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Mathematics Career Simulations: An Invitation

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Introduction

In a review of the literature surrounding motivation to learn mathematics, Middleton and Spanias (1999) report that careful design of instruction can strongly influence student motivation for mathematics achievement, which increases the likelihood students will choose to take future mathematics courses. A meta-analysis of 113 mathematics education studies found a significant influence of attitude toward mathematics upon achievement in mathematics (Ma and Kishnor, 1997). A wealth of evidence connects students' mathematics achievement with their self-beliefs in mathematics (e.g. Pajares, 1996; Pajares and Schunk, 2002). Mathematics self-efficacy is a construct describing students' beliefs about their ability to carry out mathematical tasks (Pajares, 1994). Findings suggest that self-efficacy beliefs play a role in career choices, especially in mathematics and science related fields (e.g., Hackett, 1985; Zeldin, 2000).

Pilot testing of career simulation activities began in 2005. Recently, we have scaled up the simulation, and our teaching goals have led to research questions. Can a semester-long, simulated academic career in mathematics produce differential attitudes and beliefs about mathematics or about STEM careers? Will the differences lead to different learning outcomes and motivation? How does a well-designed Career Simulation promote or enhance engagement within the classroom? Can a large-scale simulation activity serve as invitation to the students to become active co-creators of mathematics knowledge?

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A career simulation asks the students to present at class conferences, to write mathematical papers, to review and edit for class journals, and to work within teams of faculty at fictitious universities. For each activity, students earn scholarship points toward (simulated) promotion and tenure.

We describe the simulation structure that has proven successful in two simulations in a Game Theory course during the Spring semesters of 2008 and 2010, and report on the findings from a study conducted at the conclusion of the more recent term.

What is a Mathematics Career Simulation?

Our Mathematics Career Simulation invites students to perform tasks related to teaching, researching and professional service. The tasks authentically mimick tenure track academic jobs. Mathematics research communities form around conversations full of conjectures, questions, generalizations and proofs. The simulation targets this rich culture in hopes the students will experience the personal creation of course content knowledge, similar to the way in which research mathematicians develop new ideas. Conjecture-proof conversations are a natural outgrowth of the simulation activities that occur during conference meetings and throughout the editorial processes involved with publishing the class journals.

Thirty-two students completed the simulation in Game Theory in Spring 2010, eleven females and 21 males. The class has a pre-requisite of Calculus I, either the 4-hour calculus course for mathematics and science majors or the 3-hour business calculus course. One business student completed the course. The rest were STEM majors: four computer science majors, seven physics majors, nine mathematics majors and eleven mathematics secondary education majors.

Conferences and journals drive the simulation. Four students served as editors of class journals. The editors found two peer reviewers for each article. After acceptance, the editor gave the reviews to the journal publisher who in turn worked with authors to get the article into print (online class website). About half of the class meetings each week were conferences. A conference organizer recruited a slate of presenters. The organizer served as master of ceremonies for the conference and published precedings (class website). Journal articles were hyperlinked to full-text files, and conference proceedings linked to PowerPoint presentations and notes for each speaker.

The Career Simulation has developed over the years, and now includes a Faculty Handbook and Help Guides for the key tasks of Conference Organizer, Journal Editor and Journal Reviewer. Tasks are explained along with the range of scholarship points that may be awarded. Faculty teams led by their respective Department Heads ensure their university's conferences and journals run smoothly by offering presentations and service as reviewers. The scholarship points flex realistically. Presentations and papers that address more difficult proofs earn more scholarship points. Student-professors choose from a variety of professional "career" interests: writing papers, making several presentations or service-oriented careers filled with reviewing and organizing conferences.

What is mathematical discovery? How do we advance mathematical knowledge? How is an inquiring mathematical spirit developed? These are the instructor-posed questions for which the simulation was engineered. In our view, the elements of mathematics research that resonate with our students are:

1. Discovering new mathematics machinery
2. Applying existing machinery to new problems and applications
3. Extending or generalizing basic machinery to handle more difficult or advanced problems
4. Exporting machinery to other disciplines
5. Conjecturing

An example of the process may be helpful.

