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The Effects of Preceding Stimuli Formats on Proportional Reasoning Ability in Elementary School Students

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THE EFFECTS OF PRECEDING STIMULI FORMATS ON PROPORTIONAL REASONING ABILITY IN ELEMENTARY SCHOOL STUDENTS

by

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(Under the direction of Ty W. Boyer)

ABSTRACT

The National Mathematics Advisory Panel (2008) described fraction knowledge as the most important, yet most underdeveloped foundational skill among students. Due to the complex nature of fraction education, this study sought to understand the underlying fraction problem-solving skill of proportional reasoning in the hopes of gaining insight into children's problem-solving strategies in order to implement more focused educational designs. The current study examined the effects of stimuli formats on children's proportional reasoning ability by presenting four conditions involving two formats (continuous and discrete). Previous research indicates that students perform better on continuous stimuli and the goal of this study was to determine if preceding format type has any effect on subsequent trials. It was expected that children would perform better on discrete trials if preceded by continuous trials and would do worse on continuous trials if preceded by discrete trials, however, this finding was not found. Overall the study provided further evidence that children perform better with age and are more successful on trials with a continuous format rather than a discrete format.

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CHAPTER 1

INTRODUCTION

The ability to reason proportionally and an understanding of fractions are essential for children’s mathematical development as they progress through their elementary to middle school years and into their secondary education. Fractions and proportional reasoning are crucial in algebra and are important concepts in advanced mathematics and sciences (Bailey et al., 2015; Jordan et al., 2013; NMAP, 2008; Siegler, Thompson, & Schneider 2011; Siegler, Fazio, Bailey, & Zhou, 2012; Siegler & Pyke, 2013). Students in their elementary school years have been found to have great difficulty reasoning with proportions (Mack, 1990), and children’s knowledge of fractions in elementary school predicts their performance in advanced mathematics in high school (Bailey et al., 2015; Namkung & Fuchs, 2015). Therefore, there is a need to find ways in which to improve their understanding of proportional reasoning so that perhaps we can help progress their understanding of fractions, in turn, furthering their overall success in mathematics.

Both useful and difficult to master, proportional reasoning should be considered and presented as a multi-faceted activity, involving different teaching methods for different contexts and number structures (Tourniaire & Pulos, 1985). As one of the most important concepts in mathematics for elementary students to develop (Lamm & Pugalee, 2010), proportional reasoning has been researched for decades in hopes of finding some sort of insight into how to better educate students who rely on the skills of their teachers to promote their understanding of such difficult concepts. In a study examining proportional reasoning in college students, Lawton (1993) pointed out that the context in teaching such concepts is key to success. For instance, emphasis should be placed on the fact that the mathematical relationship between items does not depend on the physical similarity of those items, indicating that a focus on the quantitative relationship is necessary.

Banker (2012) also expressed the importance of presentation saying the lack of proportional reasoning development can be traced to the way problems are presented by teachers, specifically indicating that familiar contexts must be used to develop proportional reasoning understanding. Additionally, the National Mathematics Advisory Panel (2008) recommended that curriculum should allow enough time for students to sufficiently grasp and fully understand fraction and proportional reasoning concepts. Further research also needs to be conducted in order to strengthen the preparation of elementary educators and give them plenty of time to completely understand the concepts before they attempt to teach them (NMAP, 2008). For instance, Owens (1993) pointed out that there is a mismatch between what teachers are instructed to teach and what students are evaluated on, indicating a need for some sort of solidarity between the two. Moreover, Fisher (1988) conducted a study in which secondary teachers solved proportional reasoning problems with the finding that even the teachers needed to know more about what they were doing (i.e. what they were trying to teach).

As a method for clarification on educational requirements, the Georgia Standards of Excellence for Mathematics provide detailed standards for teachers to easily identify what needs to be taught and understood at each level of education. For instance, the standards indicate that students in the third grade should have gained an understanding of fractions such that they are able to explain fraction equivalence. By fourth grade students should be able to explain why fractions are equivalent, compare fractions with different numerators and denominators, and understand addition, subtraction, and multiplication of fractions. In fifth grade students are expected to understand all aspects of fractions and by seventh grade, they should be able to apply their fraction knowledge to solve real-world scenarios. This is also in line with the American Common Core State Standards for Mathematics expectations to begin developing an understanding of fractions in the third grade and to begin applying that proportional reasoning understanding to real-world and mathematical problems by middle school.

With the expectations of students to fully understand fractions and proportional reasoning by the time they are in middle school, and the research that indicates students have major difficulties with these concepts, even into adulthood, it should be postulated that there is some disconnect in the education of proportional concepts for young students. The National Council of Teachers of Mathematics even reported in 2006 that only half of eighth graders in America could properly order the proportions 2/7, 1/12, and 5/9 from least to greatest. Yet, we have expected them, and constructed their education accordingly, to have essentially mastered the concepts of proportional reasoning and fraction problem-solving by eighth grade. Based on this, it is clear that more research is needed to further understand the ways in which proportions can be presented and taught so that students can get a better understanding of the concepts earlier on. It is a hope of the current study to fulfil a portion of that research need and provide some insight into how education can better serve young students learning needs.

DIFFERENTIATING CONCEPTS

Fraction problem-solving could be characterized as performing explicit and formal mathematical operations with symbolically presented alphanumeric notation composed of a part and a whole (i.e. numerator/denominator). Proportional reasoning on the other hand is more of an ability to compare and contrast relative quantities, not necessarily represented with symbolic notation, composed of parts that make up a whole. In general, proportional reasoning is a more rudimentary ability than fraction problem-solving, and might be possible without any formal education. Alternatively, fraction problem-solving requires a more developed understanding of mathematical structures and terms. Therefore, proportional reasoning might be a necessary pre-cursor for fraction problem-solving. The current study was conducted with non-symbolic stimuli, and, therefore, focusses on the development of proportional reasoning abilities, but with hope that the findings will have relevance and implications for fraction problem-solving and mathematical education.

