs, Stuebing, et al., 2008). Additionally, instructional strategies asking students to visualize elements of the problem may also improve conceptual understanding and improve students' learning outcomes (Wendling & Mather, 2009). Faculty may also consider supplementing traditional mathematics instruction with activities that emphasize the use of students' visual-spatial skills. This involves supplementing formula-based instruction with diagrams, figures, and visual cues. An example of this is A Visual Approach to Calculus Problems (Apostol, 2000), which is a free webpage showing how to use diagrams and figures to teach and solve calculus problems.

As students age, there is also a decline in the growth of working memory and processing speed (Geary, 2007; McGrew & Woodcock, 2001). Instructional strategies that assist students experiencing difficulty holding information in immediate memory for problem solving may benefit from compensatory or instructional strategies that require students to *show their work*, thus avoiding dependency on mental calculation (Swanson & Beebe-Frankenberger, 2004). Similar strategies will also benefit students who have inefficient mental processing. Both working memory and processing speed are components of the central executive. The central executive may be thought of as

the air traffic controller of our mind. It is responsible for many meta-cognitive activities including resource allocation, purposeful attention, and evaluation. Interventions and instructional strategies that require students to demonstrate knowledge of mathematics, while at the same time, limiting the amount of information suspended in immediate awareness for problem solving/transformation/transposition should reduce students' dependency on the central executive.

References

Apostol, T. M. (2000, October). A visual approach to calculus problems. *California Institute of Technology*. Retrieved from

http://www.easybib.com/reference/guide/apa/website

Arbuckle, J.L., (2007). *Amos 7.0* [Computer software]. Chicago, IL: Smallwaters.

Bok, D. (2006). *Our Underachieving colleges: A candid look at how much students learn and why they should be learning more*. Princeton, NJ: Princeton University Press.

Benson, N., Hulac, D. & Kranzler, J. (2010). Independent examination of the Wechsler Adult Intelligence Scale-Fourth Edition (WAIS-IV): What does the WAIS-IV measure? *Psychological Assessment*, 22 (1), 121-130. Doi: 10.1037/a0017767

Bull, R., & Johnston, R. S. (1997). Children's arithmetic difficulties: Contributions from processing speed, item identification, and short-term memory. *Journal of Experimental Child Psychology, 65,* 1–24.

Carroll, J. B. (1993). *Human cognitive abilities: A survey of factor-analytic studies*. New York: Cambridge University Press.

Cerbin, B. (2013). Emphasizing learning in the scholarship of teaching and learning. *International Journal for the Scholarship of Teaching and Learning.* Retrieved from http://digitalcommons.georgiasouthern.edu/ij-sotl/vol7/iss1/5

Cronbach, L. J., & Snow, R. E. (1977). *Aptitudes and instructional methods.* New York: Irvington.

Dewar, J. & Bennett, C. (2010). Situating SoTL within the disciplines: Mathematics in the United States as a case study. *International Journal for the Scholarship of Teaching and Learning.* Retrieved from http://digitalcommons.georgiasouthern.edu/ij-sotl/vol4/iss1/14/

Floyd, R. G., Evans, J. J., & McGrew, K. S. (2003). Relations between measures of Cattell-Horn-Carroll (CHC) cognitive abilities and mathematics achievement across the school-age years. *Psychology in the Schools, 40*, 155–177. Doi: 10.1002/pits.10083

Floyd, R. G., Keith, T. Z., Taub, G. E., & McGrew, K. S. (2007). Cattell-Horn-

Carroll cognitive abilities and their effects on reading decoding skills: g Has indirect effects, more specific abilities have direct effects. *School Psychology Quarterly.* 22 (2), 200-233. Doi: 10.1037/1045-3830.22.2.200

Fuchs, L. S., Compton, D. L., Fuchs, D., Paulsen, K., Bryant, J. D., & Hamlett, C. L. (2006). The prevention, identification, and cognitive determinants of math difficulty. *Journal of Educational Psychology, 97,* 299–319. Doi: 10.1037/0022-0663.97.3.493

Fuchs, L. S., Fuchs, D., Stuebing, K., Fletcher, J. M., Hamlett, C. L., & Lambert, W. (2008). Problem solving and computational skill: Are they shared or distinct aspects of mathematical cognition? *Journal of Educational Psychology*, *100*, 30-47.

Geary, D. C. (2007). An evolutionary perspective on leaning disability in mathematics. *Developmental Neuropsychology,* 32, 471-519.

Grawe, N. D. (2011). The potential for teaching quantitative reasoning across the curriculum: Empirical evidence. *International Journal for the Scholarship of Teaching and Learning*. Retrieved from http://digitalcommons. georgiasouthern.edu/ij-sotl/vol5/iss1/14/

Gustafsson, J. E., & Balke, G. (1993). General and specific abilities as predictors of school achievement. *Multivariate Behavioral Research, 28 (40)*, 407-434.

