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Improving Education of Mathematics Majors

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Abstract

In this talk we will explore and discuss different ways of improving instruction of upper level mathematics classes. Several case studies will be presented, including in Calculus and Operations Research courses. We will also discuss the importance of extracurricular activities in education of mathematics majors. In particular, we will describe activities related to undergraduate mathematics competitions.

Outline

1. Introduction
2. Curriculum framework
3. Improving course instruction
4. Undergraduate research
5. Extracurricular activities
6. Concluding remarks

1. Introduction

Successful education of math majors rests on four building blocks

1. Balanced curriculum
2. High quality classroom instruction
3. Diverse undergraduate research
4. Engaging extracurricular activities

2. Curriculum

Some elements of well designed curriculum

- Coursework that effectively covers the foundations and promotes critical and logical thinking.
- Balanced offering of “theoretical” and “applied” upper level courses.
- Curriculum framework that supports undergraduate research.
- Interdisciplinary concentrations; making our students more ”marketable”.

3. Instruction

Some elements of a good instruction:

- Start with motivating example building intuition about the topic.
- Concentrate at the beginning on “Why” rather than “How”.
- Follow with developing the topic and illustrate with well chosen examples.
- A ”big picture” - structural and synthetic approach.
- More practice – Group and collaborative work.
- Hands on – Class Projects.
- Better less but better.

3. Instruction - Calculus case study

Calculation of Derivatives

In most textbooks first we have

Calculation by definition

DEFINITION The **derivative** of the function $f(x)$ with respect to the variable x is the function f' whose value at x is

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h},$$

provided the limit exists.

Argument: This is difficult for complicated functions.

Alternative: Differentiation using “rules”.

3. Instruction - Calculus case study

Calculation of Derivatives

Calculations using rules

Derivative of a Constant Function

If f has the constant value $f(x) = c$, then

$$\frac{df}{dx} = \frac{d}{dx}(c) = 0.$$

Power Rule (General Version)

If n is any real number, then

$$\frac{d}{dx}x^n = nx^{n-1},$$

for all x where the powers x^n and x^{n-1} are defined.

3. Instruction - Calculus case study

Calculation of Derivatives

Calculations using rules

Derivative Constant Multiple Rule

If u is a differentiable function of x, and c is a constant, then

$$\frac{d}{dx}(cu) = c\frac{du}{dx}.$$

Derivative Sum Rule

If u and v are differentiable functions of x, then their sum $u + v$ is differentiable at every point where u and v are both differentiable. At such points,

$$\frac{d}{dx}(u + v) = \frac{du}{dx} + \frac{dv}{dx}.$$

3. Instruction - Calculus case study

Calculation of Derivatives

Calculations using rules

Derivative Product Rule

If u and v are differentiable at x, then so is their product uv, and

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}.$$

Derivative Quotient Rule

If u and v are differentiable at x and if $v(x) \neq 0$, then the quotient u/v is differentiable at x, and

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}.$$

3. Instruction - Calculus case study

Calculation of Derivatives

The right approach but with some deficiencies.

- No “binding” example that illustrates the two approaches.
- Simple example: $f(x) = 2x^2 + 1$

