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We attempt to answer the problem "When approaching word problems, do students prefer certain real life application over others?" through two studies involving classroom research. Each study involved students in four college algebra sections in two two-year campuses. Students were presented with a selection of word problems covering a spectrum of application areas and asked to select three problems to solve. Problems in study one covered quadratic functions while those in study two covered exponential and logarithmic functions. The problems were categorized in to three categories. Category R represented problems to which the students can easily relate to while category I represented problems which had a certain level of intrigue. Category U represented problems which did not fall in to the R or I categories. The results conclusively showed that students significantly preferred problems in the I and R categories over those in the U category.

Keywords

College algebra, Word problems, Quadratic, Exponential

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Abstract

We attempt to answer the problem "When approaching word problems, do students prefer certain real life application over others?" through two studies involving classroom research. Each study involved students in four college algebra sections in two two-year campuses. Students were presented with a selection of word problems covering a spectrum of application areas and asked to select three problems to solve. Problems in study one covered quadratic functions while those in study two covered exponential and logarithmic functions. The problems were categorized in to three categories. Category *R* represented problems to which the students can easily relate to while category *I* represented problems which had a certain level of intrigue. Category *U* represented problems which did not fall in to the *R* or *I* categories. The results conclusively showed that students significantly preferred problems in the *I* and *R* categories over those in the *U* category.

Keywords: college algebra, word problems, quadratic, exponential

Introduction

A key objective in making college algebra a core requirement for most college curricula is to provide students an opportunity to see how mathematics can be used in a variety of ways to benefit their future careers as well as their everyday lives. Teachers of mathematics use word problems as the mechanism to bring these real life applications to the classroom. However, it is no secret that most students dislike word problems to an extent that their aversion has even reached popular cartoons, an example of which is the famous Gary Larson cartoon "Hell's Library" which shows a library full of word problem books amidst hellfire. Bruer [1994] describes word problems as the black hole in mathematical teaching -- immense energy is inserted into the process with very little light escaping. The fact that word problems, which play a huge role in justifying the teaching of college algebra, are creating such distaste is indeed sad. Many researchers in math education who have studied student difficulties encountered in the problem solving process have offered solutions to overcome them.

There are many reasons for student difficulties with word problems. Arzarello [1998] provides evidence to show that problems in reading comprehension and translating English words to mathematical symbols being two such. There are several findings on student difficulties in the actual problem solving process which refer to the pioneering contribution by Pólya[1957] to the mathematical problem solving process such as the findings by Marshell [1995], which show that struggles in devising plans of attack contribute significantly to student woes in this domain. He also showed that these difficulties are also due to the lack of exposure that students have to problem solving strategies and skills during their educational career. Cobb, Yackel and McClain [2000], too connect word problem difficulties to Pólya's four steps showing faulty implementation of the solution plan (due to mathematical deficiencies in students) as one reason. They also show that the lack of knowledge among students on control processes prevent students from developing the habit of looking back and validating their answers as being another reason for their word problem difficulties. Solutions to this crisis have also have been proposed. Jacobs and Ambrose [2008] propose a collection of interventions that teachers can use as a toolbox to pick and use at appropriate times during the word problem solving process of a student to remove certain bottlenecks. However, we feel that prior to implementing the problem solving strategies it is important that students attain at least some level of motivation to start attempting the problem without shying away from it.

In this article, we study this motivational aspect of the word problem. Why a statement from the teacher such as "Now we are going to do some problems" is usually greeted with a groan could be due to several reasons such as bad experiences that students have had with word problems in the past or stories and legends that they may have heard about how difficult word problems are.

Either way, this discouragement causes students to reject word problems even before attempting them or to attempt them with little motivation and/or confidence. This is indeed sad, as most of these students could, under proper guidance, master the problem solving process to a reasonable degree. Therefore we feel that it is important that educators try to find ways of motivating students to attempt word problems.

