Numerical Simulation of Natural Convection in Rectangular Enclosures of Varying Aspect Ratios and its Feasibility in Environmental Impact Management Studies

Charles R. Walker

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NUMERICAL SIMULATION OF NATURAL CONVECTION IN RECTANGULAR ENCLOSURES OF VARYING ASPECT RATIOS AND ITS FEASIBILITY IN ENVIRONMENTAL IMPACT MANAGEMENT STUDIES

by

CHARLES WALKER

(Under the Direction of Mosfequr Rahman)

ABSTRACT

Numerical simulation was used to investigate natural convection in horizontal and vertical enclosures with and without an internal heat source. Natural convection in rectangular enclosures is found in many real-world applications. Included in these applications are the energy efficient design of buildings, operation and safety of nuclear reactors, solar collector design, passive energy storage, heat transfer across multi-pane windows, thermo-electric refrigeration and heating devices, and the design-for-mitigation of optical distortion in large-scale laser systems. Considering all these applications, especially controlling heat transfer in nuclear power plants, knowledge and research results of natural convection in enclosure play a vital role in environmental impact management studies. This study simulated horizontal enclosures heated from below (configuration 1) and vertical enclosures heated from the side (configuration 2) with a variety of different aspect ratios (AR) and Rayleigh numbers (Ra). Each aspect ratio (1, 2, 4, 6, 8, and 10) was examined using different sets of Rayleigh numbers. The first numerical experiment used only external Rayleigh number \( \text{Ra}_E = 2 \times 10^4, 2 \times 10^5, \text{ and } 2 \times 10^6 \) which simulated natural convection in enclosures for outside temperature gradient only. The second case used a constant external Rayleigh number \( \text{Ra}_E = 2 \times 10^5 \) with a changing internal Rayleigh number \( \text{Ra}_I = 2 \times 10^4, 2 \times 10^5, \text{ and } 2 \times 10^6 \). The third simulation used a constant internal Rayleigh
number (\(Ra_l = 2 \times 10^5\)) and a changing external Raleigh number (\(Ra_E = 2 \times 10^4, 2 \times 10^5,\) and 
\(2 \times 10^6\)). All three cases were simulated for each configuration and at each aspect ratio. The streamline and isotherm flow patterns were created to reflect each case. The average heat flux ratio and convection strength were also calculated. Tests with the external temperature gradient only confirmed previous studies. There were many notable outcomes in this study which are discussed in the main body of this thesis work. When \(Ra_E > Ra_l\), the results were similar to the study with a varying external Rayleigh number (\(Ra_E\)) and no internal heat source.

INDEX WORDS: Natural convection, Rectangular enclosure, Numerical simulation, Average heat flux ratio, Convection strength, Rayleigh number.
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by

CHARLES WALKER

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by

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DEDICATION

“Thank you” to my family for supporting me in my educational endeavors. I would also like to thank my professors for their guidance throughout my graduate work.
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List of Symbols

AR – Aspect Ratio (L/H)
Ra – Rayleigh Number
Ra₁ - Internal Rayleigh Number
Raₑ - External Rayleigh Number
Re – Reynolds Number
Gr – Grashof Number
Pr – Prandlt Number
Nu – Nusselt Number
CS – Convection Strength
q – Heat flux (W/m²)
g – Acceleration due to gravity (m/s²)
T – Temperature (°K)
ν - kinematic viscosity (m²/s)
v – mean velocity (m/s)
β – compressibility
k – thermal conductivity (W/(m·K))
ρ – mass density (kg/m³)
α - linear expansion coefficient
CHAPTER 1

INTRODUCTION

1.1 Problem Statement

Natural convection in enclosures has been the topic of research for many years. Researchers have focused on natural convection in enclosures because of its presence in engineering applications. A common industrial application of natural convection is free air cooling without the aid of fans, and can happen on small scales such as computer chips all the way to large scale process equipment. Natural convection in rectangular enclosures is found in many real-world applications. These applications can be organized into two categories of study; the first is the classical field of study which is the nontraditional flow where an enclosure is heated from the bottom and cooled from the top. The second is traditional where an enclosure is heated from one vertical side and cooled on the other. The most common difference between the two categories is the nontraditional enclosure has to reach a critical value before any flow can be observed. However in a traditional setup flow is present when a small temperature difference is observed. Traditional enclosures have many applications such as, solar collectors, operation and safety in nuclear reactors, double wall or window pane insulations, air circulation in rooms and energy efficient electronic cooling. An example of a use for nontraditional setup is flow through an attic space. Judging from the number of possible engineering applications, the enclosure phenomena can loosely be organized in to two large classes:

1. Horizontal enclosures heated from below
2. Vertical enclosures heated from the side
Configuration one (horizontal enclosure heated from below) represents the natural convection in fluid layers heated from below (the flow) and is a classical subject. However, the current applications in thermal insulation engineering, solar technology, rotating fluid machinery, and energy management in architectural design demand an emphasis on configuration 2 (vertical enclosures heated from side), which is a much newer subfield in the research of convective heat transfer. The ratio of the length of the isothermal wall to the distance of separation, designated as the aspect ratio (L/H) of the enclosure, also plays an important role in establishing the convection and heat transfer process in the enclosure.

This study numerically simulates both configurations with a variety of different aspect ratios (AR) and Rayleigh numbers (Ra). The aspect ratios used in the study are 1, 2, 4, 6, 8, and 10. Each aspect ratio was examined using a different set of Rayleigh numbers (Ra). Two types of Rayleigh number are used in this study: external Rayleigh number (Ra_E), which denotes the effect due to the differential heating of the sidewalls defined by \( \text{Ra}_E = \frac{g \beta (T_h - T_c) H^3}{\nu \alpha} \) and internal Rayleigh number (Ra_I), which represents the strength of the internal heat generation defined by \( \text{Ra}_I = \frac{g \beta GH^5}{\nu \alpha k} \). The first numerical experiment used only an external Rayleigh number (Ra_E) to simulate an outside temperature gradient. The second case used a constant external Rayleigh number (Ra_E) with a changing internal Rayleigh number (Ra_I). The third used a constant internal Rayleigh number (Ra_I) and a changing external Rayleigh number (Ra_E). The third case simulated a constant internal heat source with a changing external temperature gradient. All three of these cases were simulated for each category at each aspect ratio.
1.2 Objective of the Present Study

The primary objective of the present study is to investigate the natural convection in a horizontal and vertical enclosure of various aspect ratios with and without an internal heat source. The flow is assumed to be laminar, steady, and two dimensional. The study has the following objectives:

1. To determine the suitable external Rayleigh number (Ra_E) for different aspect ratios for both configurations to initiate natural convection.

2. To use those Rayleigh numbers to conduct three separate computational simulations for both configurations under the following three conditions:
   a. Changing external temperature gradient without internal heat source; i.e. changing external Rayleigh number (Ra_E) with zero internal Rayleigh number (Ra_I),
   b. Constant external temperature gradient with a changing internal heat source; keeping external Rayleigh number constant (Ra_E) with changing internal Rayleigh number (Ra_I),
   c. Changing external temperature gradient with a constant internal heat source; i.e. changing external Rayleigh number (Ra_E) with constant internal Rayleigh number (Ra_I).

To observe the following effects for the above conditions:

   i. Generate isotherms and streamlines for each condition and each configuration for all aspect ratios.

   ii. Observe the effect of the changing Rayleigh number and aspect ratios on the average heat flux ratio (convective heat flux/corresponding conduction heat flux) along hot and cold walls.
iii. Observe the behavior of convective strength with the variation of internal Rayleigh number, external Rayleigh number, and aspect ratios.

1.3 Outline of the Thesis

Chapter two provides a review of current literature on the subject of natural convection in rectangular enclosures with different boundary conditions, different geometry of the enclosure, both experimental and numerical. Chapter three outlines the governing equations, and methodology used for this numerical study. Chapter four presents the final results and discussion of said results. Lastly, the conclusion of current studies and recommendations for further studies are addressed in chapter five.
CHAPTER 2

REVIEW OF LITERATURE

Natural convection is a mechanism, or type of heat transport, where the fluid motion is not generated by any external source, but only by density differences in the fluid occurring due to temperature gradients. In natural convection, fluid surrounding a heat source receives heat, becomes less dense and rises. The surrounding cooler fluid then moves to replace it. The cooler fluid is then heated and the process continues, forming convection current. The process transfers heat energy from the bottom of the convection cell to the top (Absolute Astronomy 2011). Natural convection has attracted a great deal of attention from researchers because of its presence in engineering applications. A common industrial application of natural convection is free air cooling without the aid of fans and can happen on small scales such as computer chips to large scale process equipment (Absolute Astronomy 2011).

Natural convection in rectangular enclosures is found in many real-world applications. Included in these applications are the energy efficient design of buildings, operation and safety of nuclear reactors, solar collector design, passive energy storage, heat transfer across multi-pane windows, thermo-electric refrigeration and heating devices, and the design-for-mitigation of optical distortion in large-scale laser systems.

