Modeling the Stock Market Through Game Theory

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MODELING THE STOCK MARKET THROUGH GAME THEORY

An Honors Thesis submitted in partial fulfillment of the requirements for Honors in Mathematical Science.

by

KYLIE HANNAFEY

Under the mentorship of Hua Wang

ABSTRACT

Game Theory is used on many occasions to help us understand interactions between decision-makers. The famous Nash equilibrium is a steady state in a model that shows the interaction of different players, in which no player can do better by choosing a different action if the actions of the other players do not change. These two concepts can be applied to numerous situations that vary in types of players, but for our research, we are focusing on businesses in the stock market. The main objective is to use Game Theory to analyze data collected from the stock market, model our findings, predict decisions made by businesses, and understand what scenarios will produce a stable stock market. In particular, we will provide a thorough analysis of the stock market behavior between the three leading competitors in technology. We will first use statistical models to analyze and report data collected on Apple, Microsoft, and Google from the stock market. To begin, we will use the website Nasdaq, a detailed online record of the stock market, to record the daily price of stock and share volume for each company. Then, we will use Excel to statistically analyze and model our findings. As a result, we should be able to evaluate the stability of the stock market and discuss the relationships between our companies. More specifically, we expect to find the impact changes in the stock market have on each business and predict the behavior, or the “best next move,” they should have.

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Honors Director: __________________________________________________________________ Dr. Steven Engel

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<td>25</td>
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<td>25</td>
</tr>
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<td>MSFT Stock Price and Volume Percentage</td>
<td>26</td>
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CHAPTER 1
INTRODUCTION

The majority of the theoretical background for this thesis can be found in [5].

1.1 Financial stability

Financial stability can be defined as such: “A financial system is in a range of stability whenever it is capable of facilitating (rather than impeding) the performance of an economy, and of dissipating financial imbalances that arise endogenously or as a result of significant adverse and unanticipated events” [8]. Now let us unpack that definition. First of all, “the stock market is an institution that connects potential buyers and sellers of companies’ stocks” [3]. The stock market is essentially a business. Using our definition of financial stability, a business is considered financially stable if it is efficient with its resources, profiting, making smart investments, and increasing in wealth. Essentially, a business must be successful to improve the performance of an economy. In addition, the system must be able to handle financial imbalances that occur due to internal issues or as a result of external, unanticipated events. For a business in the stock market, this means if they made poor investment choices or another business they have competition with does something, they must be able to make the best next move to overcome the obstacle and stay in the game. Game Theory strategy will help businesses in the stock market achieve financial stability. Players in the game are making decisions that are in their best interest while also taking into account what the other players are possibly doing. A player wants to succeed, but also wants to survive, which is essentially what our definition of financial stability implies.
1.2 Game Theory and Nash Equilibrium

1.2.1 Background

In the 1920s, Emile Borel and John Von Neumann developed Game Theory, which aims to help us understand interactions between decision-makers [5]. It is formally defined as the study of mathematical models of strategic interactions among rational decision-makers. Von Neumann described two types of games: “In the first type, rule-based games, players interact according to specified ‘rules of engagement’. In the second type, freewheeling games, players interact without any external constraints” [2]. Business, specifically the stock market, is a combination of these two types of games.

Game theory is very versatile and can be applied to a variety of situations. According to Franklin Allen, “game theory has provided a methodology that has led to insights into many previously unexplained phenomena by allowing asymmetric information and strategic information to be incorporated in the analysis” [1]. The most common uses of game theory include economic theory, political science, and psychology.

An important feature of game theory is the Nash equilibrium. The Nash equilibrium was developed by the famous game theorist John Nash. He showed that “in any finite game (i.e., a game in which the number of players $n$ and the strategy sets $S_1, \ldots, S$ are all finite) there exists at least one Nash equilibrium” [4]. A Nash equilibrium is a steady state in a model in which no player can do better by choosing a different action if the actions of the other players do not change [5]. Essentially, a Nash equilibrium is the best response a player can make to any of the other players’ actions. There are also cases of multiple Nash equilibria: “McLennan shows that standard normal-form games can have enormous numbers of equilibria” [7]. An example of such a case is shown below in A Second Example.
1.2.2 Formal definitions

**Definition 1.** Game Theory: The study of mathematical models of strategic interactions among rational decision-makers.

**Definition 2.** Payoff Function: It essentially ranks each action according to its preferability. It is another way to represent the preferences of players.