The Larssen conjecture developed in the second conference of the term (Spring 2010). Having completed his calculations for the correct strategy mixture for the row-player, Student-Professor Larssen stated that he did not need to show the calculations for the column-player because the zero-sum game matrix was symmetric. When questioned, he suggested (correctly) that the book used the same technique. However, the book does not prove the result due to its intuitive nature.

Asked to propose a conjecture which the class could consider for proof attempts, Professor Larssen stated that, for symmetric game matrices, the strategy mixtures for both players were identical, and the value of the game was zero. The conjecture was proven false on the spot, almost before it was written on the board. Larssen's classmates realized many symmetric games have non-zero values. Professor Larssen proceeded undeterred. He could see bits of mathematical truth, but could not yet explain fully.

The matrix Professor Larssen was analyzing was actually skew symmetric, and the class spent ten minutes comparing and contrasting the properties of symmetric and skew-symmetric matrices. The instructor organized the discussions into two conjectures, one for symmetric matrices (which was named the Cook Conjecture), and an updated Larssen Conjecture for the skew symmetric case. Both conjectures were proven true at later conferences.

The textbook (Straffin, 1993) states the oddments method (2x2 zero-sum matrix games) is equivalent to the method equalized expectations. No proof is offered, though a rudimentary outline is sketched. The Oddments Conjecture was added to the Open Questions document as soon as a student used the result in a conference presentation. (The "open questions" are instructor-suggested proof attempts and problems. Bonus or "double" scholarship points are offered for key results that are needed move the class content forward, and the document is updated weekly.) The Oddments Conjecture developed over time as the students investigated, with partial results contributed by several presenters over three different conferences. The class determined that the method of equalized expectations, when used incorrectly, fails transparently, with impossible probabilities turning up (e.g. negative probabilities or probabilities greater than one). Oddments fail opaquely, producing reasonable-seeming yet incorrect solutions.

Another series of open questions resulted: describe and categorize how each method fails when used incorrectly. Can we prove these methods always fail transparently or opaquely? Can we prove they don't? Can we categorize all 2x2 game matrices and explicate the difference? The eventual answer to all three questions came in comprehensive fashion, in a presentation by one of the quietest members of the class. She used the graphical method of solving 2xn games to categorize the types of failure, and she tied the explanation into another series of recently-settled conjectures about the graphical method.

In the final weeks of the simulation, tedious but correct proofs of the 3x3 cases for both the Larssen and Cook conjectures were presented at conferences, and journal articles relating their proofs were submitted to journals. Student-Professor Crossfield worked for three weeks, often in the instructor's office hours, to prove the general nxn case of the Cook Conjecture, and he produced an elegant (and terse) linear algebra proof. His journal article, barely a page long, earned one of the highest scores of all 32 journal articles published.

We recommend the Career Simulation structure for application-rich courses. Though tested for mathematics, any course with a multiplicity of approachable applications would work just as well. In a class meeting three times weekly, the instructor lectured once per week. The remaining two meetings were conferences with six or seven presenters in a 55 minute period. With time taken for logistical matters, tests and quizzes, about half the class meetings were conferences. Lectures summarized and organized the information the speakers demonstrated, provided new conjectures and open problems, and attempted to provide the "big picture" for the course. We find the students are most receptive to making presentations when they have access to a wide selection of homework problems and conjectures that lead to multiple applications and investigations. Yet, this wide array of different open problems left some student confused about ''What's on the test?''

Six different game theory texts in addition to the adopted text were available in class each week, including a general interest text, a textbook geared toward economics and MBA programs and even *The Mathematics of Poker* (Chen, 2006). The Open Questions document listed several problems and investigations from each. Students borrowed the books or copied pages to help them investigate topics that interested them. The *Mathematics of Poker* introduced variants of the classic Roshambo finger game. After two Roshambo conference presentations, a student presented the Rock-Paper-Scissors-Lizard-Spock variant from the TV show "Big Bang Theory" (complete with YouTube video clip). Internet searches quickly produced many useable links.

Student opinions about the simulation are discussed below, but one student reported valuing the mathematical discussions the simulated community generated:

> The class was very informative and allowed for discussions to grow on themselves. The career simulation allowed you to teach your self because you had to explain it to your peers in a way [that] they would also understand the ideas and concepts. The class was taught in a way that you got out of it what you put into it. The more you dove into a subject, the more you would understand and the more questions that would come about that need to be answered ... Later on you must teach yourself and explore you own ideas based off others'.