Natural convection in two-dimensional rectangular enclosures has received a lot of attention over the last 40 years. Many combinations and permutations of boundary conditions have been investigated. Various geometries of fluid-filled rectangular enclosures have been theoretically and experimentally modeled in order to look at the effects of some design parameter on the thermal performance of simulated systems. Davis (1968) presented the application of
finite element in numerical methods for investigating natural convection in rectangular cavities. He also investigated natural convection of air in a square cavity using numerical methods. His work became the benchmark solution referenced by other researchers.

The effects of an internal heat generation source on the natural convection in enclosures have also been a topic of research. The existence of a heat source in the cavities simulates engineering problems such as electronic chips cooling or heat transfer in buildings having a heat source. Ju and Chen (1996) found that a rectangular enclosure having five discrete protruding heaters mounted on one vertical wall showed a good agreement between the numerical simulation and experimental data for the flow pattern and temperature profile within the entire enclosure for various power inputs. Another study by Wroblewski and Joshi (1994) extended the study of natural convection in square enclosures to three-dimensional simulations. The study examined the effects of geometry and boundary conditions on the liquid immersion cooling of a substrate mounted protrusion in a three-dimensional enclosure.

Turan, Chakraborty, and Poole (2010) studied two-dimensional steady-state simulations of laminar natural convection in square enclosures with differentially heated sidewalls where the enclosures are considered to be completely filled with a yield stress fluid obeying the Bingham model. They found that the mean Nusselt number increases with increasing values of the Rayleigh number for both Newtonian and Bingham fluids. However, they also discovered the Nusselt numbers obtained for Bingham fluids are smaller than those found in the case of Newtonian fluids with the same values of nominal Rayleigh number. The Nusselt number was found to decrease with increasing Bingham number. For large values of Bingham number, the value of mean Nusselt number settled to unity as the heat transfer took place mainly by conduction.
An analysis of a numerical unsteady two-dimensional heat transfer and fluid flow in a square enclosure with a centered square obstruction was done by Faghri, Asako, Berard, and Chaboki (1994). Their study revealed that isotherm and streamline demonstrated a significant difference with the addition of radiation into the solution. Fluid in the enclosure-obstruction gap was heated or cooled more significantly due to the radiation heating or cooling of the obstruction wall from the enclosure wall. They concluded that the ideal case is a large enclosure, large Rayleigh number and radiation/conduction number, high thermal diffusivity ratio and large aspect ratio. The larger the enclosure, the more heat flux is being input into the system. The least ideal case would be a small enclosure, low thermal diffusivity ratio and large aspect ratio.

In another study by Raos (2001), laminar natural convection in square enclosures was successfully simulated. His study revealed that complex flow patterns heat transfer rates with orientation above 90° adversely affected the convection process particularly when increased to 180°. Orientation under 20° showed inconsistent flow pattern.

Sarris, Grigoriadis and Vlachos (2010) conducted a numerical study of the laminar regime of free convection flow due to spatially varying magneto-convection and thermo-convection in a square enclosure. They found that a suitable combination of the magnetic and gravitational forces may enhance heat transfer up to 40% over the usual natural convection heat transfer rates. A stratification of the flow may happen when the two forces have an opposing action on the fluid. They also found for the range of flow parameters considered in their study, the intensity of the flow circulation and the heat transfer reach asymptotically a plateau because the limit of transition-to-unsteadiness is reached for large enough Hartmann numbers. Finally, they concluded that the magnetic Reynolds number was found to be a significant flow parameter.
since it controlled the portion of the magnetic energy that was transformed into fluid kinetic energy.

According to Tong, Sparrow, and Abraham (2008) flat, wide rectangular enclosures have been studied because they are prone to instabilities when the temperature of the lower bounding surface exceeds the upper bounding surface. Tall, narrow, enclosures may also be unstable and create flow patterns which differ from those encountered in flat, wide enclosures. Natural convection does not occur in either of these enclosures until a threshold value of the Rayleigh number is exceeded. There is a continuum of such threshold Rayleigh numbers that are shared by the two enclosure categories. The investigation by Tong, Sparrow, and Abraham (2008) was aimed at unifying the treatment of both tall, narrow enclosures and flat, wide enclosures. They found the threshold Rayleigh numbers marking the onset of natural convection flow varied only moderately for aspect ratios that characterize flat, wide enclosures. However, the threshold Rayleigh numbers increased markedly with increasing aspect ratio for the tall, narrow enclosures. The threshold values of the Rayleigh numbers for the bottom-heated case were approximately 12 times greater than those for heating and cooling at the respective side walls. Thus, for flat, wide enclosures heated from below, an array of recirculation zones deployed across the width of the enclosures appeared to be the forerunner of Rayleigh-cells.

Moghimi et al, (2009) studied natural convection in rectangular enclosures heated from below and cooled from above. They studied numerically for width-to-height aspect ratios of the enclosure in the range between 0.25 and 1, and for values of the Rayleigh number based on the cavity height. They found that with the correct parameters, such as configurations and dimensions of the enclosure, they could control the heat transfer and obtain the optimal heat transfer in it.
Caronna, Corcione, and Habib (2009) investigated natural convection inside air-filled, square and tall rectangular enclosures subjected to a combination of two types of heating/cooling conditions. One condition involved rectangular cavities with two or more adjacent walls differentially heated. The other condition involved rectangular cavities with two portions of the same wall maintained at different temperatures. These conditions were studied for different values of the height-to-width aspect ratio of the enclosure, the heated fractions of both sidewalls, and the Rayleigh number of the enclosure.

Aydin and Yang (2000) also investigated natural convection of air in a two-dimensional, rectangular enclosure with localized heating from below and symmetrical cooling from the sides. They found that the flow and temperature fields were symmetrical about the mid-length of the enclosure due to the symmetry of the boundary conditions in the vertical direction. For small Ra, the heat transfer is dominated by conduction across the fluid layer, while at high Ra the process is primarily one of convection, and the effect of conduction vanishes. Increasing nondimensional length of the heat source enhances the heat transfer, especially for high values of Raleigh number.

Calcagni, Marsili, and Paroncini (2005) conducted a study on free convective heat transfer in a square enclosure characterized by a discrete heater located on the lower wall and cooled from the lateral walls. Their study concentrated on the effects of the dimensions of the heat source on the convective heat exchange. The partial heating at the lower surface simulated electronic components. Their experimental method used the holographic interferometry technique in real-time and double-exposure, to obtain respectively the visualization of possible oscillation of the plume and steady-state temperature distribution inside the cavity. The objective of the heat transfer analysis was the investigation of the Nusselt number distribution on the heat
source and on the cooling surfaces at various Raleigh numbers. Both the experimental and numerical investigation pointed out a heat transfer prevalently conductive for $Ra \leq 10^4$ while the convective phenomenon developed completely for $Ra = 10^5$. The results of the investigation by Calcagni, Marsili, and Paroncini (2005) indicated that an increase of the heat source dimension produces a rise in heat transfer particularly for high $Ra$.

In another study by Ganzarolli and Milanez, (1995) a rectangular enclosure heated from below and symmetrically cooled from the sides was analyzed. The boundary condition for the cavity floor was uniform heat flux while the side walls were cooled at a uniform temperature. The results showed a little influence of the Prandtl number on the heat transfer and on the flow circulation inside the cavity. For the square cavity, uniform surface temperature did not affect the flow or the isotherm contours. However, for the shallow cavity, there are differences when the surface temperature or the heat flux is prescribed. In the case of uniform temperature at the cavity floor, the cavity is not always thermally active along its whole extension and the flow does not fill it uniformly in those cases. When the heat flux was prescribed, the isotherms and the streamlines occupy the whole cavity more uniformly, even for low values of the Rayleigh number.

Basak et al (2009) investigated the influence of linearly heated vertical walls or cooled right wall with uniformly heated bottom wall on flow and heat transfer characteristics due to mixed convection within a square cavity. They found that multiple circulation cells appeared inside the cavity with the increase of Prandtl number (Pr) in the case of linearly heated side walls and only two circulation cells were formed inside the cavity. As Pr increased to 0.7, three circulation cells formed inside the cavity. A further increase in Pr to 10 lead to the formation of four circulation cells inside the cavity. On the other hand, the researchers noted that only two
circulation cells are formed inside the cavity for the case of cooled right wall. A detailed analysis of flow pattern showed that as the value of Raynolds number (Re) increased from 1 to $10^3$, there was a transition from natural convection to forced convection depending on the value of Grashof number (Gr) irrespective of Prandlt number (Pr). They further observed that the secondary vortex at the top left corner disappeared. Due to enhanced motion of the upper lid in the case of the cooled right wall, a small secondary vortex existed at the bottom right corner. The local Nusselt number plot showed that heat transfer rate is equal to 1 at the edges for the case of linearly heated side walls case and it was zero at the left edge and thereafter increased the case of cooled right wall. Basak et al (2009) also observed that the Nusselt number was large within the enclosure due to compression of isotherms. Average Nusselt numbers at the bottom and right walls were strong functions of Grashof number (Gr) at larger Prandtl numbers (Pr), whereas average Nusselt number (Nu) at the left wall at a specific Pr was a weaker function of Gr.