We attempted to answer the question: Given a choice, would students prefer word problems in certain real life application areas over others? We feel that an affirmative answer to this question could provide an instructor a window of opportunity to motivate students towards word problems by offering problems based on preferred application areas. We considered two application categories as candidates for possible preferred areas. The first category, which we denote by R, has applications to which students can easily relate to from their everyday lives. A good example of a category R problem would be the use of quadratic functions to find the speed which optimizes the gas mileage of a car. This calculation provides a possible solution to a crisis that almost anybody could encounter at some point if running out of gas while driving. Students can easily relate to this application of mathematics. Finding the time it takes for money to double is also such an example for an application in the R category, this time using exponential functions.

The other category, which we denote by I, consists of applications which have some intrigue or mystery. An example of a problem from this category would be to find the time of death from a murder scene using exponential functions. The "CSI" type nature of this problem would attach a level of intrigue to this problem. We denote by U problems which do not fall in to either one of these categories. An example of a problem which would fall in to the U category would be a classic quadratic function problem which asks the students to maximize the revenue from the sales of a given commodity. We conducted this study to test when given a choice of word problems, whether students would prefer to select problems from categories I and R over the category U.

Our main rationale for creating the preferred categories is to trigger situational interest among students. The novelty of the applications (Hidi, 1990), any suspense related to the applications (Jose and Brewer, 1984) and vividness (Garner et al., 1992) help create situational interest in students and these are some reasons for the creation of the *I* category. The justification for the *R* category comes from the fact that students do have some prior knowledge of applications that they can easily relate to and such prior knowledge is also a way of creating situational interest (Alexander and Jetton, 1996). As students usually do not expect such familiar situations to be mathematically modeled, a sense of unexpectedness is also attached to problems in this category further enhancing situational interest.

Our findings should help college teachers in two situations. Most teachers in Science and Social Science disciplines follow the practice of teaching a concept first and then showing its applications in the "real world." For example, a mathematics teacher would first teach students how to solve systems of equations and then show, as an application, how to spend a fixed amount of dollars buying a given total of apples and oranges. A physics teacher would teach the theory of electricity and give applications in lightning protection. As an introductory application, we feel that it is important to present example problems which generate student motivation and interest so that students will look forward to other applications and examples or at the least will remember the application for a reasonable length of time. Our findings will provide guidelines to help the teacher in selecting such motivating examples. At the end of a section it is customary to give a set of exercises for the students to work themselves. It is important to create this exercise set in a manner which will maintain the student's motivation and interest. Our findings will also help the teacher to create such exercises. In this paper we present findings of two studies to check student preference for problems in the two categories described above.

Method

The studies were carried out on four college algebra sections in two liberal arts colleges in the Midwest. The activities involved in the study were part of the regular classroom activities. Students in all the sections gave their consent prior to participating in the study. However students did not have knowledge of the study prior to registering in the particular section of the course. So our samples while being convenience samples are representative of the population of students taking college algebra in the region. Study 1 concerns student preference in application areas surrounding the topic of quadratic functions and study 2 concerns student preference in exponential and logarithmic function applications. A total of 130 students took part in study 1 and a total of 103 students took part in study 2. In both studies, students were given a number of word problems covering a variety of application areas and asked to select three problems that they liked better than the others. The problems were selected from standard college algebra textbooks. Once the selection was done, students were asked to answer the problems. In study 1, where the emphasis was on quadratic functions, students chose their three preferred problems from a list of twelve problems while making their preferences out of fourteen problems in study 2 (exponential and logarithmic functions). A summary of the problem focus was inserted before each problem, so that the student would be able to identify the problem focus without difficulty. For example, a summary titled "Maximizing grape crop yield" would appear before a problem which would give the per acre grape yield as a quadratic function of the number of trees and would ask the students to find the number of trees which would maximize the grape crop yield. Three of the sections (denoted A1, A2 and A3) are from one campus

(Campus A) and the fourth section (denoted B1) is from campus B. Sections A1 and A2 were taught by a single teacher. Section A3 and Section B were taught by two other teachers.