By definition: $\lim_{h \to 0} \dfrac{(2(x+h)^2 + 1) - (2x^2 + 1)}{h} = \cdots$

By rules: $(2x^2 + 1)' = (2x^2)' + (1)' \to |sum\,rule|$

$= 2(x^2)' + (1)' \to |const.\ mult.rule|$

$= 2 \cdot (2x) + 0 \to |basic\,der.|$

3. Instruction - Calculus case study

Calculation of Derivatives

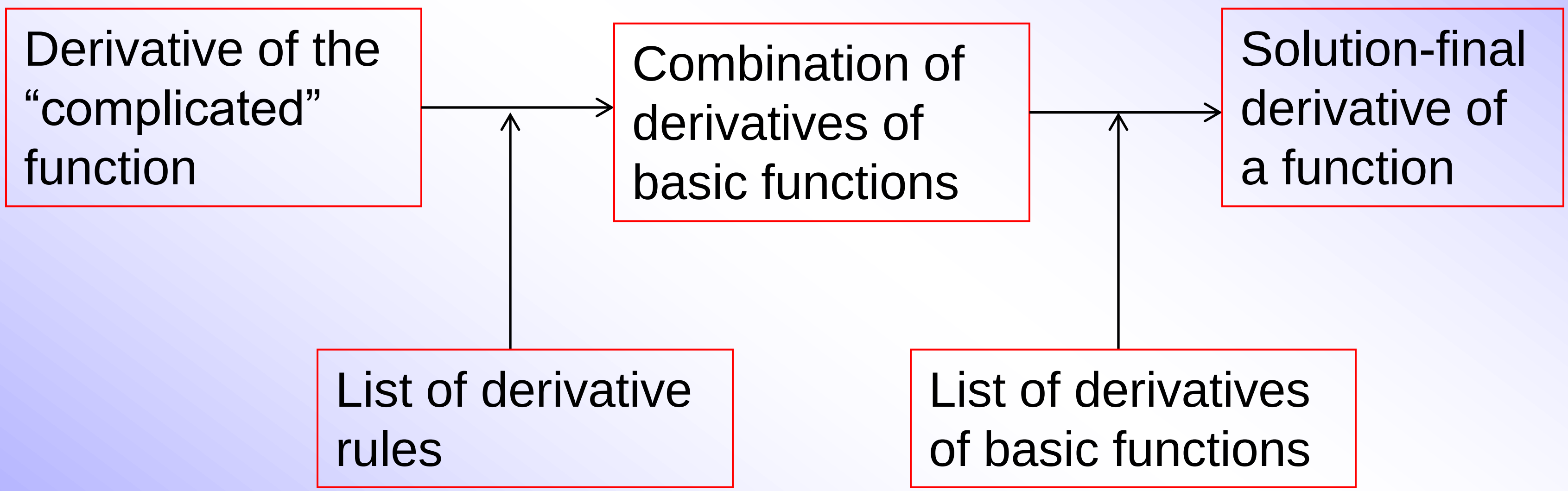
- More important deficiency : There is an important distinction between
  - Constant function rule, Power (function) rule,

  and
  - Sum rule, Constant multiple rule, Product rule, Quotient rule.
- First group represents derivatives of the <u>basic functions</u> while the second group represents <u>actual "rules"</u>, that is, how the concept of derivatives relates to algebraic operations.

3. Instruction - Calculus case study

Calculation of Derivatives

“Calculus” of derivatives: seeing a “big picture”.



3. Instruction - Calculus case study

- Similar “calculus” diagrams can be made for Limits and Integrals.
- They represent the main idea of calculus “machinery”.
- Although important for the understanding of Calculus, they are not mentioned in textbooks.

3. Instruction - OR case study

Interior –Point Methods

- Interior-point methods (IPM) developed in last two decades have been called a “revolution” in the field of Optimization and more generally Operations Research (OR).
- Most commercial codes now include IPMs as a standard option in addition to Simplex method.
- Consequently, there is a need to introduce IPMs in introductory Optimization, OR, and Industrial Engineering type classes.

3. Instruction - OR case study

Interior –Point Methods

- IPMs originated with the seminal paper of Karmarkar in 1984.
- Karmarkar’s method is based on the concept of projective transformations.
- Soon it was shown that Karmarkar’s method can be connected to a Newton-type methods and that became the prevailing framework in which IPMs have been developed.
- Newton-type approach is simpler and easier to implement than the original projective transformation approach.
- Almost all commercial IPM codes are based on the Newton-type approach.

3. Instruction - OR case study

Interior –Point Methods

Deficiency: Most of the popular OR textbooks still introduce IPMs using Karmarkar’s method.

- Hilier, Lieberman, Introduction to Operations research, 9th edition; Section 7.4, An Interior-Point Algorithm.
- Winston, Venkataramanan, 4th edition; Section 10.6, Karmarkar’s Method for Solving LPs.

3. Instruction - OR case study

Interior –Point Methods

In my OR classes I introduced IPM using Newton-type approach.

- Developed the class notes
- Talked at several conferences on this topic
- Published a paper:
  - G. Lesaja, Introducing Interior-Point Methods for Introductory Operations Research Courses and/or Linear Programming Courses, The Open Operational Research Journal, 2009, 3, 1-12
  - http://www.benthamscience.com/open/toorj/articles/V003/1TOORJ.pdf

4. Undergraduate Research

Some elements of well organized undergraduate research program:

- Faculty willing to offer interesting and diverse undergraduate research projects.
- Reward mechanism for faculty working with students on undergraduate research projects.
- Reward mechanism for students working on undergraduate research projects (venues to present and publish research, travel grants).
- Curriculum framework that supports undergraduate research (Undergraduate seminar, Senior thesis project).
- Organize undergraduate research conferences, symposiums, etc.