**Definition 3.** Best Response Function: The best action a player can make given the other player’s action. It is the action that will yield player A the highest payoff value given player B’s action.

**Definition 4.** Nash Equilibrium: A steady state in a model in which no player can do better by choosing a different action if the actions of the other players do not change.

1.2.3 A First Example

The best game theory example is the Prisoner’s Dilemma. In this problem, there are two prisoners and they each have two choices: stay quiet or confess. If they both stay quiet, then they each get one year of jail. If one stays quiet, but the other confesses, then the person who confessed gets 0 years and the person who stayed quiet gets 4 years and vice-versa. Finally, if they both confess, then they both get 3 years of jail time. This is modeled in the table below.

<table>
<thead>
<tr>
<th>Prisoner 1</th>
<th>Quiet</th>
<th>Confess</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quiet</td>
<td>(1, 1)</td>
<td>(4, 0)</td>
</tr>
<tr>
<td>Confess</td>
<td>(0, 4)</td>
<td>(3, 3)</td>
</tr>
</tbody>
</table>

The number of years spent in jail can be represented using payoff values. Payoff values essentially rank each action according to its preferability. In this case, each action
will be ranked from 0-3, with 3 meaning it is the most preferred action. So, four years in jail would have a payoff value of 0. Three years in jail would have a payoff value of 1. One year would have a payoff value of 2. Finally, zero years in jail would have a payoff value of 3. This can be modeled in the table below. We also can mark a player’s best response to each action of the other player using “∗”. The box that ends up with two “∗” is the Nash equilibrium.

<table>
<thead>
<tr>
<th></th>
<th>Quiet</th>
<th>Confess</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quiet</td>
<td>(2, 2)</td>
<td>(0, 3∗)</td>
</tr>
<tr>
<td>Confess</td>
<td>(3∗, 0)</td>
<td>(1∗, 1∗)</td>
</tr>
</tbody>
</table>

From looking at this table, one can see that the Nash equilibrium is (Confess, Confess). This is because, given that Prisoner 2 chooses Confess, Prisoner 1 is better off choosing Confess rather than Quiet (looking at the first entries in the right column, Quiet yields Prisoner 1 and payoff value of 0 and Confess yields them a payoff value of 1), and vice versa. Given that Prisoner 1 chooses Confess, Prisoner 2 is better off choosing Confess rather than Quiet (looking at the second entries in the second row, Quiet yields Prisoner 2 a payoff value of 0 and Confess yields them a payoff value of 1).

### 1.2.4 A Second Example

Another great game theory example is the Battle of the Sexes. In this problem, there is a man and woman trying to decide where to go out for the night, either to a football game or a ballet performance. They would both rather spend the night together than apart. However, the woman prefers they both go to the ballet performance, but the man prefers they both go to the football game [6]. Their preferences can be modeled below.

<table>
<thead>
<tr>
<th></th>
<th>Football</th>
<th>Ballet</th>
</tr>
</thead>
<tbody>
<tr>
<td>Football</td>
<td>(2∗, 1∗)</td>
<td>(0, 0)</td>
</tr>
<tr>
<td>Ballet</td>
<td>(0, 0)</td>
<td>(1∗, 2∗)</td>
</tr>
</tbody>
</table>
In this problem, (Football, Football) and (Ballet, Ballet) are Nash equilibria [6].

1.3 COURNOT AND BERTRAND GAME THEORY MODELS

1.3.1 COURNOT’S MODEL

In general, there is a single good being produced by \( n \) firms. The cost for firm \( i \) to produce \( q_i \) units of the good is \( C_i(q_i) \), where \( C_i \) is increasing. Also, the firms’ total output is \( Q \) and the market price is \( P(Q) \) (\( P \) is a decreasing function when positive) [5].

Cournot’s oligopoly game consists of firms as the players and the actions for each firm are its possible outputs. The preferences of each firm are represented by its profit (1.1) [5].

\[
\pi_i(q_1, \ldots, q_n) = q_iP(q_1 + \ldots + q_n) - C_i(q_i). \tag{1.1}
\]

Example 1.1. We can look at an example from [5]. Suppose there are two firms with the same cost function: \( C_i(q_i) = cq_i \) for all \( q_i \). The inverse demand function is given by (1.2).

\[
P(Q) = \begin{cases} 
\alpha - Q & \text{if } Q \leq \alpha \\
0 & \text{if } Q > \alpha 
\end{cases} \tag{1.2}
\]

where \( \alpha > 0 \) and \( c \geq 0 \) are constants. We can see the graph of the inverse demand function in Figure 1.1.
In general, there is a single good being produced by $n$ firms. The cost for firm $i$ to produce $q_i$ units of the good is $C_i(q_i)$, where $C_i$ is increasing. Also, $p$ is the price and demand is $D(p)$. If all the firms set different prices, then consumers will buy the goods at the lowest price. Firms also only produce what is demanded [5].