We created the problem sets so that they had problems from each of the areas, *I*, *R* and *U* described above. It is understood that attributes such as intrigue and curiosity can change from student to student and the things that students can relate to can vary significantly, so authors tried hard to look at the problems "through student eyes" while doing the categorization.

We strongly believe that all the problems were more or less of the same difficulty within each study. Each of the problems in study 1 (quadratic functions) required students to perform the exact same process, which was to extract coefficients from the given quadratic function, substitute these in to the vertex formula, and use the calculator to get the answer. As such, all problems considered in study 1 can be regarded as having the same level of difficulty. As far as study 2 is concerned, the problems represent two topics: exponential functions and logarithm functions. All problems on exponential function required students to solve an exponential equation for a variable like time and therefore can be regarded as having the same level of difficulty. All problems in logarithmic functions involve students substituting values in a function involving logarithms and once again have the same level of difficulty for students. The computations in the logarithm problems are less involved than those in the exponential problems, however any bias towards these problems would be offset by the greater familiarity that students have with exponents needed in solving exponential function problems. Thus all problems in study 2 may be regarded as being equally difficult.

The following tables show the problems given in study 1 and study 2.

Study 1 (Quadratic Functions)

Table 1
Problems given in study 1 (Quadratic functions) and their categorization

Problem Number	*Problem Summary*	*Category*
Q1	Maximizing effectiveness of a commercial	*U*
Q2	Maximizing fuel efficiency of a car	*R*
Q3	Maximizing grape crop yield	*R*
Q4	Finding height of a volcano	*I*
Q5	Finding height of an arc	*U*
Q6	Finding the maximum height of a space plane	*I*
Q7	Finding the maximum height of a basketball from the free throw line	*R*
Q8	Maximizing revenue of a commodity	*U*

Q9	Maximizing revenue from the sale of calculators	*U*
Q10	Minimizing the cost of producing digital cameras	*R*
Q11	Fencing a horse corral to get the maximum area	*R*
Q12	Maximizing the cross sectional area of a rain gutter	*U*

Study 2 (Exponential and logarithmic functions)

Table 2
Problems given in study 2 (Exponential and Logarithmic functions) and their categorization

Problem Number	*Problem Summary*	*Category*
Q1	Finding the depth of a lake which has the given light intensity.	*U*
Q2	Finding the time it takes for the population of California to reach 50 million	*U*
Q3	Finding the time it takes for a Radium sample to reduce to a given amount?	*U*
Q4	Finding the time it takes for a deer population of a PA county to reach 100000?	*R*
Q5	Finding the time it takes for bowl of soup to cool?	*U*
Q6	Finding the pH value of lemon juice	*U*
Q7	Finding the time it takes for a bacteria culture to reach a count of 50,000	*R*
Q8	Comparing the magnitudes of the 1906 San Francisco earthquake and the 1964 Alaska earthquake	*I*
Q9	Finding the age of an artifact from an ancient tomb	*I*
Q10	Finding the pH value of sea water	*U*
Q11	Finding the time of death at a crime scene	*I*
Q12	Finding the time it takes for my money to double	*R*
Q13	Finding the number of months of training that will be needed for a pole-vaulter to vault a certain height?	*U*
Q14	Finding the ion concentration in Wine	*U*

Observations

The table below shows student preference for quadratic function problems by a section-wise breakdown.

Table 3
Student preference for quadratic function problems by a section-wise breakdown

	Problems											
Sections	Maximizing effectiveness of a commercial	Maximizing fuel efficiency of a car	Maximizing grape crop yield	Finding height of a volcano	Finding height of an arc	Finding the maximum height of a space plane	Finding the maximum height of a basketball from the free throw line	Maximizing revenue of a commodity	Maximizing revenue from the sale of calculators	Minimizing the cost of producing digital cameras	Fencing a horse corral to get the maximum area	Maximizing cross sectional area of a rain gutter.
A1	5	20	9	12	9	6	12	3	3	12	10	4
A2	7	18	10	12	8	5	10	3	1	5	10	4
A3	11	23	4	19	3	9	12	6	2	6	5	3
B1	3	7	6	9	1	5	13	9	6	5	22	3
Total	26	68	29	52	21	25	47	21	12	28	47	14