FRACTION IMPLICATIONS

Providing evidence of the important implications that fraction problem-solving has on children's other mathematics skills, Siegler and Pyke (2013) used a multi-task study to examine developmental and individual differences between higher and lower achieving 6th and 8th grade students. Through their examination of the students' number line estimation, magnitude comparison, fraction arithmetic, whole number division and executive functioning, Siegler and Pyke (2013) noted that all were related to the students' overall mathematical ability based on standardized test scores. Specifically, on the number line estimation task, students saw a number line from 0-1 along with a fraction. The students then estimated where on the line that fraction belonged. Performance on the fraction number-line estimation task was predictive of all of the other mathematics tasks and assessments (Siegler & Pyke, 2013).

Due to this connection between fractions and other aspects of mathematics, it is rather important to ensure that the education of such knowledge is thorough and effective. Mack (1990) provided an example of this need by spending several weeks working one-on-one with 6th grade students to access and further develop their fraction understanding. Mack (1990) indicated that all of the students came into the assessment with multiple misconceptions regarding fraction symbols and procedures and had difficulty solving problems using such symbols. Even in trying to explain their solutions, students had faulty reasoning. For example, when asked which is larger, 1/6 or 1/8, many students responded that 1/8 is greater because 8 is greater than 6, even after having answered that 1/6 of a pizza would be greater than 1/8 of a pizza (Mack, 1990). This example points out the extremely common error of applying whole number rules to fractions. As Mack (1990) highlights, the students had an informal understanding of fractions that can be used in real-world scenarios but had difficulty when applying that understanding to symbolic representations.

FRACTION DIFFICULTY

This difficulty with fractions seems to stem from a variety of possible sources. One potential culprit is the knowledge and understanding of whole numbers that children have prior to learning concepts of fractions; the basic and fundamental rules of whole numbers do not apply to fractions and in fact cause error when attempting to solve fractions using such rules, i.e. the whole number bias (Braithwaite & Siegler, 2017; Siegler & Pyke, 2013; Namkung & Fuchs, 2015). For instance, a rather simple error can be made when children apply basic math rules when attempting to compare fractions. A child learns that 4>2 so when they see a comparison of 1/2 and 1/4, they might mistakenly believe that 1/4 is larger because 4 is greater than 2. They have not quite grasped the concept that 1 is being divided into 4 pieces rather than only 2 pieces making 1/4 a smaller fraction of a whole than 1/2. Additionally, children learn how to add and subtract whole numbers early on, but the guidelines that apply to adding and subtracting whole numbers do not apply to fractions in the same way. Based on the whole number rules, children might initially believe that 1/3 + 3/4 = 4/7, when in fact they must first find a common denominator of each and create essentially new fractions to add together (1/3 = 4/12, 3/4 = 9/12, so 4/12 + 9/12 = 13/12 or 1 1/12). Even multiplication and division are different; with whole numbers, multiplication increases the quantity and division decreases it, however, with fractions it is the opposite. According to Braithwaite and Siegler (2017), this whole number bias does decrease with age.

ROLE OF PROPORTIONAL REASONING

Given the importance of fraction knowledge to children's overall mathematical education, there is an immense need to improve their ability to understand fractions. As noted above, proportional reasoning is necessary to fraction problem-solving, and, therefore, it is important to also consider the development of basic proportional reasoning processes and consider implications for fraction problem-solving and mathematics education. In much of the research examining children's proportional reasoning processes, a manipulation of the presentation of the stimuli is used so that children might be more likely to understand and be able to correctly solve the task at hand. For example, Sophian (2000) examined children's sensitivity to proportional properties using imaginary animal stimuli. Sophian (2000) presented children with three images of four different imaginary creatures. One of the images was designated as a target. One of the other images was a proportional match for the target, in that it had the same features (e.g., head, body, arms and legs) in the same configuration and relative proportion to one another as displayed in the target, but overall was smaller. The third image was proportionally different from the target, in that the relative size of some of the features varied from the relative size displayed in the target (e.g., a different relative head to tail proportion). Children's task was to identify, between the second and third images, that which was the proportional match for the target image of the creature. Through this study, Sophian (2000) determined that children as young as 5 years of age were able to make proportional matches.

Additionally, Mix, Levine and Huttenlocher (1999) examined young children's proportional reasoning ability. Mix et al. (1999) showed children white sponge pieces that could be divided into quarters and reattached. The experimenter showed a fraction in the form of the sponge pieces to the child (e.g. ¾ of a whole sponge) then covered the sponge pieces so the child could no longer see them. The experimenter would then either add or subtract a piece of sponge (e.g. adding ¼ piece of sponge) in such a way that the child knew what the initial sponge was and knew what portion of sponge was added or removed, but did not see the outcome of the change. Then, the child saw four images of solid white circles representing the sponge, and then chose which circle they believed represented the correct outcome proportion (e.g. the whole circle). Through their study using these sponge pieces to represent fractions, Mix et al., (1999) were able to determine that there is some level of competence in proportional reasoning even at the age of four.

Similarly, Singer-Freeman and Goswami (2001) used models of pizzas and boxes of chocolate to determine children's emerging proportional reasoning ability. The children received either a model pizza or a model box of chocolates to resemble the experimenter's pizza or box of chocolates. The children in this study removed portions of their model pizza or box of chocolate in order to mirror the experimenter's relative amount of pizza or relative amount of chocolate. The children's models were divided into eighths while the experimenter's models were divided into fourths creating the need for the child to figure out what portion of pizza or chocolate would be proportionally equivalent to the experimenter's. In the study, Singer-Freeman and Goswami (2001) found that the preschoolers performed at a level greater than chance and demonstrated an informal and intuitive knowledge of proportional reasoning seemingly based on their ability to identify relational similarities.