Bertrand’s oligopoly game consists of firms as the players and the actions for each firm are the possible prices. The preferences of each firm are represented by its profit (1.3) [5].

$$\pi_i(p_1, p_2) = \begin{cases} 
    p_1D(p_1) - C_1(D(p_1)) & \text{if } p_1 < p_2 \\
    1/2p_1D(p_1) - C_1(1/2D(p_1)) & \text{if } p_1 = p_2 \\
    0 & \text{if } p_1 > p_2 
\end{cases} \quad (1.3)$$

**Example 1.2.** We can look at an example from [5]. Suppose there are two firms with the same cost function: $C_i(q_i) = cq_i$ for $i = 1, 2$. The demand function is $D(p) = \alpha - p$ for $p \leq \alpha$ and $D(p) = 0$ for $p > \alpha$, and $c < \alpha$. To find the Nash equilibria of the game, we must find the firms’ best response functions. Following [5], “we can look at firm i’s payoff
as a function of its price \( p_i \) for various values of the price \( p_j \) of firm \( j \).” One can see the graphs of each payoff function in Figure 63.1 in [5].
CHAPTER 2
DATA COLLECTION AND ANALYSIS

2.1 INTRODUCTION

Our data will be collected from an online stock market database called Nasdaq and we will record the stock price for each business, and the share volume as well, to obtain payoff values.

Stock price is the price at which each stock is sold. We chose to record it because we believe it will give us a great deal of information as to how the company is doing on a surface level. Share volume is the number of shares traded in a given time period. We chose to record it because share volume will give us more insight into how the company is doing on a deeper level. Share volume measures the effort behind movement in stock prices. It shows how many investors, stockholders, etc. were involved in that move.

2.2 COLLECTION

Here, we show off some exemplary data of the stock prices and volume of shares of Apple, Microsoft, and Google, over the month of March 2020, in Tables 2.1, 2.2, 2.3.

<table>
<thead>
<tr>
<th>Date</th>
<th>Stock Price</th>
<th>Vol of Shares</th>
<th>Variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>3/5</td>
<td>292.92</td>
<td>46,893,220</td>
<td>8.14</td>
</tr>
<tr>
<td>3/10</td>
<td>285.34</td>
<td>71,322,520</td>
<td>17.07</td>
</tr>
<tr>
<td>3/16</td>
<td>242.21</td>
<td>80,605,870</td>
<td>19.08</td>
</tr>
<tr>
<td>3/20</td>
<td>229.24</td>
<td>100,423,300</td>
<td>23.83</td>
</tr>
<tr>
<td>3/25</td>
<td>245.52</td>
<td>75,900,510</td>
<td>13.95</td>
</tr>
<tr>
<td>3/30</td>
<td>254.81</td>
<td>41,994,110</td>
<td>6.12</td>
</tr>
</tbody>
</table>

Table 2.1: Apple
<table>
<thead>
<tr>
<th>Data type\Date</th>
<th>3/5</th>
<th>3/10</th>
<th>3/16</th>
<th>3/20</th>
<th>3/25</th>
<th>3/30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock Price</td>
<td>166.27</td>
<td>160.92</td>
<td>135.42</td>
<td>137.35</td>
<td>146.92</td>
<td>160.23</td>
</tr>
<tr>
<td>Volume of Shares</td>
<td>47,817,250</td>
<td>65,354,390</td>
<td>87,905,870</td>
<td>84,866,220</td>
<td>75,638,220</td>
<td>63,420,330</td>
</tr>
<tr>
<td>Variation</td>
<td>5.18</td>
<td>8.45</td>
<td>14.35</td>
<td>11.24</td>
<td>9.89</td>
<td>10.59</td>
</tr>
</tbody>
</table>