Abu-Nada and Oztop (2009) conducted research that presented the effects of inclination angle on flow field and temperature distribution in differentially heated and nanofluid filled square enclosures. Their results showed that heat transfer enhanced with increasing of Rayleigh number almost linearly but the effect of nanoparticles concentration on Nusselt number is more pronounced at low Rayleigh number than at high Rayleigh number. They also found that lower heat transfer is formed for $=90^\circ$. However, higher values of volume fraction become insignificant from the fluid flow point of view at that inclination angle. Abu-Nada and Oztop’s (2009) research showed that the effects of inclination angle on percentage of heat transfer enhancement become insignificant at low Rayleigh number, but it decreases the enhancement of heat transfer with nanofluid.
Studies on the natural convection due to the external heating in inclined cavities mainly concentrated on the cavities with differentially heated opposite sidewalls. In realistic situations, that is not always the case. Aminossadati and Ghasemi (2005) conducted research that looked at the natural convection in inclined enclosures where two adjacent walls were at different temperatures. They found low Rayleigh numbers did not cause any significant change in the flow field with the inclination angle. However, dissimilar fashions were observed at high Rayleigh numbers because of the strengthening in buoyancy forces. As the inclination angle increased, the intensity of the flow field remained unchanged at low Rayleigh numbers and increased at high Raleigh numbers. Basically, at all Rayleigh numbers, as the inclination angle rose, the average temperature in the enclosure started to increase until it reached its maximum and then decreased.

An investigation of two-dimensional laminar natural convection in externally heated rectangular enclosures with and without heat generation was carried out by Rahman and Sharif (2003). The investigation was numerically by a finite-difference procedure for a range of aspect ratios at various angles of inclination. They found the flow patterns and isotherms showed only a slight shift and changes in stream function and isotherm values. Rahman and Sharif (2003) also discovered the influence of inclination angle is particularly strong in the upstream regions on the isothermal walls. Their study showed the convection strength increased as the aspect ratio increased; this implies the convective flow inside the enclosure gets more and more vigorous as the enclosure shape changed from slender through square to shallow at any particular inclination.

The most common types of nuclear power plants use water for cooling to convey heat from the reactor core to the steam turbine and to remove and dump surplus heat from the steam circuit. (Cooling Power Plants 2011) The cooling function to condense the steam can be done by direct or once-through cooling, recirculating or indirect cooling, or dry cooling. The majority of
plants use once-through cooling. According to the Nuclear Energy Institute (Institute 2011), once-through cooling withdraws water from a water body and circulates it within the plant to condense the steam from the turbine into water through heat absorption. A large amount of water runs through the condensers in a single pass and discharge it back into the sea, lake, or river a few degrees warmer and without much loss from the amount withdrawn. (Cooling Power Plants 2011) A typical nuclear power plant supplies 740,000 homes with all of the electricity they use while consuming 13 gallons of water per day per household in a once-through cooling system. Using seawater means that higher-grade materials must be used to prevent corrosion, but cooling is often more efficient. (Niiler 2011) Some scientists also believe that when the water used for cooling is returned to its source, it can be harmful to the environment. For example, in California, the Water Resources Control Board recently testified that once-through cooling systems kill 2.6 million fish a year. (Standen 2012) Michael Podowski, a visiting professor at Massachusetts Institute of Technology’s department of nuclear engineering believes that most of the existing United States nuclear reactors are reaching the end of their lifespans. They need to be replaced, or the American public has to find another source for 20 percent of its power. In 2010 the Electric Power Research Institute (EPRI) studied 428 U.S. power plants with once-through cooling systems which were potentially subject to revised US Environmental Protection Agency regulations to protect aquatic life from being caught up in the cooling water intake structures. The EPRI found that the total cost of retrofitting U. S. power plants to preserve and protect aquatic life would exceed $95 billion. (Cooling Power Plants 2011) This study of natural convection could help guide researchers to more efficient ways to cool reactors while new generations of reactors are being developed.
CHAPTER 3
NUMERICAL MODEL

3.1 Governing Equations

For the mathematical description of this numerical study, the following assumptions are made.

1. The flow is incompressible, steady or laminar.
2. The fluid is Newtonian with constant properties.
3. The effects of heat radiation are negligible.

Based on the above assumptions, the governing equations are as follows:

Continuity equation:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{3.1}
\]

Momentum equation in \(x\) direction:

\[
u \frac{\partial u}{\partial x} + u \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + g\beta(T - T_0) \sin \theta \tag{3.2}\]

Momentum equation in \(y\) direction:

\[
u \frac{\partial v}{\partial x} + u \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + g\beta(T - T_0) \cos \theta \tag{3.3}\]

Energy Equation:

\[
\rho C_p \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + G \tag{3.4}\]

By defining a modified pressure \(p^*\) as
\[ p^* = p + \rho_0 g x \sin \theta + \rho_0 g y \cos \theta \]  \hspace{1cm} (3.5) 

and using the Boussinesq approximation 

\[ \rho = \rho_0 [1 - \beta(T - T_0)] \]  \hspace{1cm} (3.6) 

the governing equations in dimensionless form become 

\[ \frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \]  \hspace{1cm} (3.7) 

\[ U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} + \frac{Ra_E}{Pr} \phi \sin \theta \]  \hspace{1cm} (3.8) 

\[ U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} + \frac{Ra_E}{Pr} \phi \cos \theta \]  \hspace{1cm} (3.9) 

\[ U \frac{\partial \phi}{\partial X} + V \frac{\partial \phi}{\partial Y} = \frac{1}{Pr} \left( \frac{\partial^2 \phi}{\partial X^2} + \frac{\partial^2 \phi}{\partial Y^2} \right) + \frac{Ra_I}{Ra_E} Pr \]  \hspace{1cm} (3.10) 

In obtaining the above equations, the following dimensionless variables have been used 

\[ X = x / H, \quad Y = y / H \]  \hspace{1cm} (3.11a) 

\[ U = u / (v / H), \quad V = v / (v / H), \quad P = p^* / \rho (v / H)^2 \]  \hspace{1cm} (3.11b) 

\[ \phi = (T - T_0) / (T_h - T_c) \text{ where } T_0 = (T_h + T_c) / 2 \]  \hspace{1cm} (3.11c) 

Only rigid walls are considered, so that no-slip boundary conditions are assumed on all walls. 

The sidewalls are assumed to be adiabatic while the hot wall is maintained at a temperature \( T_h (\phi = 0.5) \) and the cold wall has a temperature \( T_c (\phi = -0.5) \).
3.2 Methodology

The enclosures are sub-divided into many rectangular shaped control volumes and contain a nodal point at the center of each volume. The governing equations are integrated over the control volume. These equations produce sets of algebraic equations. The computational method used is based on a method used by Ferziger and Peric (1997). A simple method for pressure-velocity coupling previously used by Patankar (1980) was used. The algebraic equations produced are solved using Strongly Implicit Procedure or SIP of Stone (1968). The convergence of the sequential iterative solution is achieved when the sum of the absolute differences of the solution variables between two successive iterations fall below a pre-specified small number, which is $1 \times 10^{-3}$ in this study based on work by Rahman (2003). The grid used in this experiment was based on the grid use by Rahman (2003).

In order to produce the results for this experiment the input file was altered for the purposes of this study. The input file was plugged into the first program that ran tests at different points along the grid and produced a matrix of output data. The matrix of data was then plugged into the plot program to produce the streamlines, isotherms and the average heat flux at the hot and cold wall and to determine the convection strength in the enclosures. The data was then compiled and used to create displays to show the average heat flux for the different Rayleigh numbers and aspect ratio and also for the convection strength at each aspect ratio.
CHAPTER 4

RESULTS AND DISCUSSION

4.1 Introduction

The results presented in this section include isotherm, streamline, average heat flux across hot and cold walls, and convection strength with change in aspect ratio. These values are calculated for aspect ratios 1, 2, 4, 6, 8, and 10 for the class one and two configurations for three different situations: varying external temperature gradient only, constant external temperature gradient with varying internal heat source, and varying external temperature gradient and constant internal heat source.

4.2 Rayleigh Number Selection

The purpose of this experiment is to investigate natural convection in rectangular enclosures in the following two configurations: enclosures heated from below and enclosures heated from the side. This numerical study was used to see what the effects an internal heat source would have on the natural convection caused by the external temperature gradient. Rayleigh numbers were selected to ensure that the Rayleigh number was above the critical value for natural convection. The selection of the external Rayleigh number, $Ra_E$, at which significant convection occurs for all aspect ratios was determined by numerical experimentation. Three different values were selected to be used for both $Ra_E$ and $Ra_I$ are $2 \times 10^4$, $2 \times 10^5$ and $2 \times 10^6$. 
4.3 Effect of External Temperature Gradient Only

In this section the results of both class one and class two configurations are presented for all aspect ratios and external Rayleigh numbers (RaE).

4.3.1 Streamlines and Isotherms

The computed isotherms and streamlines show the temperature and flow inside the enclosure for the different aspect ratios of 1, 2, 4, 6, 8, and 10. Below, each aspect ratio has a set of three isotherms and three streamlines representing each external Rayleigh number (RaE) used for the value of the external temperature gradient. The computation will then be carried out on the class one configuration. The enclosure will then be rotated 90 degrees and the computation will be performed. The results are illustrated for the class two configuration below. This will show how the value of the external temperature gradient and aspect ratio effect the natural convection occurring in the enclosure.