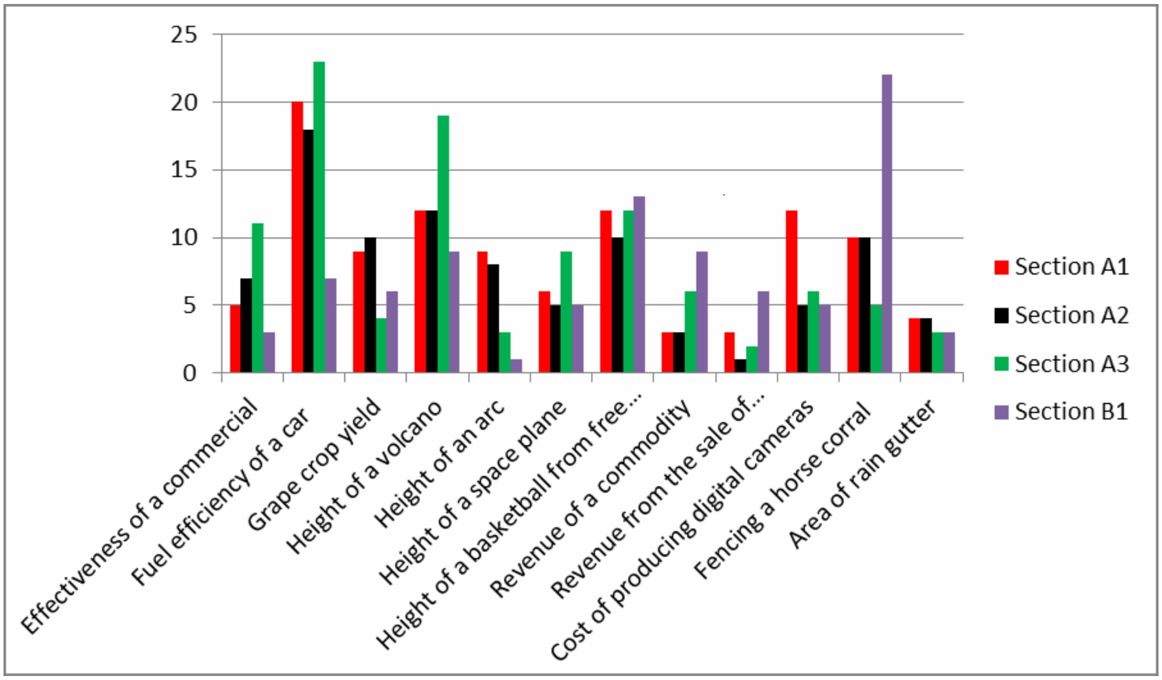

Figure 1. Preference for problems given in study 1 (Quadratic Functions) by students in the different sections (Sections A1, A2, A3 and B1)