Hale, J. B., Fiorello, C. A., Kavanagh, J. A., Hoeppner, J. B., & Gaither, R. A. (2001). WISC-III predictors of academic achievement for children with learning disabilities: Are global and factor scores comparable? *School Psychology Quarterly, 16,* 31-55.

Hecht, S. Close, L., & Santisi, M. (2003). Sources of individual differences in fractional skills. *Journal of Experimental Child Psychology*, *86*, 27-302.

Hu, L. & Bentler, P. M. (1999). Cutoff criteria for fit indexes in covariance structure analysis: Conventional criteria versus new alternatives. *Structural Equation Modeling*, *6*, 1-55.

Huber, M. T. & Hutchings, P. (2005). The advancement of learning: Building the teaching commons. San Francisco, CA: Jossey-Bass.

Hunter , J. E., & Hunter, R. F. (1984). Validity and utility of alternate predictors of job performance. *Psychological Bulletin, 96,* 72-98.

Hutchings, P., Huber, M. T., & Ciccone, A.(2011). Feature essays: Getting there: An integrative vision of the scholarship of teaching and learning. *International Journal for the Scholarship of Teaching and Learning.* Retrieved From http://digitalcommons.georgiasouthern.edu/ij-sotl/vol5/iss1/31

Jensen, R. A. (1984). Test validity: g versus the specificity doctrine. *Journal of Social and Biological Structure, 7,* 93-118.

Jensen, A. R. (1998). *The g factor.* Westport, CT: Preager.

Keith, T. Z. (1999). Effects of general and specific abilities on student achievement: Similarities and differences across ethnic groups. *School Psychology Quarterly, 14*, 239-262.

Keith, T. Z., Fine, J. G., Taub, G. E., Reynolds, M. R., & Kranzler, J. H. (2006). Higher-Order, Multi-Sample, Confirmatory Factor Analysis of the Wechsler Intelligence Scale for Children—Fourth Edition: What Does it Measure? *School Psychology Review*, 35, 108-127

Keith, T. Z., Reynolds, M. R. (2010). Cattell-Horn-Carroll abilities and cognitive tests: What we've learned from 20 years of research. *Psychology in the Schools, 47*, 635-650.

MacCallum, R. C., Roznowski, M., Mar, C. M., & Reith, J. V. (1994). Alternate strategies for cross-validation of covariance structure models. *Multivariate Behavioral Research, 29*, 1-32.

Mather, N., & Woodcock, R. W. (2001). *Examiner's manual: Woodcock-Johnson III Tests of Cognitive Abilities.* Itasca, IL: Riverside Publishing.

McGrew, K. S. (2009). CHC theory and the human cognitive abilities project: Standing on the shoulders of the giants of psychometric intelligence research. *Intelligence*, 37, 1-10. Doi:10.1016/j.intell.2008.08.004

McGrew, K. S. (2005). The Cattell—Horn—Carroll theory of cognitive abilities: Past, present, and future. In D. P. Flanagan & P. L. Harrison (Eds.), *Contemporary intellectual assessment: Theories, tests, and issues* (2nd ed., pp. 136—177). New York: Guilford Press.

McGrew, K. S. (1997). Analysis of the major intelligence batteries according to a proposed comprehensive Gf-Gc framework. In D. P. Flanagan, J. L. Genshaft, & P. L. Harrison (Eds.), Contemporary intellectual assessment: Theories, tests, and issues (pp. 131—150). New York: Guilford Press.

McGrew, K. S., & Flanagan, D. P. (1998). *The intelligence test desk reference (ITDR): Gf-Gc Cross-Battery assessment.* Boston: Allyn & Bacon.

McGrew, K. S., Flanagan, D. P., Keith, T. Z., & Vanderwood, M. (1997). Beyond g: The impact of *Gf-Gc* specific cognitive abilities research on the future use and interpretation of intelligence tests in the schools. *School Psychology Review, 26,* 189-210.

McGrew, K. S., & Hessler, G. L. (1995). The relationship between the WJ-R *Gf-Gc* cognitive clusters and mathematics achievement across the life-span. *Journal of Psychoeducational Assessment, 13*, 21-38.

McGrew, K. S., & Woodcock, R. W. (2001). Technical Manual. *Woodcock-Johnson III Tests of Achievement.* Itasca, IL: Riverside Publishing.