5. Extracurricular Activities

Engaging and diverse opportunities for extracurricular activities:

- Mathematics clubs (MAA student chapter)
- Undergraduate Mathematics Competitions
  - Putnam Mathematics Competition
  - Mathematical Contest in Modeling
  - Departmental Mathematics Contest
  - Mathematics Jeopardy

5. Extracurricular Activities

- We have been participating in two prestigious national undergraduate mathematics competitions since 2000:
  - Putnam Competition
  - Mathematical Contest in Modeling (MCM)
- In what follows we briefly review experiences, and successes of participating in these competitions.

5. Extracurricular Activities- Putnam competition

Basic facts:

- Long tradition: This year was 72^{nd} year
- First initiated and sponsored by William Lowell Putnam Intercollegiate Memorial Fund
- Organized by MAA (since 1935)
- Date: First Saturday in December
- Problems are sent to the department
- Format: 6 problems during the 3 hr morning session and 6 problems during the 3 hr afternoon session

5. Extracurricular Activities-Putnam competition

- Problems are challenging and difficult
- More “pure” math oriented. Require high level of mathematical knowledge and maturity
- Require preparation and training
- Our approach
  - Organize weekly practice sessions in the fall
  - Provide literature
  - Advisement by appointment
  - Proctoring the exam
- Further enhancement would be problem solving course (elective)

5. Extracurricular Activities- Putnam competition

Some statistics:

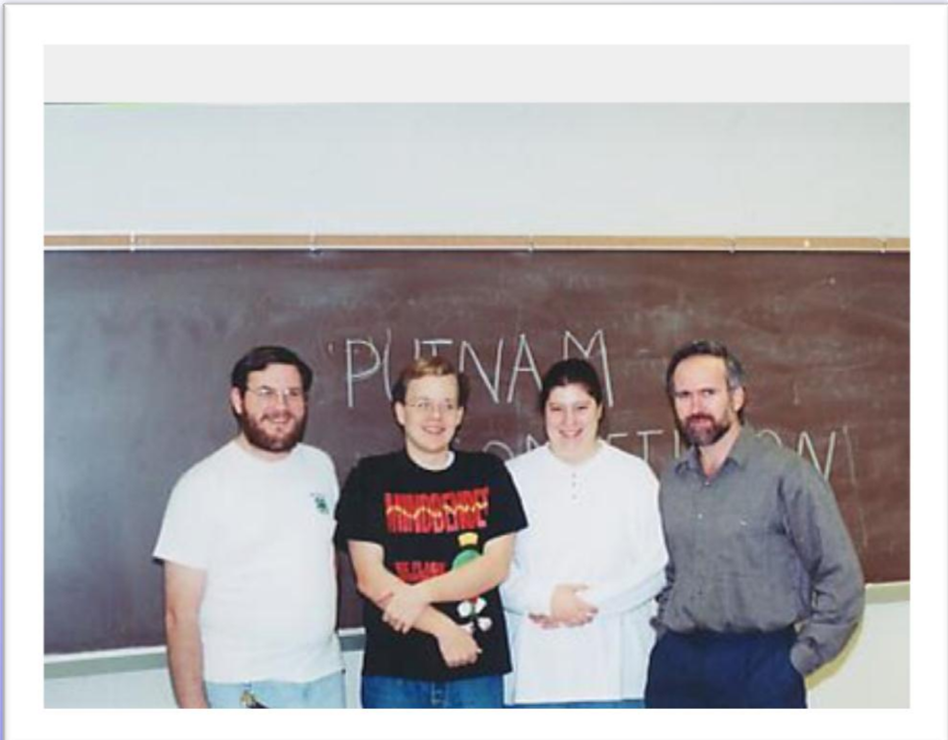
- Each problem is worth 10 points so total is 120 points
- In 2010 a total of 4296 students from 546 colleges and universities in Canada and the United States participated in the competition
- Due to the difficulty of the problems majority of students do not score any points. Scoring even a few points is considered a success.
- At 2010 completion only about half of the contestants score more than 1 point