4.3.2 Class One Configuration

In the isotherms for aspect ratio 1, all of the walls are an equal distance apart. For aspect ratio 1 a single rotational flow pattern was formed as the external temperature gradient increased. However, the isotherms began to disintegrate when the Rayleigh number increased to \( R_{E} = 2 \times 10^6 \). The isotherms are contained in Figures 4.1 to 4.6. The streamlines observed represented a recirculating pattern that grew as the external temperature gradient increased. These are represented in Figures 4.7-4.12.

As the aspect ratio increased, the hot and cold walls became closer, and the convection slightly increased. As the aspect ratio increased, the number of cellular convective rolls
increased also. The convective dominance decreased as the external temperature gradient increased.
Figure 4.01 Isotherms for three different external Rayleigh number (Ra_E) for class one configuration with aspect ratio 1

Ra_E = 2\times10^4

Ra_E = 2\times10^5

Ra_E = 2\times10^6

Figure 4.01 Isotherms for three different external Rayleigh number (Ra_E) for class one configuration with aspect ratio 1
Figure 4.02 Isotherms for three different external Rayleigh number ($Ra_E$) for class one configuration with aspect ratio 2

$Ra_E = 2 \times 10^4$

$Ra_E = 2 \times 10^5$

$Ra_E = 2 \times 10^6$
Figure 4.03 Isotherms for three different external Rayleigh number ($Ra_E$) for class one configuration with aspect ratio 4

$Ra_E = 2 \times 10^4$

$Ra_E = 2 \times 10^5$

$Ra_E = 2 \times 10^6$
Figure 4.04 Isotherms for three different external Rayleigh number ($Ra_E$) for class one configuration with aspect ratio 6

$Ra_E = 2 \times 10^4$

$Ra_E = 2 \times 10^5$

$Ra_E = 2 \times 10^6$
Figure 4.05 Isotherms for three different external Rayleigh number ($Ra_E$) for class one configuration with aspect ratio 8

$Ra_E = 2 \times 10^4$

$Ra_E = 2 \times 10^5$

$Ra_E = 2 \times 10^6$
Figure 4.06 Isotherms for three different external Rayleigh number (Ra_E) for class one configuration with aspect ratio 10
Figure 4.07 Streamlines for three different external Rayleigh number ($Ra_E$) for class one configuration of aspect ratio 1

$Ra_E = 2 \times 10^4$

$Ra_E = 2 \times 10^5$

$Ra_E = 2 \times 10^6$
Figure 4.08 Streamlines for three different external Rayleigh number ($Ra_E$) for class one configuration with aspect ratio 2

$Ra_E = 2 \times 10^4$

$Ra_E = 2 \times 10^5$

$Ra_E = 2 \times 10^6$
Figure 4.09 Streamlines for three different external Rayleigh number ($Ra_E$) for class one configuration with aspect ratio 4

$Ra_E = 2 \times 10^4$

$Ra_E = 2 \times 10^5$

$Ra_E = 2 \times 10^6$
Figure 4.10 Streamlines for three different external Rayleigh number (Ra_E) for class one configuration with aspect ratio 6.
Figure 4.11 Streamlines for three different external Rayleigh number \((Ra_E)\) for class one configuration with aspect ratio 8

\[
Ra_E = 2 \times 10^4
\]

\[
Ra_E = 2 \times 10^5
\]

\[
Ra_E = 2 \times 10^6
\]
Figure 4.12 Streamlines for three different external Rayleigh number ($Ra_E$) for class one configuration with aspect ratio 10
4.3.3 Class Two Configuration

For this section the same enclosures that were used in section 4.3.2 are used, but the enclosures were rotated 90° counter clockwise. This situation placed the hot wall on the right side of the enclosure and the cold wall on the left side of the enclosure. Each enclosure is plotted showing the isotherms and then the streamlines at each external Rayleigh number (RaE). The isotherm illustrations are contained in Figures 4.13-4.18 and the streamlines are in Figures 4.19-4.24.

For aspect ratio 1, the isotherms became more parallel as the RaE increased (Figure 4.19). This remained true for aspect ratios 2 and 4 in Figures 4.20 and 4.21, respectively.
Figure 4.13 Isotherms for class two configuration with aspect ratio 1 at three different Ra$_E$:

- $Ra_E = 2 \times 10^4$ 
- $Ra_E = 2 \times 10^5$ 
- $Ra_E = 2 \times 10^6$
Ra_E = 2 \times 10^4

Ra_E = 2 \times 10^5

Ra_E = 2 \times 10^6

*Figure 4.14 Isotherms for class two configuration with aspect ratio 2 at three different Ra_E*
Figure 4.15 Isotherms for class two configuration with aspect ratio 4 at three different $Ra_E$

$Ra_E = 2 \times 10^4$

$Ra_E = 2 \times 10^5$

$Ra_E = 2 \times 10^6$
Figure 4.16 Isotherms for class two configuration with aspect ratio 6 at three different $Ra_E$

- $Ra_E = 2 \times 10^4$
- $Ra_E = 2 \times 10^5$
- $Ra_E = 2 \times 10^6$
Figure 4.17 Isotherms for class two configuration with aspect ratio 8 at three different $Ra_E$
Figure 4.18 Isotherms for class two configuration with aspect ratio 10 at three different $Ra_E$

$Ra_E = 2 \times 10^4$  

$Ra_E = 2 \times 10^5$  

$Ra_E = 2 \times 10^6$
Figure 4.19 Streamlines for class two configuration with aspect ratio 1 at three different $Ra_E$

$Ra_E = 2 \times 10^4$

$Ra_E = 2 \times 10^5$

$Ra_E = 2 \times 10^6$
Figure 4.20 Streamlines for class two configuration with aspect ratio 2 at three different $Ra_E$

$Ra_E = 2 \times 10^4$

$Ra_E = 2 \times 10^5$

$Ra_E = 2 \times 10^6$
Figure 4.21 Streamlines for class two configuration with aspect ratio 4 at three different $Ra_E$

- $Ra_E = 2 \times 10^4$
- $Ra_E = 2 \times 10^5$
- $Ra_E = 2 \times 10^6$
Figure 4.22 Streamlines for class two configuration with aspect ratio 6 at three different $Ra_E$

- $Ra_E = 2 \times 10^4$
- $Ra_E = 2 \times 10^5$
- $Ra_E = 2 \times 10^6$
Figure 4.23 Streamlines for class two configuration with aspect ratio 8 at three different $Ra_E$.
Figure 4.24 Streamlines for class two configuration with aspect ratio 10 at three different $Ra_E$

- $Ra_E = 2 \times 10^4$
- $Ra_E = 2 \times 10^5$
- $Ra_E = 2 \times 10^6$
4.3.4 Average Heat Flux Ratio variation with Aspect Ratio

The overall heat transfer characteristics in the enclosure can be described by an average heat flux ratio along the hot surface \( \bar{q}_{r,h} \) and along the cold surface \( \bar{q}_{r,c} \). The average value is computed from a local heat flux ratio

\[
q_{r,h} = \frac{q_h}{q_{h,\text{cond}}} \quad (4.1)
\]

\[
q_{r,c} = \frac{q_c}{q_{c,\text{cond}}} \quad (4.2)
\]

where \( q_h \) and \( q_c \) are the convective heat flux at the hot and cold surfaces and \( q_{h,\text{cond}} \) and \( q_{c,\text{cond}} \) are the heat flux at the hot and cold surfaces by conduction only. In dimensionless form, the local heat flux ratio along the hot surface \( q_{r,h} \) and along the cold surface \( q_{r,c} \) may be written as

\[
q_{r,h} = -\left(\frac{\partial \phi}{\partial Y}\right)_{y=0} \left(1 - 0.5 \frac{Ra_I}{Ra_E}\right) \quad (4.3)
\]

and

\[
q_{r,c} = -\left(\frac{\partial \phi}{\partial Y}\right)_{y=1} \left(1 + 0.5 \frac{Ra_I}{Ra_E}\right) \quad (4.4)
\]

The average heat flux ratio along the hot and cold walls has been calculated using the simulation results due to the temperature gradient effect for each aspect ratio of both configurations.

4.3.4.1 Class one configuration

Variation of the average heat flux ratio along hot and cold walls with aspect ratio has been plotted for three different external Rayleigh number \( Ra_E \) (Figures 4.25 – 4.27). Results indicate with the increase of aspect ratio, average heat flux ratio along both hot walls and cold walls also increases. When these three plots are compared, one understands that with regard to each aspect ratio the magnitude of heat flux ratio increased with the increase of \( Ra_E \).
Figure 4.25 Variation of average heat flux ratio along hot and cold wall with aspect ratio at $Ra_E = 2 \times 10^4$

Figure 4.26 Variation of average heat flux ratio along hot and cold wall with aspect ratio at $Ra_E = 2 \times 10^5$
Variation of average heat flux ratio along hot and cold walls with aspect ratio has been plotted for three different external Rayleigh number ($Ra_E$) (Figures 4.28 – 4.30). Results indicate with the increase of aspect ratio average heat flux ratio along both hot walls and cold walls decreased. In comparing these three plots, the magnitude of heat flux ratio increases with the increase of $Ra_E$ which corresponds with the aspect ratio. Also, for this class two configuration the magnitude of the average heat flux ratio corresponding to each aspect ratio remains the same for both hot and cold walls at all three $Ra_E$. 