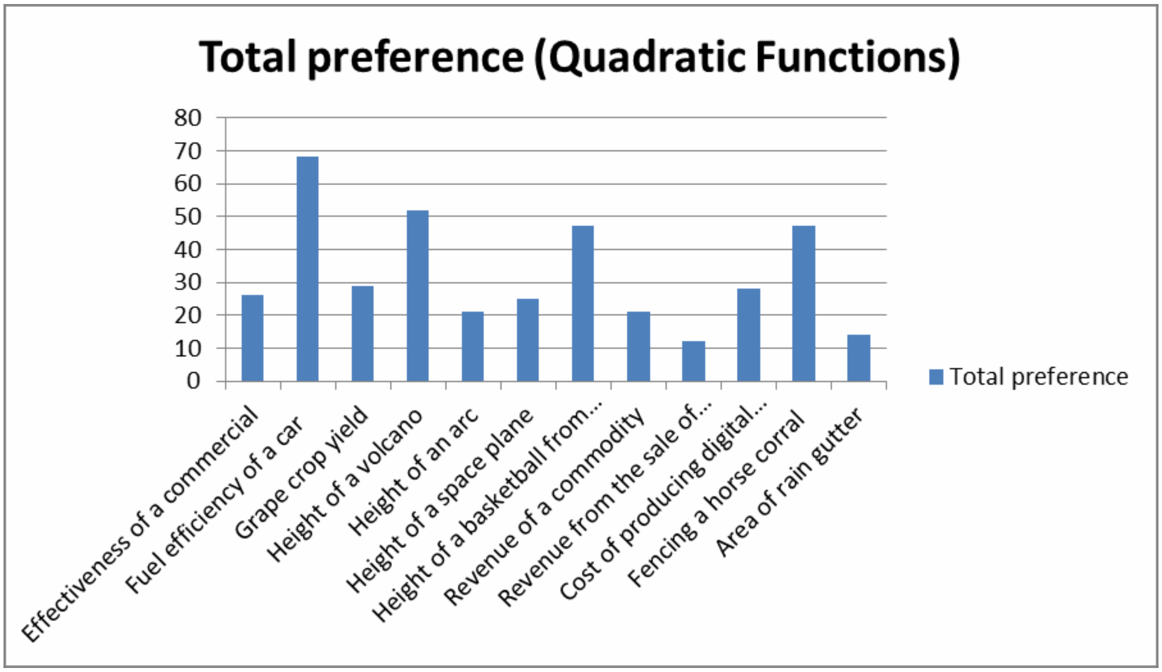

Figure 2. Total preference (all sections) for problems in study 1 (Quadratic Functions)

The student preference for exponential and logarithmic function problems through a section-wise breakdown is shown in the table below as well as in the chart that follows the table.

Table 4
Student preference for exponential and logarithmic functions problems through a section-wise breakdown

	Problems													
Sections	Lake depth with given light	California population	Radium Decay	Deer Population of PA county	Cooling time for Soup Bowl	pH of lemon juice	Bacteria Growth	Comparing Earthquake magnitudes	Age of ancient artifact	pH value of sea water	Time of death at a crime scene	Doubling money	Paul Vaulter training	Ion concentration in Wine
A1	2	5	1	7	6	3	9	1	5	6	8	14	4	4
A2	4	4	1	4	10	6	6	3	10	4	6	11	0	3
A3	3	7	2	4	1	2	8	5	12	3	14	14	9	2
B1	0	6	3	8	3	8	2	5	3	7	6	10	2	3
Total	9	22	7	23	20	19	25	14	30	20	34	49	15	12