5. Extracurricular Activities- Putnam competition

Some of our recent results

- 2008: Mark Hanna, Kahee Bowman, Richard Trimm -2 points each
- 2009: Anh Tran- 19 points, Goeal Tushar 10 points, Nathan Farmer 8 points
- 2010: Goel Tushar and Charles Dedrickson- 10 points each

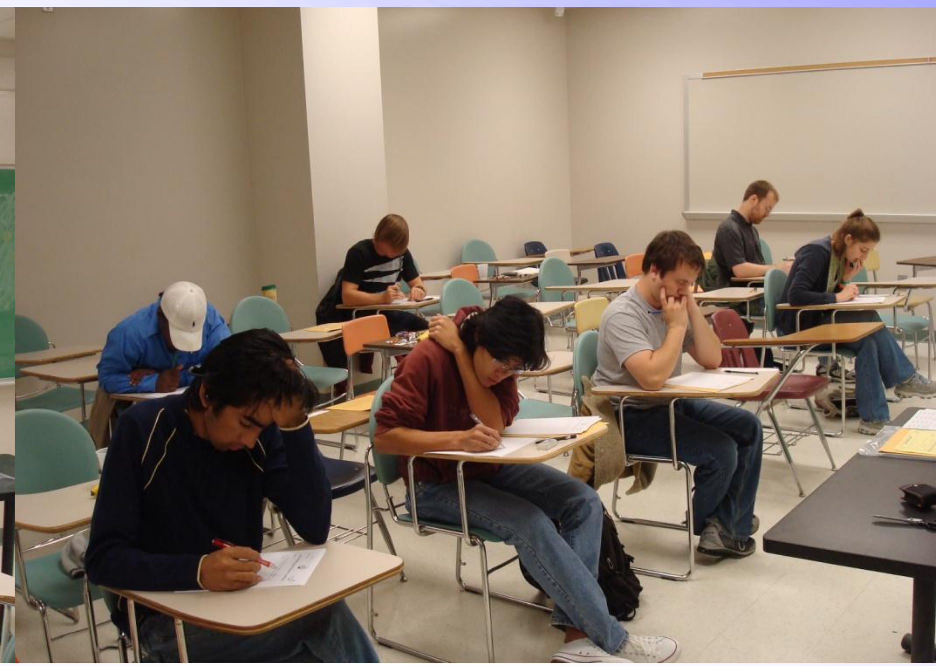
5. Extracurricular Activities- Putnam competition



Billy Jackson, Jacob Warren, Elizabeth Carver, Sean Craig

Putnam 2000 – The first one

5. Extracurricular Activities- Putnam competition



Elizabeth Lowe, Nathan Farmer, Toby Sanders, Nathan Dunn, Trey Banani, Tushar Goel, Anh Tran

Putnam 2009 – The most successful one

5. Extracurricular Activities-Mathematical Contest in Modeling

- Initiated by Ben Fusaro in 1985
- Organized by COMAP (Consortium for Mathematics and Its Applications**)**
- Date: Long weekend in February
- Format:
  - Teams of up to three students
  - Choose among two open-ended applied real-world problems
  - Allowed to use internet and literature
  - “Solution” of the problem is a paper

5. Extracurricular Activities- Mathematical Contest in Modeling

Our approach:

- Team meetings and discussion as needed
  - Read samples of previous wining papers
  - How to research the literature
  - Good modeling strategies
  - Good practices for project write-up, i.e. writing a paper
- Organization of the competition

5. Extracurricular Activities-Mathematical Contest in Modeling

Some statistics:

- Categories: Outstanding, Finalist, Meritorious, Honorable mention, Successful participant
- In 2010, 2254 teams participated (15 high school teams, 358 US Teams, 1890 Foreign Teams) and Winners were:
  - 9 Outstanding (1/2%)
  - 12 Finalists (1/2 %)
  - 431 Meritorious (19%)
  - 542 Honorable Mention (24%)
  - 1245 Successful Participant (55%)

5. Extracurricular Activities- Mathematical Contest in Modeling

Our results:

- Almost every year we have had a team that received an Honorable Mention placement
- The biggest success: In 2010 one of our teams (Elizabeth Lowe, Goel Tushar, and Anh Tran) won the designation of Finalist and placed 10th (!) overall out of all teams that selected to work on problem A.
- This really was an outstanding achievement!