Table 2.2: Microsoft

<table>
<thead>
<tr>
<th>Data type\Date</th>
<th>3/5</th>
<th>3/10</th>
<th>3/16</th>
<th>3/20</th>
<th>3/25</th>
<th>3/30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stock Price</td>
<td>1319.04</td>
<td>1280.39</td>
<td>1084.33</td>
<td>1072.32</td>
<td>1102.49</td>
<td>1146.82</td>
</tr>
<tr>
<td>Volume of Shares</td>
<td>2,561,288</td>
<td>2,611,373</td>
<td>4,252,365</td>
<td>3,601,750</td>
<td>4,081,528</td>
<td>2,574,061</td>
</tr>
<tr>
<td>Variation</td>
<td>53.81</td>
<td>62.38</td>
<td>77.83</td>
<td>78.50</td>
<td>62.89</td>
<td>55.15</td>
</tr>
</tbody>
</table>

Table 2.3: Google

As an example of more detailed changes of the market, Table 2.4 shows the data for the same three stocks at an hourly rate on April 7 2020.

<table>
<thead>
<tr>
<th>Stock\Hour</th>
<th>8am</th>
<th>9am</th>
<th>10am</th>
<th>11am</th>
<th>12pm</th>
<th>1pm</th>
<th>2pm</th>
<th>3pm</th>
<th>4pm</th>
<th>5pm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Apple</td>
<td>268.30</td>
<td>270.10</td>
<td>269.43</td>
<td>264.25</td>
<td>265.96</td>
<td>266.04</td>
<td>263.13</td>
<td>260.17</td>
<td>259.43</td>
<td>258.45</td>
</tr>
<tr>
<td>MSFT</td>
<td>169.36</td>
<td>169.81</td>
<td>167.95</td>
<td>165.50</td>
<td>166.33</td>
<td>167.30</td>
<td>166.34</td>
<td>164.48</td>
<td>163.46</td>
<td>163.08</td>
</tr>
<tr>
<td>GOOGL</td>
<td>1219.09</td>
<td>1220.50</td>
<td>1214.18</td>
<td>1191.63</td>
<td>1198.98</td>
<td>1207.19</td>
<td>1202.69</td>
<td>1196.55</td>
<td>1186.51</td>
<td>1184.00</td>
</tr>
</tbody>
</table>

Table 2.4: Hourly Data
2.3 Normalization

To analyze the behavior of these different but related stocks, we take data similar to the above, but from previous years. As the goal is to observe the behavior of these three stocks and their impact upon each other, we often normalize our collected data.

Over the same period of time, we use \( p_A \), \( p_M \) and \( p_G \) to denote the vectors of prices over time. For example, from Tables 2.1, 2.2, 2.3 we have

\[
p_A = \langle 292.92, 285.34, 242.21, 229.24, 245.52, 254.81 \rangle,
\]

\[
p_M = \langle 166.27, 160.92, 135.42, 137.35, 146.92, 160.23 \rangle,
\]

and

\[
p_G = \langle 1319.04, 1280.39, 1084.33, 1072.32, 1102.49, 1146.82 \rangle.
\]

For such a “price vector”, \( p = \langle p_1, p_2, \ldots, p_n \rangle \), let

\[
\bar{p} = \frac{1}{n} \sum_{i=1}^{n} p_i / n
\]

and the “normalized price vector” be

\[
P = \left\langle \frac{p_1}{\bar{p}}, \frac{p_2}{\bar{p}}, \ldots, \frac{p_n}{\bar{p}} \right\rangle.
\]

Taking the above data as an example, we have

\[
P_A = \begin{pmatrix}
292.92 & 285.34 & 242.21 & 229.24 & 245.52 & 254.81 \\
258.34 & 258.34 & 258.34 & 258.34 & 258.34 & 258.34
\end{pmatrix}
\]

\[
= \langle 1.1339, 1.1045, 0.9376, 0.8874, 0.9504, 0.9863 \rangle.
\]

\[
P_M = \begin{pmatrix}
166.27 & 160.92 & 135.42 & 137.35 & 146.92 & 160.23 \\
151.19 & 151.19 & 151.19 & 151.19 & 151.19 & 151.19
\end{pmatrix}
\]

\[
= \langle 1.0997, 1.0644, 0.8957, 0.9085, 0.9718, 1.0598 \rangle.
\]
\[ P_G = \left\langle \frac{1319.04}{1167.57}, \frac{1280.39}{1167.57}, \frac{1084.33}{1167.57}, \frac{1072.32}{1167.57}, \frac{1102.49}{1167.57}, \frac{1146.82}{1167.57} \right\rangle = \langle 1.1297, 1.0966, 0.9287, 0.9184, 0.9443, 0.9822 \rangle. \]

As for the “volume vectors” \( v_A, v_M \) and \( v_G \), we let
\[
(V_\alpha) = (v_\alpha) / v
\]
for \( \alpha = A, M, G \) respectively, where
\[
v_i = (v_A)_i + (v_M)_i + (v_G)_i
\]
for \( i = 1, 2, \ldots, n \).