Simulation Logistics

If you are considering a Career Simulation, we encourage you to visit the web site where full versions of all the Career Simulation materials are housed permanently.

https://sites.google.com/a/northgeorgia.edu/game-theory/

A workable points system uses a standard grading rubric for all presentations and papers, with points adjusted by a difficulty multiplier.

During class conferences, the instructor's role was two-fold: evaluate all presentations and update the Open Problems document. The instructor worked on a laptop during class since the classroom computer was typically in use for presentations. Before each conference, the organizer completed and shared an evaluation spreadsheet that included an entry for each presenter with title and topic, a brief abstract, and a blank scoring rubric.

With practice, the instructor was able to follow and grade each presentation while simultaneously updating the open questions. The most difficult days were the most rewarding, when several good presentations led to interesting new questions and conjectures. Grading the work and directing thought-provoking investigations required a great deal of instructor efficiency and intensity.

Students kept abreast of their career simulation points by viewing a spreadsheet. A log page showed each activity, date and the student-professor who completed it. Summary pages kept individual tallies. Students corrected mistakes and omissions in their simulated vitae and tracked their progress. The log page updates mostly occurred during class conference days. Setup required about an hour, with an additional hour each week to add activities and points.

The transparency of scholarship points was one source of class frustration. Students looked through each other's scholarship points and complained that some jobs requiring little work were worth lots of points. While vitae-envy is not uncommon in professional circles, this authentic aspect of the simulation led to a difficulty: correctly incentivizing all the tasks so the simulation would run efficiently. Anticipating at the beginning of the term how readily students will volunteer is difficult, especially for tasks they have never attempted (or heard of) before. The exact number of presentations and journal articles (and their depth of analysis) depends upon class interest and the energy invested. In the last week of class, the points value for all articles were doubled because they were much longer (and better!) than anticipated. The points for reviewers were also doubled due to the attention to detail that was obvious in their work.

While we feel the points awarded were reasonable and fair, especially after adjustments, some students felt differently. Those views are summarized below, but this structural challenge exists indigenously within such a pervasive simulation. The semester-long evolution of the simulation requires adjusting points which, almost by definition, unequally benefits certain students. We do not agree that unequal equates to unfair, but some of our colleagues lay out very precise syllabi and view them as unalterable contracts with the students. In our view, the grading scheme must retain some flexibility to help a simulation run properly.

To facilitate adoption, we suggest introducing one component at a time. Having peer-reviewed papers, for example, is an idea we've seen others use. Many professors use student presentations already, so introducing the conference organizer role may be an easy adaption. To facilitate the simulation, the class were divided into four university faculties, each with a department head. A weekly department heads meeting ensured they understood what tasks needed to be accomplished by each team. For any needed task, a service role can be created with scholarship points offered. Abundant volunteers offered to do extra tasks to maximize their scholarship points.

Results

The evaluation of results from the course was led by a qualitative research specialist (not the instructor) using ethnographic methodology. The investigation was not traditional ethnography due to the fact the classroom culture was imposed by the conditions of the simulation. Of interest was the classroom culture that emerged due to the constraints imposed by the simulation. Ethnographic methods are well-suited to the study of cultures (Patton, 2001).

For 10% of their final exam grade, students were required to annonymously submit a response to one of the following prompts:

1. Describe the things about the class that most interested you and helped you learn best. Contrast those with the things that you found least interesting and degraded your ability to learn Game Theory.

2. What aspect of Career Simulation did you think offered the most points for the least amount of work? What aspect could be better incentivized by offering more points? Was career simulations overall a valuable aspect of your learning experience with Game Theory?

3. What do you think of Game Theory as a subject? Was I able to convey some of the interesting applications and amazing contributions of folks like Nash, van Neumann, Morgenstern and Rappaport? Do you now think of Game Theory as valuable and interesting? If so, why? If not, why not?

The responses to these questions were invaluable as they highlighted student perspectives and thoughts on the career simulation. To promote honesty and thoughtfulness in their responses, the students were assured that gramatically correct submissions that had some evaluative aspects would receive 100% of the points. Only two of the 32 student responses comment solely on Game Theory content. The remaining 30 students addressed aspects of the simulation. The students' critiques were submitted electronically and are expatiated below based on the emergent themes from the work of two qualitative analysts. Coding occurred independently, and the triangulated results below were confirmed by the lead investigator.