PRESENTATION OF INFORMATION

Additional research indicates that the way in which the proportional information is presented affects how children interpret and solve those problems. For instance, Mohring, Newcombe, Levine and Frick (2015) used images of blue and red rectangles representing water and cherry juice to test 8- to 10-year-old children's proportional reasoning skills. The blue and red rectangles were presented above a horizontal line that children used to provide an estimate of the amount of cherry flavor present in the rectangles; at the left end of the line was one cherry indicating slight cherry flavor, and at the right end of the line there was a pile of cherries to indicate a strong cherry flavor. One group of children saw the red and blue rectangles stacked on top of one another to form on larger rectangle, while the other group saw the two rectangles side-by-side as two separate rectangles. The children were asked to place a peg at the point on the line that best designated the amount of cherry flavor the presented rectangle would have. Mohring et al. (2015) determined that children were more successful in choosing the appropriate amount of cherry flavor when in the stacked condition rather than the side-by-side comparison condition.

Some research has indicated that young children are more successful at solving proportional reasoning problems if those problems are presented using continuous, unmarked proportions rather than discrete units that can be counted (Boyer & Levine, 2012/2015; Boyer, Levine, & Huttenlocher, 2008; Jeong, Levine, & Huttenlocher, 2007; Mix, Levine, & Huttenlocher, 1999; Singer-Freeman & Goswami, 2001; Spinillo & Bryant, 1999). For example, Spinillo and Bryant (1999) studied the potential effects of continuous and discrete quantities on children's equivalence judgements using images of pizzas. In the study, children took part in two tasks (sliced and non-sliced pizza image). The children saw an image of a pizza and were presented with two sets of wooden triangles that represented pizza slices. They were to choose which set of slices corresponded with the image of the pizza. Spinillo and Bryant (1999) found that the children overall performed better on the tasks presented in a continuous format (non-sliced pizza images) than on those presented using a discrete format (sliced pizza images).

Moreover, in the study noted above, Singer-Freeman and Goswami (2001) found that the children overall performed better when the stimulus was the pizza which was presented as a continuous circle rather than the boxes of chocolates which were always divided into pieces. This performance difference seems to be the case due to children applying a more intuitive comparison strategy when solving continuous stimuli and using a more explicit, but incorrect, absolute quantity comparison strategy on discrete problems (Boyer et al., 2008). Specifically, children appear to consider the continuous stimuli as more of a whole and therefore compare it to the alternatives in such a way that allows them to more accurately identify the correct choice. In the case of the discrete stimuli, children seem to have a tendency to get consumed in counting and comparing to find the exact number match rather than the proportionate match.

Additionally, Boyer and Levine (2015) examined students' proportional reasoning using a computerized juice mix task originally used in Boyer et al. (2008). In the task, students are shown three columns, each depicting juice parts and water parts, and with each either presented in a continuous format with only a separation between the water part and juice part, or in a discrete format with both juice and water parts divided into smaller, countable units. The initial column on the left side of the screen represented the desired juice-water proportion, and the other two columns, which were choice alternatives in the judgment task, were presented on the remainder of the screen. Students were asked which of the two juice mix alternatives would be the proportionately correct choice to match the target juice mix in terms of the relative proportion of juice and water parts.

As mentioned above, previous research has shown that children are more likely to succeed on proportional reasoning tasks presented with continuous than discrete stimuli, theoretically because they engage in relative quantity comparisons instead of absolute number matching, and the aim of Boyer and Levine (2015) was to determine if students would perform better on the discrete task (marked, countable units) if presented with stimuli in a continuous format (unmarked, whole units) first. Perhaps the children would transfer the strategy they used to solve the continuous stimuli onto the discrete stimuli allowing for the accurate decision of the choice that contains the same ratio of juice mix to water as the target. Rather than seeing individual, countable units on the discrete trials, maybe the students would continue looking at the rectangles as a whole as they did in the continuous format. The results indicated that students did perform better on the continuous trials, and the older students in the study performed better on the discrete trials if preceded by the continuous trials (Boyer & Levine, 2015).

CURRENT EXPECTATIONS

The current study, an extension of Boyer and Levine (2015), sought to further examine how children are affected by the stimulus format of the preceding trials when attempting to solve proportional reasoning problems. In the study, students were given a series of trials of a proportional reasoning task, either with stimuli presented with a continuous format throughout, with stimuli presented in a discrete format throughout, or in one stimulus format for the first half of the trials followed by the other format for the second half of the trials. The hypothesis was that elementary school aged children are vulnerable to contextual clues; therefore, their performance might be better on proportional reasoning trials that were preceded by continuous trials rather than a series of trials that are preceded by discrete trials that elicit counting. Specifically, it was expected that participants in the conditions with preceding continuous trials would do better on the secondary series of trials, which was anticipated to more likely occur in the later elementary school years. This expectation was based on the previous research indicating that overall, students perform better on trials presented in continuous units, and the expectation that students will apply those successful strategies to solving the secondary series of trials. With the continuous units, it was expected that students were more likely to look at the stimuli as a whole and compare it to the alternatives in that manner, providing a more successful choice.

Similarly, it was also expected that the participants in the conditions with preceding discrete trials would do worse on the secondary series of trials, which was projected to more likely occur in the younger elementary years. This expectation was based on the research indicating that students perform worse on trials that are divided into discrete units rather than continuous units and the understanding that the students would likely apply their unsuccessful, explicit strategies to the secondary series of continuous trials. Such strategies could include counting of each individual unit within the juice mixture, then choosing the foil alternative that matches the absolute quantity of the juice part or the juice and water mix as a whole, leading to the choice of the wrong option.