\textit{Figure 4.27 Variation of average heat flux ratio along hot and cold wall with aspect ratio at $Ra_E = 2 \times 10^6$} 

\textit{4.3.4.2 Class two configuration} 

Variation of average heat flux ratio along hot walls and cold walls with aspect ratio has been plotted for three different external Rayleigh number ($Ra_E$) (Figures 4.28 – 4.30). Results indicate with the increase of aspect ratio average heat flux ratio along both hot walls and cold walls decreased. In comparing these three plots, the magnitude of heat flux ratio increases with the increase of $Ra_E$ which corresponds with the aspect ratio. Also, for this class two configuration the magnitude of the average heat flux ratio corresponding to each aspect ratio remains the same for both hot and cold walls at all three $Ra_E$. 

Figure 4.27 Variation of average heat flux ratio along hot and cold wall with aspect ratio at $Ra_E = 2 \times 10^6$
Figure 4.28 Variation of average heat flux ratio along hot and cold wall with aspect ratio at $Ra_E = 2 \times 10^4$

Figure 4.29 Variation of average heat flux ratio along hot and cold wall with aspect ratio at $Ra_E = 2 \times 10^5$
4.3.5 Convection Strength variation with Aspect Ratio

Convection Strength (CS) is an overall measurement of how vigorous the fluid moves inside the enclosure. The Convection Strength is calculated by averaging the magnitudes of the velocity measurements. The equation is written as follows. 

$$CS = \frac{\sum_{i,j} \left( u_{i,j}^2 + v_{i,j}^2 \right) \frac{1}{2}}{\text{(Total number of nodes)}}$$

Convection Strength (CS) was calculated using the simulation results only due to the temperature gradient effect for each aspect ratio of both configurations.

4.3.5.1 Class one configuration

The convection strength as it changes with aspect ratio and the value of the external Rayleigh number ($Ra_E$) is shown in figure 4.31. These results illustrate that convection strength increased as the external Rayleigh number ($Ra_E$) increased. There is little change due to increasing aspect ratio for values of $Ra_E = 2 \times 10^4$ and $2 \times 10^5$. However, when $Ra_E = 2 \times 10^6$ the convection strength decreased after aspect ratio 2.

![Figure 4.30 Variation of average heat flux ratio along hot and cold wall with aspect ratio at $Ra_E = 2 \times 10^6$](image)
4.3.5.2 Class two configuration

Variation of convection strength with aspect ratio for three external Rayleigh number ($Ra_E$) is shown in figure 4.32. For this configuration the convection strength increased with the increase of both the aspect ratio and the external Rayleigh number.
4.4 Effect of Constant External Temperature Gradient with Changing Internal Heat Source

This section uses the same aspect ratios as used in the previous section but maintains a constant external Rayleigh number at \( Ra_E = 2 \times 10^5 \) and varying internal heat source values. The values used for the internal Raleigh number \( (Ra_I) \) were \( 2 \times 10^4 \), \( 2 \times 10^5 \) and \( 2 \times 10^6 \) respectively. This condition was used for both class one and class two configurations.

4.4.1 Isotherms and Streamlines

The computed isotherms and streamlines show the temperature and flow inside the enclosure for the different aspect ratios of 1, 2, 4, 6, 8, and 10. Each aspect ratio has a set of three isotherms and three streamlines illustrated for constant external Rayleigh number \( (Ra_E = 2 \times 10^5) \) and each internal Rayleigh number \( (Ra_I) \) used for varying values of the internal heat source. The computation was first carried out on the class one configuration. The enclosure was then rotated 90° counter clockwise and the computation was performed; the results are illustrated for the class two configuration. These diagrams show how the value of internal heat source and aspect ratio effect the natural convection occurring in the enclosure with a constant temperature gradient.

4.4.2 Class one configuration

The following figures represent the flow patterns and temperature fields for the different aspect ratios and different internal heat source values with constant temperature gradient (i.e. \( Ra_E = 2 \times 10^5 \)) for the class one configuration. The isotherms are presented in Figures 4.33-4.38 and the streamlines are presented in Figures 4.39-4.44.
No significant change in the isotherms and streamlines patterns can be observed when compared to the previous no heat source condition cases with the exception of when the cellular flow begins at a lower value of $Ra_I$ than the value of $Ra_E$. 
Figure 4.33 Isotherms at constant $Ra_E = 2 \times 10^5$ and three different values of $Ra_I$ with aspect ratio 1 for class one configuration.
Figure 4.34Isotherms at constant Ra_E = 2 × 10^5 and three different values of Ra_I with aspect ratio 2 for class one configuration
Figure 4.35 Isotherms at constant $Ra_E = 2 \times 10^5$ and three different values of $Ra_I$ with Aspect Ratio 4 for class one configuration
Figure 4.36 Isotherms at constant $Ra_E = 2 \times 10^5$ and three different values of $Ra_I$ with Aspect Ratio 6 for class one configuration
Figure 4.37 Isotherms at constant $Ra_E = 2 \times 10^5$ and three different values of $Ra_I$ with Aspect Ratio 8 for class one configuration
Figure 4.38 Isotherms at constant $Ra_E = 2 \times 10^5$ and three different values of $Ra_I$ with Aspect Ratio 10 for class one configuration
Figure 4.39 Streamlines at constant $Ra_E = 2 \times 10^5$ and three different values of $Ra_I$ with Aspect Ratio 1 for class one configuration
Figure 4.40 Streamlines at constant \( Ra_E = 2 \times 10^4 \) and three different values of \( Ra_I \) with aspect ratio 2 for class one configuration
Figure 4.41 Streamlines at constant $Ra_E = 2 \times 10^5$ and three different values of $Ra_I$ with aspect ratio 4 for class one configuration
Figure 4.42 Streamlines at constant $Ra_E = 2 \times 10^5$ and three different values of $Ra_I$ with aspect ratio 6 for class one configuration.
Figure 4.43 Streamlines at constant $Ra_E = 2 \times 10^4$ and three different values of $Ra_I$ with aspect ratio 8 for class one configuration
Figure 4.44 Streamlines at constant $Ra_E = 2 \times 10^5$ and three different values of $Ra_I$ with aspect ratio 10 for class one configuration
4.4.3 Class two configuration

The isotherms for all aspect ratios of class two configuration with constant external Rayleigh number at $Ra_E = 2 \times 10^5$ and varying internal Rayleigh number ($Ra_I$) are shown in Figures 4.45 – 4.50; the streamlines are shown in Figures 4.51 – 4.56. There is no significant change in the isotherms and streamlines patterns compared to the previous no heat source condition cases except the cellular flow begins at lower value of $Ra_I$ than the value of $Ra_E$. However, there was a difference in the convection strength and average heat flux ratio. This will be discussed in the next section.
Figure 4.45 Isotherms at constant $Ra_E = 2 \times 10^5$ and three different values of $Ra_I$ with aspect ratio 1 for class two configuration.
Figure 4.46 Isotherms at constant $Ra_E = 2 \times 10^5$ and three different values of $Ra_I$ with aspect ratio 2 for class two configuration.
Figure 4.47 Isotherms at constant $Ra_E = 2 \times 10^5$ and three different values of $Ra_I$ with aspect ratio 4 for class two configuration.
Figure 4.48 Isotherms at constant $Ra_E = 2 \times 10^5$ and three different values of $Ra_I$ with aspect ratio 6 for class two configuration
Figure 4.49 Isotherms at constant $Ra_E = 2 \times 10^5$ and three different values of $Ra_I$ with aspect ratio 8 for class two configuration.
Figure 4.50 Isotherms at constant $Ra_E = 2 \times 10^5$ and three different values of $Ra_I$ with aspect ratio 10 for class two configuration
Figure 4.51 Streamlines at constant $Ra_E = 2 \times 10^5$ and three different values of $Ra_I$ with aspect ratio 1 for class two configuration
Figure 4.52 Streamlines at constant $Ra_E = 2 \times 10^5$ and three different values of $Ra_I$ with aspect ratio 2 for class two configuration.

$Ra_I = 2 \times 10^4$

$Ra_I = 2 \times 10^5$

$Ra_I = 2 \times 10^6$
Figure 4.53 Streamlines at constant \(Ra_E = 2 \times 10^5\) and three different values of \(Ra_I\) with aspect ratio 4 for class two configuration
Figure 4.54 Streamlines at constant $Ra_E = 2 \times 10^5$ and three different values of $Ra_I$ with aspect ratio 6 for class two configuration
Figure 4.55 Streamlines at constant $Ra_E = 2 \times 10^5$ and three different values of $Ra_I$ with aspect ratio 8 for class two configuration.
Figure 4.56 Streamlines at constant $Ra_E = 2 \times 10^5$ and three different values of $Ra_I$ with aspect ratio 10 for class two configuration
4.4.4 Average Heat Flux Ratio Variation with Aspect Ratio

In addition to the constant external temperature gradient and aspect ratio, the effect of internal heat source on natural convection in the enclosure has been observed for both classes configurations.