Mitchell, S. M., Anderson, W. L., Sensibaugh, C. A., & Osgood, M. (2011). What Really matters: Assessing individual problem-solving performance in the context of biological sciences. *International Journal for the Scholarship of Teaching and Learning.* Retrieved from http://digitalcommons.georgiasouthern.edu/ij-sotl/vol5/iss1/17

Passolunghi, M. C., & Seigel, L. S. (2001). Short-term memory, working memory, and inhibitory control in children with specific learning disabilities. *Journal of Experimental Child Psychology, 80*, 44–57.

Richardson, R. M., & McCallum, W. G. (2004). Embedding QL across the curriculum. In L. A. Steen (Ed). *Achieving quantitative literacy: An urgent challenge for higher education*. Washington, DC: The Mathematical Association of America.

Swanson, H. L., & Beebe-Frankenberger, M. (2004). The relationship between working memory and mathematical problem-solving in children at risk and not at risk for serious math difficulties. *Journal of Educational Psychology, 96*, 471–491.

Taub, G. E. & Benson, N. (2013). Identifying the effects of specific CHC factors on college students' reading comprehension. *International Journal for the Scholarship of Teaching and Learning*. Retrieved from http://digitalcommons.georgiasouthern.edu/ij-sotl/vol7/iss2/10

Taub, G. E., Floyd, R. F., Keith, T. Z., & Mc Grew, K. S. (2008). Effects of general and broad cognitive abilities on mathematics achievement. *School Psychology Quarterly*. 23(2), 187–198. Doi: 10.1037/1045-3830.23.2.187

Taub, G. E. & McGrew, K. S. (2004). A confirmatory factor analysis of CHC Theory and cross-Age invariance of the Woodcock-Johnson Tests of Cognitive Abilities III. *School Psychology Quarterly, 1*, 72–87. doi:10.1521/scpq.19.1.72.29409

Taub, G. E., McGrew, K. S., & Witta, E. L. (2004). A confirmatory analysis of the factor structure and cross-age invariance of the Wechsler Adult Intelligence Scale-Third Edition. *Psychological Assessment,* 16, 85–89. Doi:10.1037/1040-3590.16.1.85

Thorndike, R. L., Hagen, E. P., & Sattler, J. M. (1986*). Technical Manual: Stanford-Binet Intelligence Scale: Fourth Edition.* Chicago: Riverside.

Wendling, B. J. & Mather, N. (2009). *Essentials of evidence-Based Academic Interventions*. Hoboken, NJ: John Wiley & Sons.

Woodcock, R. W., McGrew, K. S., Mather, N., & Schrank, F. A. (2003). *Woodcock-Johnson III Diagnostic Supplement to the Tests of Cognitive Abilities.* Itasca, IL: Riverside.

Woodcock, R. W., McGrew, K. S., & Mather, N.（2001）. *Woodcock-Johnson III.* Itasca, IL：Riverside.

Figure Captions

Figure 1. The Traditional model accounting for the relationship between specific CHC-based cognitive abilities and General Intelligence. g= General Intelligence; *Gf* = Fluid Reasoning, *Gv* = Visual Processing, *Gs* = Processing Speed, *Glr* = Long-Term Storage and Retrieval, *Ga* = Auditory Processing, *Gsm* = Short-Term Memory, *Gc* = Crystallized Intelligence, *g* = General Intelligence, and *Gq* = Quantitative Reasoning.

Figure 2. The Integrated model accounting for the relationship between specific CHC-based cognitive abilities and Quantitative Reasoning. g= General Intelligence; *Gf* = Fluid Reasoning, *Gv* = Visual Processing, *Gs* = Processing Speed, *Glr* = Long-Term Storage and Retrieval, *Ga* = Auditory Processing, *Gsm* = Short-Term Memory, *Gc* = Crystallized Intelligence, *g* = General Intelligence, and *Gq* = Quantitative Reasoning.

Figure 3. The Integrated Calibration model, based on CHC-theory to account for direct effects from general intelligence, and specific CHC cognitive ability factors on Quantitative Reasoning. *g* = General Intelligence *Gf* = Fluid Reasoning, *Gv* = Visual Processing, *Gs* = Processing Speed, *Glr* = Long-Term Storage and Retrieval, *Ga* = Auditory Processing, *Gsm* = Short-Term Memory, *Gc* = Crystallized Intelligence, and *Gq* = Quantitative Reasoning.

Figure 4. The Integrated Validation model, which was tested using the validation data. *g* = General Intelligence, *Gf* = Fluid Reasoning, *Gv* = Visual Processing, *Gs* = Processing Speed, *Glr* = Long-Term Storage and Retrieval, *Ga* = Auditory Processing, *Gsm* = Short-Term Memory, *Gc* = Crystallized Intelligence, and *Gq* = Quantitative Reasoning.