The goal of the study was to create a method of presenting these proportional reasoning problems that might increase understanding of challenging proportional concepts. The hope was that this study would reveal a subtle intervention that could be introduced into the current educational system to better student's understanding of proportional reasoning. Additionally, we also hoped that the findings provide some insight into the rigidness versus flexibility of children's problem-solving strategies.

CHAPTER 2

METHOD

PARTICIPANTS

The participants included in this study consisted of 288 students aged 5 to 12 (152 females, 136 males) with 96 from each kindergarten (50 females, 46 males), second- (48 females, 48 males) and fourth-grade (54 females, 42 males) from the schools in the surrounding area of Statesboro and Bulloch County. In order to include those students, I obtained approval from the Bulloch County School District to gain access to the schools within the county. At that point, I proceeded by contacting the principals of each school in hopes of gaining access to the students in the schools. Of the nine elementary schools in Bulloch County, four allowed access to their students including three public schools (n = 228) and one private school (n = 60). At each of those schools, informed consent forms were sent home to the students' parents in order to gain their permission for their child to participate in the study. Overall, there were 1,024 forms sent home with the grade breakdown as follows: 365 in kindergarten, 309 in second grade, and 350 in fourth grade. There were 420 forms returned overall with 162 in kindergarten, 131 in second grade and 127 in fourth grade, which allowed me to obtain the numbers I needed for the study.

The three public schools the participants were sampled from averaged 69% of students with free or reduced lunch, 11% of students disabled, 46% Black, 43% White, 5% Hispanic, 3.7% multi-racial, and 2.5% Asian. Additionally, these schools have an average College and Career Readiness Performance Index (CCRPI) score of 69.3. This score provides a measure of how well schools are meeting student goals and preparing them for life after school on a scale of 0 to 100. These demographics were averages of the three public schools as a whole and do not reflect the exact demographics of the sample as this data was not collected for individual participants. However, Table 1 presents a breakdown of these demographics by school allowing for a more accurate representation of the population the participants were sampled from. Additionally, it is important to note that while there was not an equal number of participants across schools, extreme care was taken to ensure that there was an equal number of participants in each grade per condition per school, as shown in Table 1. The difference in numbers at each school could be attributed to a potential bias to families that are more open to experimental studies such as this.

Table 1. Number of children per grade, per school, per experimental condition – Continuous-Continuous (CC), Discrete-Continuous (DC), Discrete-Discrete (DD) and Continuous-Discrete (CD) – with overall school demographics

School	Kindergarten	Second	Fourth	Total	% eligible for free/reduced lunch	Ethnicity Breakdown		Percent Disabled	CCRPI
Bulloch Academy	CC – 5	CC – 5	CC – 5	60	N/A	Black	N/A	N/A	N/A
	DC – 5	DC – 5	DC – 5			White	N/A		
	DD – 5	DD – 5	DD – 5			Hispanic	N/A		
						Asian	N/A		
	CD – 5	CD – 5	CD – 5			Multi-Racial	N/A		
Portal Elementary	CC – 1	CC – 1	CC – 1	12	74%	Black	34 %	13%	61.7
	DC – 1	DC – 1	DC – 1			White	54 %		
	DD – 1	DD – 1	DD – 1			Hispanic	7%		
						Asian	1%		
	CD – 1	CD – 1	CD – 1			Multi-Racial	4%		
Mill Creek Elementary	CC – 2	CC – 2	CC – 2	24	77%	Black	58 %	9%	69.1
	DC – 2	DC – 2	DC – 2			White	31 %		
						Hispanic	4%		
	DD – 2	DD – 2	DD – 2			Asian	3%		
	CD – 2	CD – 2	CD – 2			Multi-Racial	4%		
Julia P. Bryant Elementary	CC – 16	CC – 16	CC – 16	192	55%	Black	47 %	12%	77.2
	DC – 16	DC – 16	DC – 16			White	44 %		
						Hispanic	3%		
	DD – 16	DD – 16	DD – 16			Asian	2%		
	CD – 16	CD – 16	CD – 16			Multi-Racial	3%		
Totals	96	96	96	288					

PROCEDURE

The program was designed and run using E-Prime (Psychology Software Tools Inc., Pittsburg, PA, USA). The participants completed the experiment using a Dell Latitude laptop computer with a 15.6" HD LED screen and a wireless mouse in a familiar area within the school, designated by the school's administrators. The two proportional reasoning tasks (Boyer & Levine, 2012/2015; Boyer et al., 2008) were presented using a cover story in which a teddy bear character, Wally Bear, makes his own juice. As in the previous studies (Boyer & Levine, 2012/2015; Boyer et al., 2008) the juice mixtures were presented as rectangles that are divided into water and juice parts, and dependent upon condition, were either continuous, unmarked proportions or discrete, countable proportions. The participant was then provided an example of Wally Bear's juice mixes – two proportionately matching juice mixtures – and was told how important it was to maintain the correct proportional mixture so that the juice would taste just right.

The participant was then asked if he/she would be willing to help Wally Bear in making his juice by assisting in choosing the correct juice mix. During each series of trials, the participant was presented with a target juice mix that was divided into juice and water parts seen under an image of Wally Bear, along with two choice alternatives seen on the remainder of the screen. The participant was then asked which of two choice alternative juice mixes would taste the most like the juice Wally Bear wants to make. The correct of the two choice alternatives would match the juice/water mixture proportionately. However, the foil (i.e. the incorrect option) was an absolute match to the juice portion of the target's mixture (numerator foil) or to the juice/water mix as a whole (denominator foil), but not a proportionate match to the target juice/juice + water mixture. Participants then made their selection by mouse-clicking an arrow that appeared below their choice. Following each selection, the next trial appeared with another target and two choice alternatives. The juice "flavor" or color for each trail was randomly chosen from red, green, blue, yellow or purple. Participants completed a set of 16 trials in random order, each presented twice, totaling 32 trials (see Table 2). There was no performance feedback provided.