Twenty students wrote general statements that were interpreted as indicating a positive experience with the class overall and with Career Simulation. These statements are exemplified by comments such as, "I think overall Career Simulation was a fun way to learn Game Theory," or "As for the Career simulation, I feel that it was great way to convey the subject to the students." While these comments confirm enjoyment of the course and of the material, they lend little explanation as to why the enjoyment occurred. Exactly what aspects do the students like? More importantly, what facets of the class enhance student learning? Do students appreciate the course components that enhance student learning? The following comments (from two different students) were typical.

> The best thing I enjoyed about class was the freedom we were given to learn the subject. Besides the general direction and help you provided, it was up to us to learn, and then teach our fellow students. This made it much more critical and worthwhile to make sure everyone knew what they were talking about (so you didn't make a fool of yourself), and to pay attention to others when they talked.
>
> The best aspects of the course were the interesting material, the break from traditional class methods, and the ability to fully enjoy the class. I really feel as though game theory is completely fascinating, surprising applicable and relevant topic in mathematics -- it shows the usefulness of mathematics in the "real world."

These excerpts illustrate three predominate themes within the data: *self-teaching along with teaching others, freedom of choice,* and *career relevance and application*. The first theme became apparent in phases such as "…it was up to us to learn, and then teach our fellow students." The label of *self-teaching along with teaching of others* originates from the language that the students used in their comments. Students viewed preparing a presentation or paper as self-teaching, and they qualified how well they needed to know the material by explaining that they should be able to teach their classmates. We argue that a better description of the theme is engagement with mathematics and the acceptance of personal responsibility to learn with understanding. The word "self" in self-teaching does not imply students learned in a vacuum or without the consultation of others but rather implies recognition of individual accountability. Teaching others is the benchmark for how well one should know the material. Students did not view presentations and papers as regurgitation of material but instead as learning in anticipation of other's questions. During the preparation, students reported developing an understanding of connections among the material in the course. Overall the theme of self teaching and teaching others was coded with exhilaration because these responses highlight a fundamental goal of the simulation. Specifically, students accepted the invitation to engage in mathematics.

The second theme labeled *freedom of choice* demonstrates students' positive reception of the ability to select problems from homework and the Open Questions. Student comments addressed motivation levels with phrases such as "…I know for me a lot of interest was sparked, and it was definitely because I was able to learn about whatever I wanted in the game theory field as long as I learned the basics." Another student commented, "For the first time, I was allowed to think up my own theories using what I know." Freedom of choice presents mathematics as a plethora of question and conjectures. Career Simulation allows the student, maybe for the first time, to think of mathematics as the quest for learning about the unknown.

The final theme that exposes the rich environment created by Career Simulation is *career relevance and application*. Several of the secondary mathematics education majors said they felt the experience would help them become better teachers: "I believe that the Career

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Simulation as a whole was a great success and was a very good tool to prepare the students for their future teaching careers." Making presentations of mathematical material helped develop their teaching skills. Another student simply remarked on the usefulness by stating, "I do think game theory is valuable and interesting because it can be applied in all walks of life, economics, politics, sports, and some board games."

Even though Career Simulation was not a typical approach to teaching and learning for these students, we considered it to be a successful and unique experience. One student stated: "In fact, the process of working on the career simulation has led me to strongly consider attending grad school for mathematics." Another student wrote about gaining insight into faculty life: "I think that the career simulation helped me see what a true professorship at a major university would look like, because I did not know what all went into it."

Not every student was happy with the course or the simulation, though only one wrote an overwhelmingly negative critique. It reads, in part:

> As for if the class was good for learning, my honest answer is no. In fact, I would feel confident in saying it was a poorly designed class setup. At no point did I feel that students should attempt to teach each other ... I am paying for a professor to teach me, not my fellow students. I personally feel cheated out of a class by this teaching method, as well as by my fellow students. The Utopian idea that we would teach ourselves is nice, but because of the simple fact that we are paying for a service makes it nothing more than a whimsical dream that it could succeed.