With Tables 2.1, 2.2, 2.3 we now have

\[ v_A = \langle 46, 893, 220, 71, 322, 520, 80, 605, 870, 100, 423, 300, 75, 900, 510, 41, 994, 110 \rangle, \]
\[ v_M = \langle 47, 817, 250, 65, 354, 390, 87, 905, 870, 84, 866, 220, 75, 638, 220, 63, 420, 330 \rangle, \]
\[ v_G = \langle 2, 561, 288, 2, 611, 373, 4, 252, 365, 3, 601, 750, 4, 081, 528, 2, 574, 061 \rangle. \]

And
\[
\bar{v} = \langle \bar{v}_1, \ldots, \bar{v}_n \rangle
\]
\[ = \langle 97271758, 139288283, 172764105, 188891270, 155620258, 107988501 \rangle \]

So,
\[ V_A = \langle 48.2085\%, 51.2050\%, 46.6566\%, 53.1646\%, 48.7729\%, 38.8876\% \rangle \]
\[ V_M = \langle 49.1584\%, 46.9202\%, 50.8820\%, 44.9286\%, 48.6044\%, 58.7288\% \rangle \]
\[ V_G = \langle 2.6331\%, 1.8748\%, 2.4614\%, 1.9068\%, 2.6227\%, 2.3836\% \rangle \]

Take \( f_\alpha \) for the variation data, define \( \bar{f} \) accordingly (similar to \( p \)) and let

\[ R_\alpha = \frac{\bar{f}}{\bar{p}} \]

for \( \alpha = A, M, G \). This is our “risk” factor. Again, from Tables 2.1, 2.2, 2.3, we have

\[ \bar{f} = \sum_{i=1}^{n} f_i/n \]

So,

\[ \bar{f}_A = 14.6983 \]
\[ \bar{f}_M = 9.95 \]
\[ \bar{f}_G = 65.0933 \]

This means our risk factors for \( \alpha = A, M, G \) are

\[ R_A = \frac{14.6983}{258.34} = 0.056895 \]
\[ R_M = \frac{9.95}{151.19} = 0.065811 \]
\[ R_G = \frac{65.0933}{1167.57} = 0.055751 \]

We may summarize our data in the following Table 2.5.
Table 2.5: Multi-row table

<table>
<thead>
<tr>
<th></th>
<th>$P_A$</th>
<th>1.1339</th>
<th>1.1045</th>
<th>0.9376</th>
<th>0.8874</th>
<th>0.9504</th>
<th>0.9863</th>
</tr>
</thead>
<tbody>
<tr>
<td>Apple</td>
<td>$V_A$</td>
<td>48.2085%</td>
<td>51.2050%</td>
<td>46.6566%</td>
<td>53.1646%</td>
<td>48.7729%</td>
<td>38.8876%</td>
</tr>
<tr>
<td></td>
<td>$R_A$</td>
<td>0.056895</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$P_M$</td>
<td>1.0997</td>
<td>1.0644</td>
<td>0.8957</td>
<td>0.9085</td>
<td>0.9718</td>
<td>1.0598</td>
</tr>
<tr>
<td>Microsoft</td>
<td>$V_M$</td>
<td>49.1584%</td>
<td>46.9202%</td>
<td>50.8820%</td>
<td>44.9286%</td>
<td>48.6044%</td>
<td>58.7288%</td>
</tr>
<tr>
<td></td>
<td>$R_M$</td>
<td>0.065811</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$P_G$</td>
<td>1.1297</td>
<td>1.0966</td>
<td>0.9287</td>
<td>0.9184</td>
<td>0.9443</td>
<td>0.9822</td>
</tr>
<tr>
<td>Google</td>
<td>$V_G$</td>
<td>2.6331%</td>
<td>1.8748%</td>
<td>2.4614%</td>
<td>1.9068%</td>
<td>2.6227%</td>
<td>2.3836%</td>
</tr>
<tr>
<td></td>
<td>$R_G$</td>
<td>0.055751</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
CHAPTER 3
DATA ANALYSIS FOR GAME THEORY MODEL

In this chapter we show how the practical data is handled.