DESIGN

In line with the previous research, the experimental stimuli were divided into two forms; discrete and continuous. The discrete quantity proportional reasoning problems (as seen in Figure 1a) were divided into smaller units differentiated with lines. The continuous quantity proportional reasoning problems (as seen in Figure 1b) were columns divided only at the point at which the juice and water meet. The experiment was conducted using 32 proportional reasoning problems. Dependent upon which of the four conditions the participant was randomly assigned to, they either saw 16 of each form of quantity proportional reasoning problems, presented in differing orders, or 32 of the same type of proportional reasoning problem.

In the Discrete-Continuous condition, the participant completed a series of trials illustrated with a discrete stimulus format followed by a series of trials illustrated with a continuous stimulus format. In the Continuous-Continuous condition, the participant completed a series of trials illustrated with a continuous stimulus format followed by additional trials illustrated with a continuous stimulus format serving as the control for the Discrete-Continuous condition. In the Continuous-Discrete condition, the participant completed a series of trials illustrated with a continuous stimulus format followed by a series of trials illustrated with a discrete stimulus format. In the Discrete-Discrete condition, the participant completed a series of trials illustrated with a discrete stimulus format followed by additional trials illustrated with a discrete stimulus format, serving as a control for the Continuous-Discrete condition. There were 24 participants from each of the three grades randomly assigned to each condition (96 students per condition).

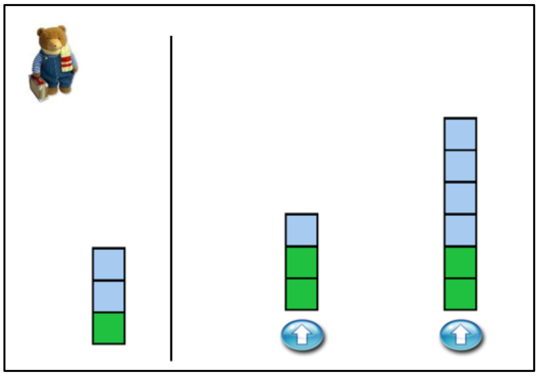

Figure 1a. An example screen shot of the discrete proportional reasoning task using the denominator foil.

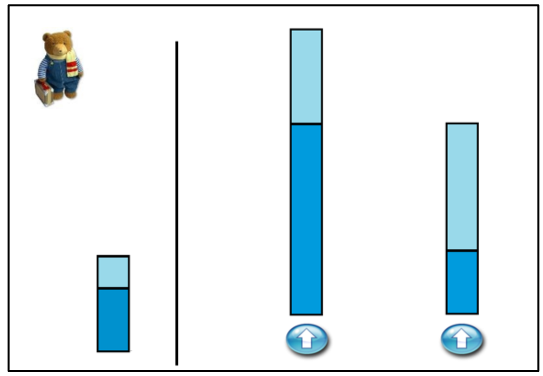

Figure 1b. An example screen shot of the continuous proportional reasoning task using the numerator foil.

The foil types, as mentioned before, were implemented based on the previous research (Boyer et al., 2008; Boyer & Levine, 2015). The numerator foil (as seen in figure 1b) consisted of an absolute match to the juice portion of the target's mixture (matched the target proportion's numerator) and the denominator foil (as seen in figure 1a) was an absolute match to the juice/water mix as a whole (matched the target proportion's denominator). In line with the previous research, each foil type was presented in half of the trials (i.e. 8 of each type in each block of 16 trials).

Additionally, based on Boyer and Levine (2015), the proportions used in the study included numerator values ranging from 1 to 9 (juice parts) and denominator values ranging from 3 to 12 total parts, as seen in Table 2. This provided proportions that ranged from 1/5 to 9/12 which could all be reduced to a multiple of thirds, fourths, or fifths. Furthermore, the foils were only presented using the juice parts rather than the water parts. This decision was also based on Boyer et al. (2008) in which a foil presented with the juice portion which alternated colors throughout the experiment was reasoned to be more perceptible than the water portion which remained the same color throughout the study.

Table 2. The exact proportions used in the study on each of the 16 trials that were repeated twice. The highlighted proportions are the examples shown in Figures 1a and 1b.

Target Proportion	**Match**	**Foil**	**Foil Type**
2/8	1/4	2/3	Numerator Foil
3/9	1/3	3/4	Numerator Foil
2/6	3/9	2/3	Numerator Foil
4/12	3/9	4/6	Numerator Foil
4/10	2/5	4/5	Numerator Foil
2/3	6/9	2/6	Numerator Foil
4/6	6/9	4/12	Numerator Foil
4/5	8/10	4/10	Numerator Foil
1/5	2/10	3/5	Denominator Foil
1/3	2/6	2/3	Denominator Foil
2/6	3/9	4/6	Denominator Foil
6/10	3/5	2/10	Denominator Foil
4/6	6/9	2/6	Denominator Foil
8/12	6/9	3/12	Denominator Foil
8/12	2/3	4/12	Denominator Foil
9/12	3/4	4/12	Denominator Foil

Note – the trials were presented in a randomized order and did not necessarily appear in the order presented in this table.