An overwhelming theme that highlights a challenge for Career Simulation was students' *inability and unwillingness to learn from their peers*. One common complaint was that too many different topics were presented each day. As one student stated, "the worst part of class was the career simulations. It was nearly impossible to comprehend six different presentations in a short fifty-five minute class." Another simply stated that, "… it was difficult to learn for my peers' presentation when the material is fairly new to them as well." One student wrote cogently about the difficulty, with a wry touch of humor.

> There seemed to be a conflict of interest when we gave presentations. Was our main interest in getting others to understand or was it in receiving scholarship points? Of course, this same issue exists in general in all teacher-student relationships. Perhaps we should use Game Theory to find an equilibrium outcome wherein the focus on student learning matches the focus on teacher compensation.

The theme code *inability to learn from their peers* comes from comments such as "…others [presentations] just sort of left me scratching my head" or "…I was unable to follow and absorb the information that was presented." Information absorbance describes the learning model that students were invoking during presentations. We purport that *unwilling to engage* is a better description than *inability to learn* because student comments and questions were encouraged. One student admits, "Without a forceful nudge, I would more than likely not spend very much time on anything class related." Another prevalent theme coincided with the lack of commitment to delve into peer presentations: *dependence upon teacher clarity*. Student comments included phrases such as "a professor could explain some of the content better than students," or "the clearest classes were those where [the instructor] got up to explain the fundamental theories behind concepts." Another exemplar criticism ties the unwillingness to learn with the dependence upon instructor clarity:

> In hindsight, I do not believe the Career Simulation worked as a teaching tool. I sat in class many days wondering what use each presentation was to me. Most of the time, the presentations felt scattered and had no application to class. I prefer a more structured and stable learning environment.

Although rare, presentations of poor quality were painful to sit through. Students expressed their frustation or *lack of tolerance* for confusion. One student commented:

> Certain speakers left plenty to be desired. It was hard to even understand certain presenters, and there was at least one instance where [the instructor] had to interject because the presenter was teaching something incorrectly.

The only other student grievance that emerged came from the second prompt where they voiced disdain of *unfairly awarded Scholarship Points* and *lack of recognition* of their work. Certain duties were perceived to earn more points than the effort required, for example, "the department head [job] offered the most points for the least amount of work, followed by the conference organizer." Another concurred:

> One final criticism (and the worst part of the class) is the department leaders and conference organizers. They literally got almost 1/2 of their points for nothing more than standing, chosen seemingly at random. That really is not fair to lottery off some people's good grades.

Twelve students indicated they felt some aspect of the Scholarship Points was inequitable, mostly pointing to the two roles these critiques mention, conference organizer and department head. The new capabilities of Google sites, Google docs and other tools made these jobs much easier in 2010 than they had been during any of the previous simulations conducted. Points promised in the Faculty Handbook were not taken away. Adjustments were made throughout the semester by adding points for certain activities like journal articles and peer review.

The selection of journal editors and publishers was based on an application process where students listed the mathematics courses they had completed with a grade of B or better. Some students perceived the assignment of these jobs as a gift from the instructor that showed preference toward particular students. The job of journal editor requires hard work and solid mathematics content knowledge, and therefore this aspect will likely not change in the future and presents a challenge to overcome.

Grade-oriented students also pined for recognition of their time and effort. One student wrote, "I wanted to take some time to devote to a proof or problem before I committed to the presentation but worried that if I did, someone else would choose the topic for their presentation and all my work would be basically for nothing." Other students simply felt that the Career Simulation was too much work for the portion of their course average it represented. This statement was typical: "With as big of a task [as] the career simulation is I believe it should have counted 50-60% of our grade rather than only 25%." We concurred. In general the vast majority of the students were hard-working and dedicated to the course.

After they submitted their critiques, three different grading schemes were offered to the class, the one in the syllabus and two others that increased the weight of the Career Simulation. Students each chose the one they preferred. In only one case did the choice hurt a student's course average, but fortunately that person's resulting letter grade was not affected.

> One final student comment:
>
> Overall, I did enjoy the Career Simulation, and I think it added to student learning for those who took it how it was intended. The best way to learn a topic is to teach it to others, and that's exactly what this simulation allowed us to do. I appreciated when you would step in and teach on topics that were still a bit confusing, but allowing us to teach as well I think enriched our individual learning.