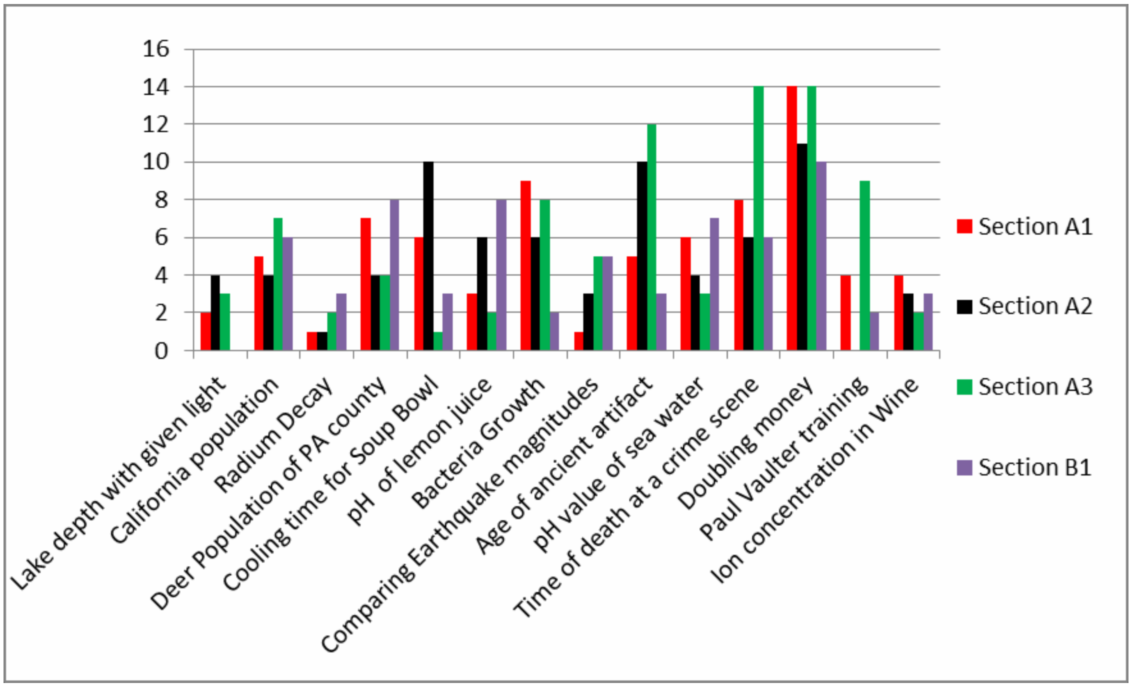

Figure 3. Preference for problems given in study 2 (Exponential and Logarithmic Functions) by students in the different sections (Sections A1,A2, A3 and B1)

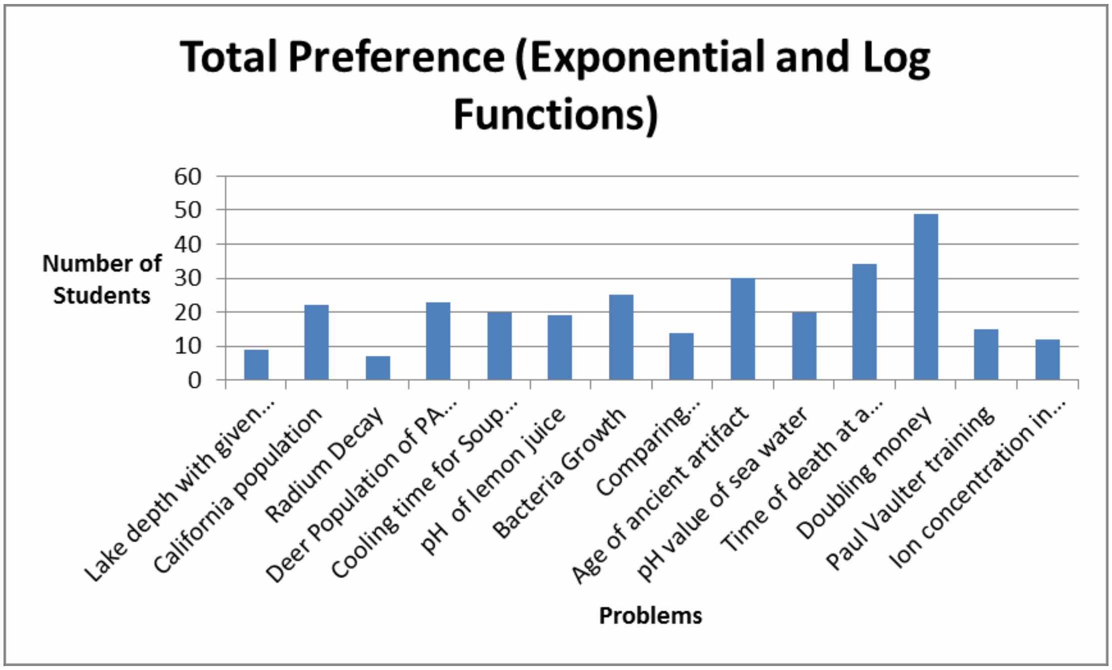

Figure 4. Total preference for problems given in study 2 (Exponential and Logarithmic Functions)

Statistical Analysis Overview

We fit mixed-effects logistic regression models for the data from both studies separately. Mixed effects logistic regression is used to model binary outcome variables, in which the log odds of the outcome variable is modeled as a linear combination of the explanatory variables that are both fixed and random. In our situation, the response to the question "Will a student rank a problem in their top three choices?" is a binary variable. Among the explanatory variables, the teacher and student variables have random effects and the category variable has a fixed effect. Hence we used the mixed effects logistic regression to model the student's responses, against the explanatory variables teacher, student and category.