4.4.4.1 Class one configuration

Figures 4.57 – 4.59 demonstrate the variation of average heat flux ratio along hot and cold walls with the aspect ratio. Figure 4.57 displays the heat flux variation pattern with aspect ratio is similar for both hot and cold walls. However, the heat flux variation is slightly higher for hot walls when \( Ra_I < Ra_E \). At \( Ra_I = Ra_E \), the average heat flux ratio decreased with an increase in aspect ratio for both hot and cold walls; but in this case average heat flux ratio is much higher in hot walls than in cold walls at each aspect ratio as shown in Figure 4.58. When \( Ra_I > Ra_E \), a completely different pattern is observed for heat flux ratio variation with aspect ratio for both hot and cold walls as shown in Figure 4.59. In this case the average heat flux ratio in the cold wall is much higher than in the hot wall at each aspect ratio. As a result, hot wall loses more heat because of the magnitude of the internal heat source being greater than the temperature gradient. Therefore, the amount of heat transferred at the hot wall is negative and becomes less as the aspect ratio increases.
Figure 4.57 Average heat flux ratio along hot and cold wall variation with aspect ratio at $Ra_E = 2 \times 10^5$ and $Ra_I = 2 \times 10^4$ for class one configuration.

Figure 4.58 Average heat flux ratio along hot and cold wall variation with aspect ratio at $Ra_E = 2 \times 10^5$ and $Ra_I = 2 \times 10^4$ for class one configuration.
Figures 4.60 – 4.62 show the variation of average heat flux ratio along hot and cold walls with the aspect ratio for the class two configuration. Figure 4.60 indicates the heat flux variation pattern with aspect ratio is similar for both hot and cold walls although there is a slight increase for hot wall when $Ra_l < Ra_E$. In this case, heat flux ratio decreased with the increase in aspect ratio. At $Ra_l = Ra_E$, average heat flux ratio decreased with the increase in aspect ratio for both hot and cold walls; but the average heat flux ratio is much higher in the hot wall than in the cold wall at each aspect ratio as shown in Figure 4.61. When $Ra_l > Ra_E$, a completely different pattern is observed in Figure 4.62 for heat flux ratio variation with aspect ratio for both hot and cold walls. The average heat flux ratio in the cold wall is much higher than in the hot wall at each aspect ratio. In this last case the hot wall loses heat because of the magnitude of the internal heat source being more than the temperature gradient. Therefore, the amount of heat transferred at
the hot wall is negative at aspect ratio 1 and 2; but it becomes positive in this configuration as the aspect ratio becomes greater than four.

*Figure 4.60* Average heat flux ratio along hot and cold wall variation with aspect ratio at $Ra_E = 2 \times 10^5$ and $Ra_I = 2 \times 10^4$ for class two configuration
Figure 4.61 Average heat flux ratio along hot and cold wall variation with aspect ratio at \( Ra_E = 2 \times 10^5 \) and \( Ra_I = 2 \times 10^5 \) for class two configuration

Figure 4.62 Average heat flux ratio along hot and cold wall variation with aspect ratio at \( Ra_E = 2 \times 10^5 \) and \( Ra_I = 2 \times 10^6 \) for class two configuration
4.4.5 Convection Strength

A graph of the convection strength for the class one configuration with constant external Rayleigh number ($Ra_E = 2 \times 10^5$) and changing internal Rayleigh number ($Ra_I = 2 \times 10^4$, $2 \times 10^5$ and $2 \times 10^6$) is shown in Figure 4.63. At all aspect ratio, the convection strength is higher and almost the same at $Ra_I \leq Ra_E$ than at $Ra_I > Ra_E$. For all three $Ra_I$, convection strength increased with the increase of aspect ratio up to 4; after that the convection strength starts to decrease slightly with the increase of aspect ratio.

Figure 4.64 shows the graph of convection strength versus aspect ratio for three $Ra_I$ with constant $Ra_E$ in the class two configuration. The convection strength increased as the aspect ratio increased for all three $Ra_I$ values.

![Figure 4.63 Convection strength versus aspect ratio at $Ra_E = 2 \times 10^5$ and $Ra_I = 2 \times 10^4$, $2 \times 10^5$, and $2 \times 10^6$ for class one configuration](image-url)
Figure 4.64 Convection strength verses aspect ratio at \( Ra_E = 2 \times 10^5 \) and \( Ra_I = 2 \times 10^4, 2 \times 10^5, \) and \( 2 \times 10^6 \) for class two configuration

4.5 Effect of Constant Internal Heat Source with Varying External Temperature Gradient

This section contains descriptions of the computational results and plots where the enclosures contain a constant internal heat source while the surrounding temperature gradient varies. The value used for the internal heat generation is internal Rayleigh number, \( Ra_I = 2 \times 10^5 \), while the values used for external Rayleigh number, \( Ra_E = 2 \times 10^4, 2 \times 10^5 \) and \( 2 \times 10^6 \) respectively. The same aspect ratios were used as in the previous cases. These computations were calculated for both class one and class two configurations.
4.5.1 Streamlines and Isotherms

The streamlines and isotherms represent the flow and temperature fields for the six different aspect ratios for both class one and class two configurations.

4.5.2 Class one configuration

The isotherms are shown in Figures 4.65-4.70. They are arranged in order by increasing aspect ratio and also by increasing external temperature gradient i.e. external Rayleigh number $Ra_E$. The streamlines are shown in Figure 4.71-4.76 and are also shown by increasing aspect ratio and external Rayleigh number.

Through reviewing the figures it is understood that with the addition of internal heat generation there is no significant difference in the isotherms when $Ra_E \leq Ra_I$. However, when $Ra_E > Ra_I$, the cellular flow patterns begins to fragment. For the figures containing streamlines when $Ra_E < Ra_I$, a negligible effect on the enclosure is shown. When $Ra_E = Ra_I$, there is little difference between it and the situation without internal heat generation. When the $Ra_E > Ra_I$, it causes larger cellular formation including the formulation of smaller cells in the corners of the enclosures.
Figure 4.65 Isotherms at constant $Ra_I = 2 \times 10^5$ and three different values of $Ra_E$ with aspect ratio 1 for class one configuration
Figure 4.66 Isotherms at constant $Ra_I = 2 \times 10^5$ and three different values of $Ra_E$ with aspect ratio 2 for class one configuration.
Figure 4.67 Isotherms at constant $Ra_I = 2 \times 10^5$ and three different values of $Ra_E$ with aspect ratio 4 for class one configuration
Figure 4.68 Isotherms at constant $Ra_I = 2 \times 10^5$ and three different values of $Ra_E$ with aspect ratio 6 for class one configuration
Figure 4.69 Isotherms at constant $Ra_I = 2 \times 10^5$ and three different values of $Ra_E$ with aspect ratio 8 for class one configuration.
Figure 4.70 Isotherms at constant $Ra_t = 2 \times 10^5$ and three different values of $Ra_E$ with aspect ratio 10 for class one configuration
Figure 4.71 Streamlines at constant $Ra_I = 2 \times 10^5$ and three different values of $Ra_E$ with aspect ratio 1 for class one configuration.
Figure 4.72 Streamlines at constant $Ra_I = 2 \times 10^5$ and three different values of $Ra_E$ with aspect ratio 2 for class one configuration.
Figure 4.73 Streamlines at constant \( Ra_I = 2 \times 10^5 \) and three different values of \( Ra_E \) with aspect ratio 4 for class one configuration.
Figure 4.74 Streamlines at constant $Ra_I = 2 \times 10^5$ and three different values of $Ra_E$ with aspect ratio 6 for class one configuration.
Figure 4.75 Streamlines at constant $Ra_1 = 2 \times 10^5$ and three different values of $Ra_E$ with aspect ratio 8 for class one configuration.
Figure 4.76 Streamlines at constant \( Ra_I = 2 \times 10^5 \) and three different values of \( Ra_E \) with aspect ratio 10 for class one configuration.
4.5.3 Class two configuration

The isotherms are shown in Figures 4.77- 4.82. They are arranged in order by increasing aspect ratio and also by increasing external Rayleigh number. The streamlines are shown in Figure 4.83- 4.88 and are shown in order by increasing aspect ratio and external Rayleigh number. There was no significant visual difference between the cases with internal heat generation and those without internal heat generation other than a small change in the streamlines. There was however a change in the heat transfer associated with the configuration. This will be discussed in later sections.
Figure 4.77 Isotherms at constant $Ra_I = 2 \times 10^5$ and three different values of $Ra_E$ with aspect ratio 1 for class two configuration
Figure 4.78 Isotherms at constant $Ra_I = 2 \times 10^5$ and three different values of $Ra_E$ with aspect ratio 2 for class two configuration
Figure 4.79 Isotherms at constant $Ra_I = 2 \times 10^5$ and three different values of $Ra_E$ with aspect ratio 4 for class two configuration
Figure 4.80 Isotherms at constant $Ra_I = 2 \times 10^5$ and three different values of $Ra_E$ with aspect ratio 6 for class two configuration
Figure 4.81 Isotherms at constant $Ra_t = 2 \times 10^5$ and three different values of $Ra_E$ with aspect ratio 8 for class two configuration
Figure 4.82 Isotherms at constant $Ra_I = 2 \times 10^5$ and three different values of $Ra_E$ with aspect ratio 10 for class two configuration
Figure 4.83 Streamlines at constant \(Ra_1 = 2 \times 10^5\) and three different values of \(Ra_E\) with aspect ratio 1 for class two configuration
Figure 4.84 Streamlines at constant \( Ra_I = 2 \times 10^5 \) and three different values of \( Ra_E \) with aspect ratio 2 for class two configuration.
Figure 4.85 Streamlines at constant $Ra_1 = 2 \times 10^5$ and three different values of $Ra_E$ with aspect ratio 4 for class two configuration
Figure 4.86 Streamlines at constant $Ra_1 = 2 \times 10^5$ and three different values of $Ra_E$ with aspect ratio 6 for class two configuration
Figure 4.87 Streamlines at constant $Ra_I = 2 \times 10^5$ and three different values of $Ra_E$ with aspect ratio 8 for class two configuration
Figure 4.88 Streamlines at constant $Ra_I = 2 \times 10^5$ and three different values of $Ra_E$ with aspect ratio 10 for class two configuration.
4.5.4 Average Heat Flux Ratio Variation with Aspect Ratio