We deem this evaluation of student comments as one that accurately portrays the benefits and challenges to Career Simulation. Career Simulation has the potential to engage students in mathematics in new and exciting ways as evident from the *self-teaching along with teaching others* theme. The challenge for the professor is creating such an environment as the students cling to more familiar, lecture-based models and resist learning from their peers.

Discussion

A mathematics class can serve as an invitation to students to join a mathematical community where they help create new knowledge and share it with others. Inquiry-based learning and problem-based learning have been suggested as viable alternatives to traditional lecture in the literature (Harel and Sowder, 1998). Describing some of the problems associated with traditional lecture-oriented classes, Harel and Sowder (1998) suggest teachers of mathematics "present proofs of well-stated, and in many cases, obvious, propositions, rather than ask for explorations and conjectures. As a consequence, students do not learn that proofs are first and foremost convincing arguments, that proofs (and theorems) are a product of human activity, in which they can and should participate."

Fields Medalist William Thurston (1995) suggested mathematicians have developed habits of communication, especially in classroom settings, that are "often dysfunctional." He describes the situation thus (p.31):

> In classrooms ... we go through the motions of saying for the record what we think the students "ought" to learn, while the students are trying to grapple with the more fundamental issues of learning our language and guessing at our mental models. Books compensate by giving samples of how to solve every type of homework problem. Professors compensate by giving homework and tests that are much easier than the material "covered" in the course, and then grading the homework and tests on a scale that requires little understanding. We assume that the problem is with the students rather than with communication: that the students either just don't have what it takes, or else just don't care. Outsiders are amazed at this phenomenon, but within the mathematical community, we dismiss it with shrugs.

Schoenfeld (1988) suggests four unintended learning outcomes of the "well-taught" traditional mathematics course, together with a corollary for each. Each presents a pattern of thought corrosive to engagement in proof-intensive mathematics coursework, or more generally, any authentic mathematical thought whatsoever (p. 151):

1. The processes of formal mathematics (e.g. "proof") have little or nothing to do with discovery or invention. Corollary: Students fail to use information from formal mathematics when they are in "problem-solving mode."

2. Students who understand the subject matter can solve assigned mathematics problems in five minutes or less. Corollary: Students stop working on a problem after just a few minutes because, if they haven't solved it, they didn't understand the material (and therefore will not solve it).

3. Only geniuses are capable of discovering, creating or really understanding mathematics. Corollary: Mathematics is studied passively, with students accepting what is passed down "from above" without the expectation that they can make sense of it for themselves.

4. One succeeds in school by performing tasks, to the letter, as described by the teacher. Corollary: Learning is an incidental by-product of "getting the work done."

Harel and Sowder (2007) suggest that it "seems possible to establish desirable sociomathematical norms relevant to proof, through careful instruction, often featuring the student role in proof-giving."

Research suggests authentic mathematical discovery can happen within the early grades classroom. Schoenfeld (2010) describes his work analyzing a tape of mathematics educator Deborah Ball teaching a third grade class. "The lesson was amazing," he reports. "The third graders argued on solid mathematical grounds; the discussion agenda evolved as a function of classroom conversations; the teacher seemed at times to play a negligible role." Schoenfeld describes Ball's primary instructional technique as "a 'debriefing routine' that involves asking questions and fleshing out answers in a particular way - and she used that technique five times in the first six minutes of class" (p. 111).

Thurston (1995) calls upon mathematicians to instruct students in clear thinking: "We mathematicians need to put far greater effort into communicating mathematical *ideas*. To accomplish this, we need to pay much more attention to communicating not just our definitions, theorems and proofs, but also our ways of thinking" (p. 32).

At the heart of inquiry-based learning for mathematics lies an invitation to the student to become a co-creator of mathematics knowledge. The Mathematics Career Simulation is an inquiry tool that adds an invitational layer, namely, an invitation to be a scholar. Students in the simulation start the semester with a tenure track faculty position and are referred to as Professor. More research is needed to clarify the results of the invitation offered by a Mathematics Career Simulation, but these results demonstrate that students will, under certain conditions, accept and embrace the invitation, and in some cases even consider for the first time the career choice of scholar.

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