CHAPTER 3

RESULTS

Overall, students responded correctly on 55% of the trials. The following analysis will examine performance in the first half of trials in order to examine effects that have been previously reported, and more specifically, grade differences and differences between continuous and discrete stimuli. A 3 x 2 x 2 x 2 mixed model Analysis of Variance (ANOVA) examined the effects of school grade (kindergarten, second-grade, fourth-grade), sex (male, female), stimulus (continuous, discrete) with foil type (numerator foil, denominator foil) as the repeated measure on the first half of the trials. Specifically, I predicted that there would be a main effect of school grade with fourth grade performing better than second grade and second grade better than kindergarten. There was a significant main effect of grade in which higher grades outperformed lower grades F (2, 286) = 21.37, p < .001, η_p^2 = .134, with fourth graders averaging 66% correct, second averaging 51% correct and kindergarten averaging 47% correct. Pairwise comparisons indicated the difference between Kindergarten and Second grade was not significant with p = .151, but the differences between Fourth and Kindergarten and Fourth and Second grade were significant with both p < .001.

Additionally, as expected, the ANOVA revealed a significant effect of stimulus, with students who were given the continuous format (M = .57, SEM = .02) outperforming those given the discrete format (M = .53, SEM = .02), F (1, 287) = 9.49, p < .005, η_p^2 = .033. There was no significant main effect of foil type F (1, 287) = .839, p = .37. Furthermore, foil type did not interact with any other factors with all $F \leq 1.4$, $p \geq .24$. Additional interactions in the first half ANOVA did not reveal any significant results.

A second omnibus analysis was conducted to examine performance across both halves and more specifically, to examine the effect of the stimulus sequence presentation manipulation. A 3 x 2 x 4 x 2 x 2 ANOVA was conducted examining grade (kindergarten, second-grade, fourth-grade), sex (male, female), condition (continuous-continuous, discrete-continuous, discrete-discrete, continuous-discrete) and foil type (numerator foil, denominator foil) between groups, with block half (first half, second half) as the repeated-measure. There was a main effect of grade F (2, 286) = 23.19, $p < .001$, $\eta_p^2 = .149$ in which students in higher grades outperformed those in lower grades (fourth graders M = 66.0%, second graders M = 51.4%, and kindergarteners M = 47.7% correct). Pairwise comparisons indicated the difference between Kindergarten and Second grade was not significant with $p = .185$, but the differences between Fourth and Kindergarten and Fourth and Second grade were significant with both $p < .001$. There was no main effect of sex F (1, 287) = 2.39, $p = .12$ nor any interactions between sex and any of the other factors with all $F \leq 2.92$, $p \geq .09$.

There was only a marginal effect of condition F (3, 285) = 2.44, $p = .07$ and no significant interactions with grade or sex, with all $F \leq 1.39$, $p \geq .25$. Foil type was also only marginally significant F (1, 287) = 2.87, $p = .09$ with students performing better on trials with the denominator foil ($M = .56$, $SEM = .015$) rather than the numerator foil ($M = .54$, $SEM = .014$). Additionally, there was only a marginally significant interaction between foil type and sex F (1, 287) = 2.92, $p = .09$ in which a larger difference between foil types was seen with the female participants. Importantly, there was also a marginally significant interaction between block half and condition F (3, 285) = 2.41, $p = .07$.

The interaction between block half and condition held particular theoretical importance in the study, and, because it was marginally significant in the overall analysis, additional analyses were conducted to explore it. Two 3 x 2 x 2 ANOVAs were conducted comparing grade, foil type and condition on the second half trials. Because there was no main effect nor any interactions with sex on the prior analysis, it was not included in this analysis. The first ANOVA compared the discrete control with the continuous-discrete condition. The expectation of this comparison was that students would perform better on discrete trials when preceded by continuous trials. However, the analysis comparing the Continuous-Discrete condition and the Discrete-Discrete condition failed to support this claim with F (1, 287) = .936, p = .335, η_p^2 = .007. As in the previous analyses, there was a significant main effect of grade with higher grades outperforming lower grades F (2, 286) = 1.167, $p < .001$, η_p^2 = .168. Specifically, pairwise comparisons indicated that significant differences occurred between fourth grade and kindergarten and fourth grade and second grade, both $p < .001$, but not between kindergarten and second grade p = .277. No other significant effects or interactions were found with all $F \leq 1.731$, $p \geq .181$.

The second of the ANOVAs compared the continuous control with the discrete-continuous condition. The expectation of this comparison was that students' performance on continuous trials would be negatively affected by preceding discrete trials. The analysis however, failed to support this claim, as the main effect of condition was not statistically significant, F (1, 287) = 1.148, p = .286, η_p^2 = .008. Again, there was a significant main effect of grade with higher grades outperforming lower grades F (2, 286) = 8.737, $p < .001$, η_p^2 = .112. More specifically, pairwise comparisons indicated the significant difference occurring between fourth grade and kindergarten $p < .001$, and between kindergarten and second grade p = .018, but only a marginally significant difference between second and fourth grade p = .079. Additionally, this analysis revealed a significant main effect of foil type F (1, 287) = 4.91, p = .028, η_p^2 = .034. Participants performed better on trials with a denominator foil (M = .57) than on trials with a numerator foil (M = .53). No other significant effects or interactions were found with all $F \leq$ 1.136, $p \geq$ .324.