The graphs below represent the change in the average heat flux ratio along hot and cold walls as the aspect ratio increases (Figures 4.89 – 4.93). The effect of the constant internal heat generation and varying external temperature gradient on natural convection can also be seen.

4.5.4.1 Class one configuration

Figures 4.89 – 4.91 show the average heat flux ratio through both the hot and cold walls variation with aspect ratio for class one configuration with a constant $Ra_I = 2 \times 10^5$ and three different values of $Ra_E = 2 \times 10^4$, $2 \times 10^5$, and $2 \times 10^6$ respectively. When $Ra_E < Ra_I$, more transfer occurred at the cold wall than at the hot wall as shown in figure 4.89. When $Ra_E = Ra_I$, there is more heat flux ratio at the hot wall than the cold wall as shown in figure 4.90. For the situation when $Ra_E > Ra_I$, the values of average heat flux ratio were inconsistent, meaning that sometimes there was more heat transfer at the hot wall and other times there was more at the cold wall as shown in Figure 4.91.
Figure 4.89 Average heat flux ratio along hot and cold wall variation with aspect ratio at $Ra_I = 2 \times 10^5$ and $Ra_E = 2 \times 10^4$ for class one configuration.

Figure 4.90 Average heat flux ratio along hot and cold wall variation with aspect ratio at $Ra_I = 2 \times 10^5$ and $Ra_E = 2 \times 10^5$ for class one configuration.
4.5.4.2 Class two configuration

For the class two configuration the transfer at the two walls follow similar trends as shown in Figures 4.92 – 4.94. This configuration was similar to class one configuration when \( R_a_E < R_a_I \), because there was more heat transfer at the cold wall than the hot wall. However, when \( R_a_E = R_a_I \), there was more heat transfer at the hot wall. It can also be seen in Figure 4.94 that the heat transfer through both walls became similar when \( R_a_E > R_a_I \).
Figure 4.92 Average heat flux ratio along hot and cold wall variation with aspect ratio at $Ra_I = 2 \times 10^5$ and $Ra_E = 2 \times 10^4$ for class two configuration

Figure 4.93 Average heat flux ratio along hot and cold wall variation with aspect ratio at $Ra_I = 2 \times 10^5$ and $Ra_E = 2 \times 10^5$ for class two configuration
4.5.5 Convection Strength

The convection strength in the class one configuration increased with the increase of $Ra_E$ and constant $Ra_I$ (Figure 4.95). The aspect ratio has a very little effect on convection strength. This is very different in the class two configuration where the convection strength increased with both the change in $Ra_E$ as well as the change in aspect ratio with constant $Ra_I$ (Figure 4.96).
Figure 4.95 Convection strength verses aspect ratio at $Ra_I = 2 \times 10^5$ and $Ra_E = 2 \times 10^4$, $2 \times 10^5$, and $2 \times 10^6$ for class one configuration

Figure 4.96 Convection strength verses aspect ratio at $Ra_I = 2 \times 10^5$ and $Ra_E = 2 \times 10^4$, $2 \times 10^5$, and $2 \times 10^6$ for class two configuration
CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

5.1 Introduction

The flow and heat transfer for two-dimensional natural convection in a rectangular enclosure with an external temperature gradient containing an internal heat source have been solved by a finite-difference procedure developed by Rahman et al. (2003). Fluid flow and heat transfer in rectangular enclosures of five different aspect ratios were analyzed in two configurations, 0° and 90°. Each aspect ratio was tested under three different conditions. The first conditions has a changing external temperature gradient only, the second has a constant external temperature gradient and changing internal heat source, and the third has a constant internal heat source with a changing external temperature gradient. The value of the varying Rayleigh numbers used in the experiment are $Ra = 2 \times 10^4$, $2 \times 10^5$, $2 \times 10^6$ and the constant Rayleigh number used was $Ra = 2 \times 10^5$. The isotherm, streamlines, average heat flux ratio, and convection strength was observed for each situation.

5.2 Conclusions

From the study, analysis and results of this research work, the following conclusions can be made:

For the class one configuration (horizontal enclosures heated from below) where the external temperature gradient was varied without an internal heat source, the isotherm for aspect ratio 1 exhibited a single rotational flow pattern as the external temperature gradient increased. As the external Rayleigh number ($Ra_E$) increased, the isotherms began to break up. The streamlines for the class one configurations represented a recirculating pattern that grew as the external
temperature gradient increased. As the aspect ratio increased the convection and the number of cellular rolls also increased, however the convective dominance decreased as the external temperature gradient increased. The average heat flux ratio corresponded to each aspect ratio; as aspect ratio increased the magnitude of the heat flux ratio also increased. In the class two configuration (vertical enclosures heated from the side), the isotherms became more parallel as the external Rayleigh number ($Ra_E$) increased across all aspect ratios, respectively. In the class two configuration, as the aspect ratio increased, the average heat flux ratio along hot and cold walls decreased. Corresponding to each aspect ratio, the magnitude of heat flux ratio increased as the external Rayleigh number ($Ra_E$) increased. So, the aspect ratio has a significant effect in fluid flow and temperature field in horizontal enclosures heated from the below and vertical enclosures heated from the side. The flow patterns and isotherms does not show any significant difference between the cases without internal heat generation other that slight shift and changes in stream function and isotherm values. In the class one configuration, the convection strength increases as the external Rayleigh number ($Ra_E$) increases. In the class two configuration, the convection strength increases as both the external Rayleigh number ($Ra_E$) and aspect ratio increases.

When the external Rayleigh number is kept constant at $2 \times 10^5$ and the internal Rayleigh number ($Ra_I$) is varied in the class one configuration there was no significant change in the isotherms and streamlines patterns can be observed when compared to the previous no heat source condition cases with the exception of when the cellular flow begins at a lower value of $Ra_I$ than the value of $Ra_E$. In the class two configuration, there was no significant visual change between the cases with internal heat generation and those without internal heat generation. The patterns in class one and class two configurations of heat flux ratio variation with aspect ratio are
similar for both the hot and cold walls; however the magnitude of heat flux ratio is slightly greater when \(Ra_I < Ra_E\). When \(Ra_I = Ra_E\), the average heat flux ratio decreases with the increase in aspect ratio for hot and cold walls in both class one and two configurations. When the \(Ra_I > Ra_E\), a completely different pattern forms in both the class one and class two configurations. In both class one and class two configurations the average heat flux ratio in the cold wall is much higher than in the hot wall. However, in the class one configuration the heat flux ratio decreases as the aspect ratio increases. In the class two configuration the heat flux ratio increases as the aspect ratio increases. In the class one configuration when \(Ra_I = 2 \times 10^4\) and \(2 \times 10^5\) the convection strength is almost identical, however when \(Ra_I > Ra_E\) the convection strength is less. The data for the class two configuration is similar to that of class one.

The last set of data was observed when the external Rayleigh number (\(Ra_E\)) was varied and the internal Rayleigh number (\(Ra_I\)) was kept constant at \(2 \times 10^5\). In the class one configuration when \(Ra_E \leq Ra_I\), no significant difference in the isotherms was observed when compared to previous result. When \(Ra_E > Ra_I\), it causes larger cellular formation including the formulation of smaller cells in the corners of the enclosure. Also, \(Ra_E = Ra_I\), more heat transfer occurred at the hot wall than at the cold wall. When \(Ra_E > Ra_I\), the average heat flux ratio values were inconsistent. The class two configuration exhibited similar results to the class one configuration. As seen in configuration one and in configuration two when \(Ra_E < Ra_I\), there was more heat transfer at the cold wall than in the hot wall. Similarly, when \(Ra_E = Ra_I\), there was more heat transfer at the hot wall than the cold wall. In both class one and class two configurations, as the Rayleigh number increased the convection strength increased. However in class one no increase was seen when the aspect ratio was increased. In class two, an increase was observed when the aspect ratio was increased.
5.3 Recommendations

The present study was conducted in a two-dimensional configuration using the same group of varying Rayleigh numbers for aspect ratios 1, 2, 4, 6, 8, and 10. Further study should include a larger difference in the chosen Rayleigh numbers. A better understanding of this subject matter could be observed if conducted in a three-dimensional configuration. Also, varying the characteristics of the fluid contained in the enclosure to more closely simulate that of fluids that can be utilized in the future, i.e. use of nanofluids.
REFERENCES

Absolute Astronomy. October 11, 2011.