In each case, we took the binary response variable (called response) to be "Will a student rank a problem in their top three choices?" The response variable assumes the value 1 if the problem is among the top three choices and 0 otherwise. We took the "category" to which a problem belongs as an explanatory variable with a fixed effect and "teacher" and "student" as explanatory variables with random effects.

We followed the same modeling and testing process for both studies. For each study, the analysis was done in two stages. In stage 1, we tested whether the variable "category" plays a significant role in the students' response. In stage 2, we tested whether students preferred certain categories more than the others.

Study 1 (Quadratic functions) statistical analysis

Stage 1 analysis (To check the association of the variable "category" to preference).

In stage 1 we used the likelihood ratio test to determine whether student preference is associated with the variable "category". We created two logistic regression models, one with the variable category and one without. The response variable in both the models was whether the problem was in the students top three choices or not. It was a binary variable taking the value 1 if the problem was in the top three and 0 otherwise.

The first model called the myqmodel had three predictor variables, category, teacher and student while the second model called the nullqmodel had only two predictor variables teacher and student as the predictor variables. The logarithm of the maximum likelihood function was calculated for each of the models.

The likelihood ratio statistic, LRT=$-2(\ln(L_{nullqmodel}) - \ln(L_{myqmodel}))$, which follows a Chi-squared distribution with 2 degrees of freedom, was used to test the null hypothesis that the category variable had no significance on students' preference. We used the software R(http://www.r-project.org/) to conduct the analysis in both the studies. The table below shows the results obtained for the stage 1 analysis.

Table 5
Results from the software R for stage 1 of study 1

Item	Value
Log Likelihood for nullqmodel $\ln(L_{nullqmodel})$	-1031.20
Log Likelihood for myqmodel ($L_{myqmodel}$)	-995.38
The likelihood ratio statistic, LRT=$-2(\ln(L_{nullqmodel}) - \ln(L_{myqmodel}))$	-2((-1031.20)-(-995.38))=71.641
p value	2.776×10^{-16}

We obtained a likelihood ratio test value (LRT) of 71.64 with 2 degrees of freedom and a corresponding *p* value of 2.776×10^{-16}, which is an extremely low value. Therefore the null hypothesis can be rejected at 0.1% level and we can conclude that the category of the problem does have an impact on student preference.

Now that we know that the category from which the problems are pulled effects student preference, the next step would be to find out whether certain categories are preferred more than others.

Stage 2 analysis of Study 1 (To analyze preference among the subcategories)

In stage 2, we tested whether any of the three categories (*I*, *R* or *U*) of problems are significantly preferred by students over others. In order to do that, we took pairs of categories at a time and tested the null hypothesis that the odds of selecting a problem from the two categories in the pair are the same. In other words we tested whether the pair-wise odds ratios between two categories in a given pair is one. We used the fact that the logarithms of the odds ratios approximately follow a normal distribution. The table below shows the z statistics for this distribution and the adjusted p values, correcting for multiple comparisons, obtained using the R library package multcomp to make pairwise comparisons testing.

Table 6

Results from the software R for stage 2 of study 1

Pairwise comparison	Odds ratio	Z value	*p*-value
R vs *U*	2.85	7.930	$< 10^{-5}$
I vs *U*	2.49827	5.203	$< 10^{-5}$
R vs *I*	1.142364	0.867	0.657

The extremely low p value obtained for *R* vs *U* shows that the null hypothesis is rejected at a highly significant level (.1%). This fact, along with values of the odds ratio implies that students significantly prefer problems from the *R* category over problems from the *U* category. A similar argument shows that students significantly prefer problems in the *I* category over problems in the *U* category. The *R* vs *I* case is however different. As seen from the p value, we cannot reject the null hypothesis here which means that there is no evidence to support that *R* or *I* is preferred over the other.