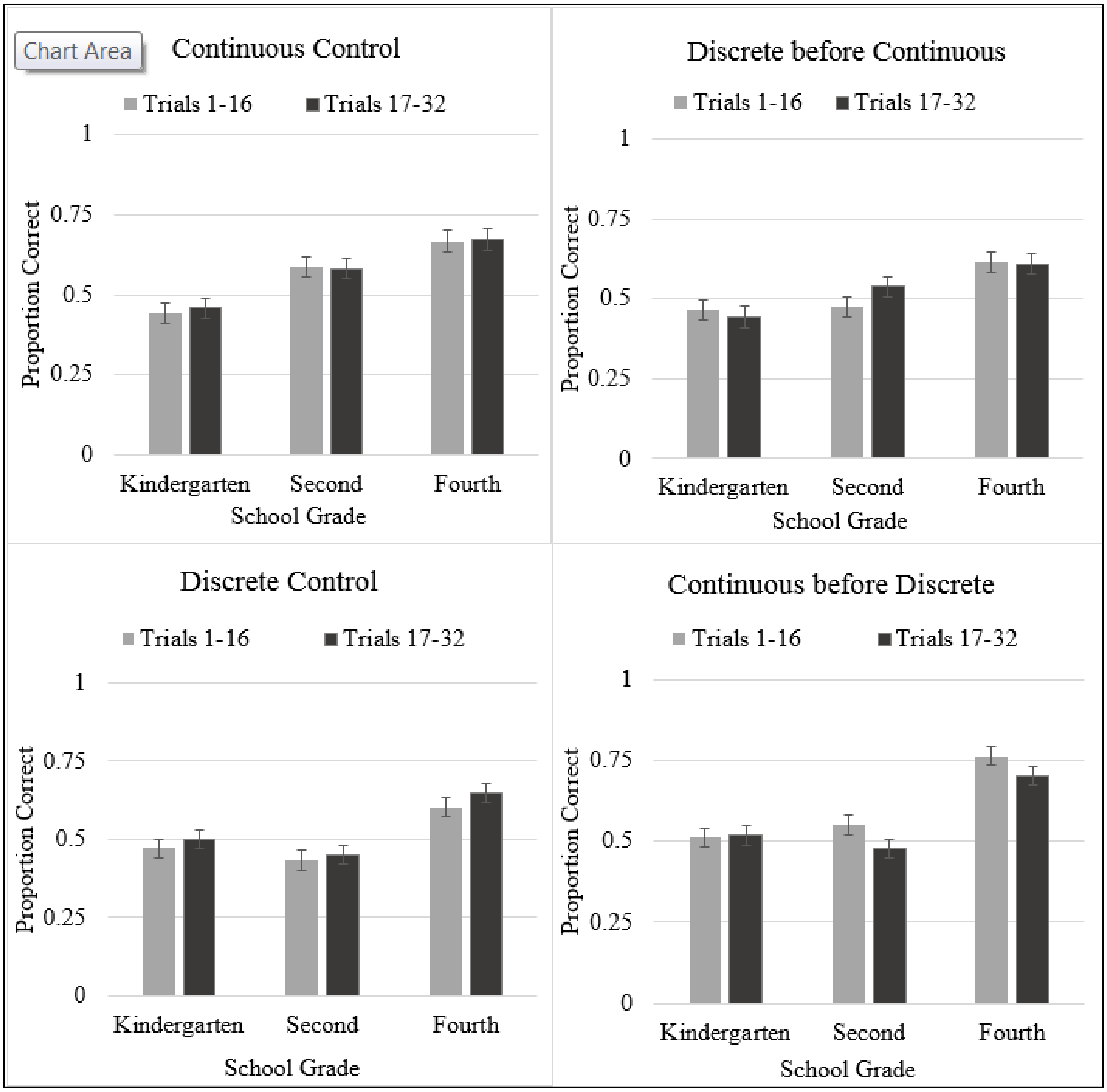

Figure 2. Mean proportion correct per school grade, block half, and experimental condition. Error bars represent SEM.

CHAPTER 4

DISCUSSION

Overall, this study replicated previous research findings indicating that students in kindergarten through fourth grade perform better when proportions are presented in a continuous format (Boyer & Levine, 2012; Boyer et al., 2008; Jeong et al., 2007; Spinillo & Bryant, 1999). This finding is believed to be due to students' use of correct and intuitive strategies on the continuous format trials and their incorrect use of counting strategies on the discrete format trials (Boyer et al., 2008; Boyer & Levine, 2012). Furthermore, proportions represented with discrete units, such as those in the current study, are particularly problematic because children have the inclination to count the individual units and, in this case, match their absolute quantities.

GRADE LEVEL

Additionally, we found a significant effect of grade with students increasing accurate performance from kindergarten to fourth grade. This is most likely due in part to the increase in focus on fractions in third grade curriculum. Based on the Georgia Standards of Excellence for Mathematics students should have a deep understanding of fractions and be able to compare them by the time they are in fourth grade (as seen in Table 3). Prior to third grade, students are not receiving any formal education on fraction problem solving, much less any focused development of proportional reasoning skills. This could indicate why the obtained data illustrate a pronounced increase in performance between second and fourth-grade participants. It is possible that the increase in familiarity beginning in third grade may have contributed to this boost in performance. At this point in their schooling, fourth graders' familiarity with formally presented fractions activates proportional reasoning skill and understanding. Moreover, this boost could actually show that the education of fractions is on an appropriate timeline. This of course can only be speculated at this point. Furthermore, Braithwaite and Siegler (2017) point out that the whole number bias that creates difficulty for children to understand fractions decreases with age. This decrease in bias could potentially explain why older students outperformed the younger students on a task that requires students to differentiate between whole number reasoning and proportional reasoning concepts and rules.