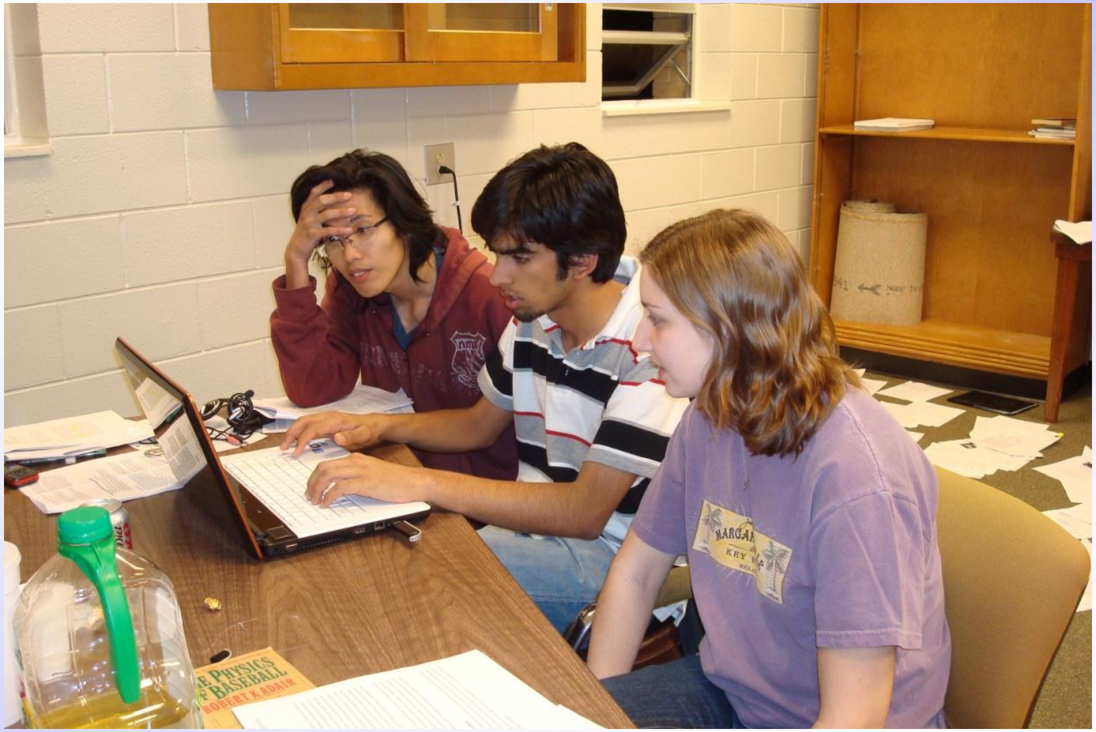
5. Extracurricular Activities-Mathematical Contest in Modeling

Jacob Warren, Richard Kilburn,

Elizabeth Carver, Emily Milette, Adrianne Dailey

MCM 2000-The first one

5. Extracurricular Activities-Mathematical Contest in Modeling



Anh Tran, Goel Tushar, Elizabeth Lowe

MCM 2010-The most successful one

5. Extracurricular activities-Benefits

- Promote active learning, synthesis of knowledge and creativity
- Help seeing mathematics as a dynamic and developing field which is both beautiful and extremely useful
- Excellent recruitment tool for both undergraduate and graduate programs
- Career determining experience

6. Concluding remarks

- Benefits of undergraduate research and extracurricular activities are numerous and evident
- Feedback from students have been positive; for some it has been a career determining experience
- Good students need attention too!
- We should emphasize more on undergraduate research and extracurricular activities being an integral and even a larger part of educational experience for our students

6. Concluding remarks

Example of career determining experience

- Jacob Warren, who is now a professor at Georgia Southern has written the following in his letter:
- "It was through these competitions that I was exposed to mathematical modeling, which eventually became my career choice. If Dr. Lesaja had not been as motivated as he is to enrich all the aspects of his students' education, I never would have found my career.”

6. Concluding remarks

Theorem: Advancement in becoming a nationally recognized student – oriented research university is equally proportional with increasing quality and quantity of undergraduate and graduate research and extracurricular activates for our students.

6. Concluding remarks

Corollary: Get busy!

THANK YOU !