These results conclusively show that in the case of applications of quadratic functions, students prefer problems to which they can easily relate to and problems which have intrigue over other problems which are more "distant" and those which lack a good level of intrigue.

Let us now analyze the data on student preference of exponential and logarithmic functions.

Study 2 (Exponential and logarithmic functions) statistical analysis

The analysis for this data was done in a similar manner to the quadratic functions case. The two models that we created for these problems are myemodel and nullemodel. Like in the study 1 analysis, the former had all three (category, student and teacher) as variables and the latter had only teacher and student as variables.

Stage 1 analysis of Study 2 (To check the association of the variable "category" to preference)

An analysis similar to that done in Stage 1 of Study 1 was done with the data from Study 2 and we obtained a likelihood ratio test value (LRT) value of 40.48 with 2 degrees of freedom and a corresponding *p* value of 1.615×10^{-9} which is an extremely low value.

Therefore the null hypothesis can be rejected at 0.1% level and we can conclude that the category of the problem does have an impact on student preference in Study 2 as well.

Stage 2 analysis of Study 2 (To analyze preference among the subcategories)

We conducted the statistical analysis of Stage 2 of Study 2 in a manner similar to Stage 2 of Study 1 by taking pairs of categories at a time and testing the null hypothesis that the odds of students selecting problems from the categories in the pair are the same. The resulting table for pairwise comparison is shown below.

Table 7

Results from the software R for stage 2 of study 2

Pairwise comparison	Odds ratio	z value	*p*-value
R vs *U*	2.5901	6.068	$< 10^{-4}$
I vs *U*	1.9092	3.957	0.000212
R vs *I*	1.3566	1.697	0.205402

Once again the very low p values obtained for the *R* vs *U* comparison and the *I* vs *U* comparison allow us to reject the null hypothesis that each of these categories are equally preferred at 0.1% level and there is no evidence to support that *R* or *I* is preferred over the other.

The analysis conducted above once again shows that problems from the R and I categories are significantly preferred by students over problems from the U category.

These results above conclusively show that in dealing with application problems from exponential and logarithmic functions too, students prefer problems to which they can easily relate to and problems which have intrigue over other problems which are more "distant" and those which lack a good level of intrigue.

Discussion

The results show that given a set of word problems covering a variety of application areas, students prefer problems which either generates intrigue or problems to which they can easily relate over those which do not fall into these categories. Does this mean that students actually like to do word problems in these areas? Not necessarily. But teachers can use problems in these areas to create some level of interest in the students which would certainly help them stay engaged. Students may not retain most of the algebra that we teach after their graduation unless they move onto a scientifically intense career. However, presenting real life modeling problems which capture the interest of students might help them retain knowledge of some of these models for at least a while. For example, a student watching a CSI episode might remember that her algebra professor showed the process of calculating the time of death and a student running out of gas while driving on a remote highway might remember that his teacher told him how to calculate the best speed at which the car should be driven (and remembering that value might actually help him to reach the next gas station safely). As a more critical example, when doing exponential decay, a motivational example on half life time of medicine might help a student assess the situation of an accidental drug overdose. In these days of easy information access, they could easily look up the relevant mathematics on the internet once they know how to model the problem.

Since the results that we obtain pertain simply to student preference to real life contexts, which need not be in mathematics, our findings should be useful to teachers in any applied discipline to find examples and problems to present in their classroom.

Our results strongly suggest that students in our region have a strong preference for mathematics problems associated with mystery and that they can relate to in their everyday life. However student preferences in other regions could be different and be influenced by the culture and society in which they live. We would like to recommend to the readers that they first identify the areas that their students would prefer and use problems from those areas to motivate and engage students in mathematics at the college algebra level.

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