Table 3. Georgia Standard of Excellence for Mathematics

Third Grade	**MCC3.NF.1 – Develop understanding of fractions as numbers.** - Understand a fraction *1/b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts (unit fraction); understand a fraction *a/b* as the quantity formed by *a* parts of size *1/b*. For example, *3/4* means there are three 1/4 parts. So *3/4 = 1/4 + 1/4 + 1/4.*
	MCC3.NF.2 – Understand a fraction as a number on the number line; represent fractions on a number line diagram. - Represent a fraction *1/b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size *1/b*. Recognize that a unit fraction *1/b* is located *1/b* whole unit from 0 on the number line. - Represent a non-unit fraction *a/b* on a number line diagram by marking off *a* lengths of *1/b* (unit fractions) from 0. Recognize that the resulting interval has size *a/b* and that its endpoint located the non-unit fraction *a/b* on the number line.
	MCC3.NF.3 Explain equivalence of fractions through reasoning with visual fraction models. Compare fractions by reasoning about their size - Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. - Recognize and generate simple equivalent fractions with denominators of 2, 3, 4, 6, and 8, e.g. Explain why the fractions are equivalent, e.g., by using a visual fraction model. - Express whole numbers as fractions and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form 3 = (3 wholes is equal to six halves); recognize that = 3; locate and 1 at the same point of a number line diagram. - Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.
Fourth Grade	**MCC4.NF.1** Explain why two or more fractions are equivalent *a/b = nxa/nxb* ex: 1/4= 3x1/3x4 by using visual fraction models. Focus attention on how the number and size of the parts differ even though the fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.
	MCC4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by using visual fraction models, by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions.
	MCC4.NF.3 Understand a fraction *a/b* with a numerator >1 as a sum of unit fractions *1/b*. - Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. - Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. Examples: 3/8 = 1/8 + 1/8 + 1/8; 3/8 = 1/8 + 2/8; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8. - Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. - Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models
	MCC4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number e.g., by using a visual such as a number line or area model. - Understand a fraction *a*/*b* as a multiple of 1/*b*. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4)*. - Understand a multiple of *a*/*b* as a multiple of 1/*b*, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)* - Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

FOIL TYPES

Based on previous research examining foil types (Boyer et al., 2008; Boyer & Levine, 2015), it was expected that a significant effect would be found for foil type in this study however, that was not the case. Specifically, foil type was manipulated and measured in the study in the hopes of examining students' ability to see past the parts of a proportion and understand the part-whole relation. In previous studies (Boyer et al., 2008; Boyer & Levine, 2015) there was an effect of foil type such that performance was better on trials with the denominator foil rather than the numerator foil. While we were able to see a marginally significant finding with a similar trend to that of the previous research mentioned above, we did not see a significant effect in the current study. Only speculations can be made as to why this trend was only marginally visible. Since the number of trials in the current study were more numerous than in previous studies of similar contexts, one possible issue could have been with the length of the study. Participants may have become bored with the trials and quit attempting to accurately respond, which would have impacted the expected foil type effects.

FORMAT: CONTINUOUS VERSUS DISCRETE

As mentioned previously, research of this nature has found that children generally perform better when tasks such as these contain stimuli that are presented in a continuous format rather than in discrete units (Boyer & Levine, 2012/2015; Boyer et al., 2008; Jeong et al., 2007; Mix et al., 1999; Singer-Freeman & Goswami, 2001; Spinillo & Bryant, 1999). A theorized potential reason for this performance difference comes down to children's inappropriate decision to count discrete units and make incorrect comparisons; whereas in a continuous format, there are no visible demarcations that would allow for children to count units (Boyer et al., 2008;

Boyer & Levine, 2015). This study did successfully replicate these previous findings regarding students' performance on continuous and discrete trials across different ages.

Boyer and Levine (2015) sought to determine if students' successful performance on stimuli presented in continuous formats could be used to increase accuracy on stimuli presented using a discrete format. As an extension, this study sought to not only see if continuous trials affected performance on discrete trials, but to see if discrete trials impacted performance on continuous trials as well. Specifically, the study was designed to examine the effects of preceding stimuli format on subsequent trials. It was expected that students would perform better on discrete trials if preceded by continuous trials and perform worse on continuous trials if preceded by discrete trials. Unfortunately, in the analysis comparing these, there were no significant findings indicating that this occurred. However, there was a clear trend that would indicate some form of this pattern emerging in the study.

Speculations could be made as to why this expectation was not met. With this study, there were no specific indicators to bring the participant's attention to the transition from the first-half to second-half set of trials. There is the possibility that this subtle change between stimuli presentation types may have negatively impacted the study's outcome. Additionally, there were more trials than is typical for this type of study, therefore, participants may have lost interest, become fatigued or bored and may have even quit trying to perform well out of disinterest. Again, these are only speculations.

IMPLICATIONS

Though crucially important for later understanding of mathematics, cognitively processing relative quantities and proportions is very demanding (Bailey et al., 2015; Jordan et al., 2013; Mack, 1990; Namkung & Fuchs, 2015; NMAP, 2008; Siegler, Thompson, & Schneider

2011; Siegler, Fazio, Bailey, & Zhou, 2012; Siegler & Pyke, 2013). Given this, the current study was designed in the hopes of deepening the understanding of how students interpret and evaluate proportional information.

The failure to show any significant findings involving the effects of preceding stimuli is unfortunate, however, fraction problem solving and proportional reasoning are very complex and challenging for students to understand and for teachers to teach. While this task was designed based on prior research (Boyer et al., 2008; Boyer & Levine, 2015), a 5-minute task such as this may not be able to overcome the limitations associated with fractions and proportional reasoning.

Additionally, the failure to obtain the predicted findings could actually be taken as an indicator for how challenging overcoming fairly rigid tendencies might be. Student's spend the first few years of their schooling learning whole numbers and becoming knowledgeable on the rules and expectations of those whole numbers. Once they finish off third grade they have begun to learn a new set of rules and instructions that are associated with fraction problem solving and proportional reasoning.

While this study did not successfully find the predicted results, it did show the trends that were expected, demonstrating that there are indeed issues associated with student's education of implementation of their fraction understanding and proportional reasoning skills. The hopes now would be that future educational resources might better prepare students in their fraction problem solving so that their proportional reasoning skills can serve them more effectively.

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