3.1 PRICE DATA SETS

First, we recorded the average stock price, the daily high and low stock price, and share volume for Apple, Microsoft, and Google during the year 2019. From there, we graphed the average stock price over 2019 for each company (see Figures 3.1, 3.2, 3.3 below).

![AAPL Stock Price](image1)

**Figure 3.1: Apple (AAPL) Stock Price**

![MSFT Stock Price](image2)

**Figure 3.2: Microsoft (MSFT) Stock Price**
Since the general stock prices have always been fluctuating, this data should be “normalized” before they can more precisely reflect the changes on the market. For this purpose, we calculated the normalized stock prices for each company by taking the daily stock price and dividing it by the yearly average stock price. The graphs for each can be seen below in Figures 3.4, 3.5, 3.6.

Figure 3.3: Google (GOOG) Stock Price

Figure 3.4: AAPL Normalized Price
Figure 3.5: MSFT Normalized Price

Figure 3.6: GOOG Normalized Price
3.2 Volume Data Sets

Similarly, the volume of stocks has been constantly growing. Hence, we need to normalize our gathered data accordingly. We calculated Apple’s volume percentage by adding up Apple, Microsoft, and Google’s share volume for each day, and dividing Apple’s daily share volume by that sum. We used a similar method to also find Microsoft and Google’s volume percentages. The graphs for the companies’ volume percentage over 2019 can be found below.

![Figure 3.7: AAPL Volume Percentage](image)

![Figure 3.8: MSFT Volume Percentage](image)
Then, to see if there was any correlation, we graphed the stock price and volume percentage together, shown in Figures 3.10, 3.11, 3.12.
Figure 3.11: MSFT Stock Price and Volume Percentage

Figure 3.12: GOOG Stock Price and Volume Percentage
There was no discernible pattern in these figures, this is partly because of the ‘normalized’ volume data and the generally increasing prices. Therefore, we developed a different approach. We used Apple’s stock price data and graphed the 1/1/2019 point and 12/31/2019 point and made a straight line, corresponding to the underlying linear progression of the price. By taking the difference of the original (normalized) price data and this new “line data” (a new line with NEW stock prices labeled as “Apple Difference Prices”), a new data set for AAPL price is obtained. We combined that graph with the volume percentage graph. In doing so, there seemed to be more of a trend or pattern going on, as shown in Figure 3.13.

![Figure 3.13: AAPL Volume Percentage vs Difference](image)

We did the same process with Microsoft and Google’s data which one can see in Figures 3.14 and 3.15.
Figure 3.14: MSFT Volume Percentage vs Difference

Figure 3.15: GOOG Volume Percentage vs Difference
CHAPTER 4
FURTHER DIRECTIONS AND CONCLUDING REMARKS

While the Volume Percentage vs. Difference graphs did show more of a trend going on, we wanted a more concrete pattern. So, we took the first (discrete) “derivative”, called the *difference vector*, of our data. By this, we mean we subtracted the Day 2 difference by the Day 1 difference and graphed it against the volume percentage (Figures 4.1, 4.2, 4.3).

![AAPL First Difference Vector](image1)

**Figure 4.1: AAPL First Difference Vector**

![MSFT First Difference Vector](image2)

**Figure 4.2: MSFT First Difference Vector**
As you can see, the *first difference vector* either mirrors or correlates with the volume percentage.

To further investigate, we found the *third difference vector* of our data. Meaning, we subtracted the Day 4 difference by the Day 1 difference and graphed it against the volume percentage (Figures 4.4, 4.5, 4.6).

Figure 4.3: GOOG First Difference Vector

Figure 4.4: AAPL Third Difference Vector
Figure 4.5: MSFT Third Difference Vector

Figure 4.6: GOOG Third Difference Vector
Again, we found that the *third difference vector* either mirrors or correlates with the volume percentage.

In this thesis, we explored potential Game Theory models and data analysis approaches for studying the stock market behavior. For future studies, one may work on combining the two topics and find a game theory model that shows the Nash equilibrium of our collected data.
REFERENCES