APPENDIX A

Input Files

'Lid-driven cavity flow at Re = 1000, single-grid

LREAD,LWRITE,LTEST,LOUTS,LOUTE,LTIME

10000 5 5 2 2 1.e-3 1.e10 0.92 MAXIT,IMON,JMON,IPR,JPR,SORMAX,SLARGE,ALFA

0. 9.8 0.001 50.15 50. 50.08 GRAVX,GRAVY,BETA,TH,TC,TREF

1. 1. 0.7 DENSIT,VISC,PRM

1. 0 0. 2.e4 aratio,theta,rai,rae

0. 0. 0. 0 1.0 1.e0 UIN,VIN,PIN,TIN,ULID,TPER

1 1 1.E20 1. ITST,NPRT,DT,GAMT

t t t t (LCAL(I),I=1,NPHI)

0.7 0.7 0.5 0.7 (URF(I),I=1,NPHI)

0.2 0.2 0.2 0.2 (SOR(I),I=1,NPHI)

1 1 6 1 (NSW(I),I=1,NPHI)

1. 1. 1. 1. (GDS(I),I=1,NPHI)

rcf NAME OF FILE FOR UNSTEADY RESULTS

Figure A-1: Input File for Configuration One, Aspect Ratio 1 Rae=2×10^4
`Lid-driven cavity flow at Re = 1000, single-grid

f t f t f t f LREAD,LWRITE,LTEST,LOUTS,LOUTE,LTIME

10000 5 5 2 2 1.e-3 1.e10 0.92 MAXIT,IMON,JMON,IPR,JPR,SORMAX,SLARGE,ALFA

0. 9.8 0.001 50.15 50. 50.08 GRAVX,GRAVY,BETA,TH,TC,TREF

1. 1. 0.7 DENSIT,VISC,PRM

1. 1.5708 0. 2.e4 aratio,theta,rai,rae

0. 0. 0.0 1.0 1.e0 UIN,VIN,PIN,TIN,ULID,TPER

1 1 1.E20 1. ITST,NPRT,DT,GAMT

  t t t t (LCAL(I),I=1,NPHI)

  0.7 0.7 0.5 0.7 (URF(I),I=1,NPHI)

  0.2 0.2 0.2 0.2 (SOR(I),I=1,NPHI)

1 1 6 1 (NSW(I),I=1,NPHI)

1. 1. 1. 1 (GDS(I),I=1,NPHI)

rct NAME OF FILE FOR UNSTEADY RESULTS

Figure A-2: Input File for Configuration Two, Aspect Ratio 1 Rae=2×10⁴
Lid-driven cavity flow at Re = 1000, single-grid

```
LREAD,LWRITE,LTEST,LOUTS,LOUTE,LTIME
10000 5 5 2 2 1.e-3 1.e10 0.92 MAXIT,IMON,IPR,JPR,SORMAX,SLARGE,ALFA
0.9 8 0.001 50.15 50. 50.08 GRAVX,GRAVY,BETA,TH,TC,TREF
1. 1. 0.7 DENSIT,VISC,PRM
1. 0 2.e5 2.e4 aratio,theta,rai,rae
0. 0. 0. 0.0 1.0 1.e0 UIN,VIN,PIN,TIN,ULID,TPER
1 1 1.E20 1. ITST,NPRT,DT,GAMT
```

```
t t t t (LCAL(I),I=1,NPHI)
0.7 0.7 0.5 0.7 (URF(I),I=1,NPHI)
0.2 0.2 0.2 0.2 (SOR(I),I=1,NPHI)
1 1 6 1 (NSW(I),I=1,NPHI)
1. 1. 1. 1. (GDS(I),I=1,NPHI)
rct NAME OF FILE FOR UNSTEADY RESULTS
```

Figure A-3: Input File for Configuration One, Aspect Ratio 1 Ra_c=2×10^4 and Ra_l=2×10^5
Lid-driven cavity flow at Re = 1000, single-grid

LREAD, LWRITE, LTEST, LOUTS, LOUTE, LTIME

10000 5 5 2 2 1.e-3 1.e10 0.92 MAXIT, IMON, JMON, IPR, JPR, SORMAX, SLARGE, ALFA

0.9 8 0.001 50.15 50. 50.08 GRAVX, GRAVY, BETA, TH, TC, TREF

1. 1. 0.7 DENSIT, VISC, PRM

1. 1.5708 2.e5 2.e4 aratio, theta, rai, rae

0. 0. 0. 0.0 1.0 1.e0 UIN, VIN, PIN, TIN, ULD, TPER

1 1 1.E20 1. ITST, NPRT, DT, GAMT

t t t t (LCAL(I), I=1, NPHI)

0.7 0.7 0.5 0.7 (URF(I), I=1, NPHI)

0.2 0.2 0.2 0.2 (SOR(I), I=1, NPHI)

1 1 6 1 (NSW(I), I=1, NPHI)

1. 1. 1. 1. (GDS(I), I=1, NPHI)

rcr NAME OF FILE FOR UNSTEADY RESULTS

Figure A-4: Input File for Configuration Two, Aspect Ratio 1 Ra_e=2\times10^4 and Ra_i=2\times10^5
'Lid-driven cavity flow at Re = 1000, single-grid

LREAD,LWRITE,LTEST,LOUTS,LOUTE,LTIME

10000 5 5 2 2 1.e-3 1.e10 0.92 MAXIT,IMON,JMON,IPR,JPR,SORMAX,SLARGE,ALFA

0.9.8 0.001 50.15 50. 50.08 GRAVX,GRAVY,BETA,TH,TC,TREF

1. 1. 0.7 DENSIT,VISC,PRM

1. 0 2.e4 2.e5 aratio,theta,rai,rae

0. 0. 0. 0.0 1.0 1.e0 UIN,VIN,PIN,TIN,ULID,TPER

1 1 1.E20 1. ITST,NPRT,DT,GAMT

0.7 0.7 0.5 0.7 0.2 0.2 0.2 0.2

(UFR(I),I=1,NPHI)

(NSW(I),I=1,NPHI)

(GDS(I),I=1,NPHI)

rct NAME OF FILE FOR UNSTEADY RESULTS

Figure A-5: Input File for Configuration One, Aspect Ratio 1 $Ra_e=2\times10^5$ and $Ra_i=2\times10^4$
Lid-driven cavity flow at Re = 1000, single-grid

LREAD,LWRITE,LTEST,LOUTS,LOUTE,LTIME

10000 5 5 2 2 1.e-3 1.e10 0.92  MAXIT,IMON,IPR,JPR,SORMAX,SLARGE,ALFA

0. 9.8 0.001 50.15 50. 50.08  GRAVX,GRAVY,BETA,TH,TC,TREF

1. 1. 0.7  DENSIT,VISC,PRM

1. 1.5708 2.e4 2.e5  aratio,theta,rai,rae

0. 0. 0.0 1.0 1.e0  UIN,VIN,PIN,TIN,ULID,TPER

1 1.1.E20 1.  ITST,NPRT,DT,GAMT

t t t t  (LCAL(I),I=1,NPHI)

0.7 0.7 0.5 0.7  (URF(I),I=1,NPHI)

0.2 0.2 0.2 0.2  (SOR(I),I=1,NPHI)

1 1 6 1  (NSW(I),I=1,NPHI)

1. 1. 1. 1.  (GDS(I),I=1,NPHI)

rct  NAME OF FILE FOR UNSTEADY RESULTS

Figure A-6: Input File for Configuration Two, Aspect Ratio 1 Ra_e=2×10^5 and Ra_i=2×10^4
Appendix B

Output Data

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### Table B-2: Class Two Configuration with External Rayleigh Number Only

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<th>Average Heat Flux Cold Wall</th>
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### Table B-3: Class One Configuration with Constant External Rayleigh Number and Varying Internal Rayleigh Number

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<th>Convection Strength</th>
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</tr>
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### Table B-4: Class Two Configuration with Constant External Rayleigh Number and Varying Internal Rayleigh Number

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<th>Average Heat Flux Hot Wall</th>
<th>Average Heat Flux Cold Wall</th>
<th>Convection Strength</th>
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<th>External Internal</th>
<th>Average Heat Flux Hot Wall</th>
<th>Average Heat Flux Cold Wall</th>
<th>Convection Strength</th>
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### Table B-5: Class One Configuration with Varying External Rayleigh Number and Constant Internal Rayleigh Number

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<th>Average Heat Flux Hot Wall</th>
<th>Average Heat Flux Cold Wall</th>
<th>Convection Strength</th>
<th>AR</th>
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<th>External Rayleigh Number</th>
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<th>Convection Strength</th>
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Table B-6: Class Two Configuration with Varying External Rayleigh Number and Constant Internal Rayleigh Number

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<th>Convection Strength</th>
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<th>External</th>
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<th>Convection Strength</th>
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