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Scalability of Microscale Fractal-Like Branching Channel Networks

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Mechanical Engineering

By

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ABSTRACT

Fractal-like branching channel networks have been shown to offer the advantages of reduced pumping power and lower maximum wall temperatures when employed in the design of microscale heat sinks. Unfortunately, previous literature has been limited to microscale flow networks. In this paper, a microscale flow network will be scaled up to result in a mesoscale flow network and a macroscale flow network. In order to compare the pressure drop across the flow networks of varying scales the results will be non-dimensionalized in the form of the Euler number. For a laminar inlet Reynolds number the microscale, mesoscale, and macroscale fractal-like flow networks result in both qualitatively and quantitatively similar Euler number distributions. This suggests that results obtained when studying flow behavior at microscale can be used at larger scales.

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Introduction

There is a growing demand in today's society for more efficient ways to transport working fluids. For example, heat sinks are often utilized to cool electronic components. Liquid cooled heat sinks must efficiently transport a working fluid to provide adequate thermal management without excessive pumping power requirements. One problem with microscale heat sinks is large pressure drops, and thus large pumping power requirements, due to the size of the channels used to transport the working fluid. A second problem is surface temperature uniformity. Both of these issues make it difficult to provide adequate thermal management of electronic devices. A large pressure drop requires bigger, more expensive pumps and uneven surface temperatures can damage the device. As traditional, parallel channel, microscale flow networks have not alleviated these concerns many researchers have looked towards biologically-inspired designs.

In biological systems, hierarchical bifurcating flow networks provide efficient fluid transport. The terms "fractal-like" and "tree-like" are often used interchangeably to describe flow networks with hierarchical bifurcating flow passages. To resolve limitations of traditional parallel channel microscale flow networks, Pence [1] proposed a fractal-like bifurcating channel network. Pence numerically-investigated the performance of fractal-like branching channel networks under three separate conditions. The first condition assumed fully developed flow with convective heat transfer areas approximated using branch level hydraulic diameters. The second condition assumed fully developed flow with true convective heat transfer areas. The third condition assumed boundary layer redevelopment following each bifurcation with true convective heat transfer areas. Pence concluded that an increase in thermal efficiency associated with fractal-like branching networks is dependent on the foot print of, and channel spacing in, an application specific heat sink.

Pence [2] numerically compared the maximum wall temperature in, and pressure drop across, a fractal-like branching channel network and a traditional parallel channel network. Pence evaluated two networks, of equal heat transfer area, based on three separate conditions: (1) identical flow rates, (2) identical pressure drops, and (3) identical pumping power requirements. Under identical flow rates the fractal-like branching channel network resulted in a 117 kPa decrease in pressure drop and a 16 K decrease in maximum wall temperature when compared with the traditional parallel channel network. Similarly, with the pressure drop held constant the fractal-like branching channel network resulted in a 40 K decrease in maximum wall temperature. Finally, under identical pumping power the fractal-like branching channel network resulted in a 42 kPa decrease in pressure drop and a 30 K decrease in maximum wall temperature. Pence concluded that for the same surface area the use of a fractal-like branching channel network resulted in lower maximum wall temperatures when compared with parallel channel networks if the flow networks were operating under the same flow rate, pressure drop, or pumping power.

Pence and Enfield [3] compared fractal-like flow networks embedded in disk-shaped heat sinks and parallel channel networks embedded in rectangular-shaped heat sinks and concluded for identical applied heat flux, heat sink surface area, and flow rate a parallel channel heat sink resulted in lower maximum wall temperatures. However, for identical convective surface areas a fractal-like flow network resulted in lower maximum wall temperatures. Pence and Enfield found that if the ratio of the fractal-like convective area over the parallel channel convective area was greater than 0.5 a fractal-like heat sink would result in lower maximum wall temperatures than a parallel channel heat sink with a square cross-section. Pence and Enfield also evaluated performance based on a benefit-to-cost ratio which is defined as the ratio of the energy advected from the system over the flow power. The benefit-to-cost ratio was found to increase with channel width, height, and length. However, the number of parent branches emanating from the inlet plenum decreases as channel widths increase. This results in a decrease in convective surface area and thus higher maximum wall temperatures.

Alharbi et al. [4, 5] performed a three-dimensional computational fluid dynamics simulation of flow through a microscale fractal-like branching channel network with heat transfer in order assess the validity of the assumptions utilized in Pence's [reference] one-dimensional numerical model. Alharbi et al. concluded: (1) while the thermal boundary layer does not redevelop at each channel wall following bifurcations the assumption of boundary layer redevelopment should be retained in the one-dimensional model, (2) bifurcation angles should be considered independently of the model during the design of a fractal-like flow network, and (3) temperature dependent fluid properties should be incorporated in the one-dimensional model for high heat flux applications.

Wang et al. [6] computationally investigated the impact of bifurcation angle on pressure drop and surface temperature uniformity. Wang et al.'s three-dimensional computational fluid dynamic model incorporated temperature dependent fluid properties. An inlet Reynolds number of approximately 200 was used in the analysis. The channel walls in the bifurcating flow network were subject to a uniform heat flux. Wang et al. investigated a range of bifurcation angles between 30° and 180°. It should be noted that fillets were used at all bifurcations but the radius of the fillet was not specified nor optimized. Wang et al. found that the pressure drop through the tree-like flow networks was lower than through parallel channel flow networks of equal convective area and mass flow rate regardless of bifurcation angle. Wang et al. concluded that the pressure drop increases with increasing bifurcation angle. Smaller bifurcation angles are thus desirable if the goal is to reduce pressure drop. For smaller bifurcation angles maximum surface temperatures were observed at higher order branch levels. However, as the bifurcation angle increased surface temperature spikes could also be observed at lower order branch levels.

Pence [7] performed a comprehensive literature review of fractal-like flow networks and corresponding one and two-dimensional predictive models. Literature for both single phase and two phase flows in fractal-like flow networks was surveyed. In addition, numerous applications of fractal-like flow networks were presented. Pence also presented a methodology to design single phase fractal-like flow networks based on geometric and fabrication constraints. The literature review was current as of 2009.

To date, and to the knowledge of the author, there has not been an investigation into the scaling ability of the results obtained from studying microscale flow networks. If the fractal-like flow network system is proved to be scalable, then the assumption can be made that other advantages of bifurcating flow networks will also be present in meso- and macroscale flow networks. While microscale fractal-like flow networks are often used in the thermal management of electronic devices a mesoscale or macroscale flow network could be used to increase the efficiency of heat exchangers or solar flat plate collectors.

Description of Fractal-Like Flow Networks

The nomenclature and coordinate system describing the fractal-like flow networks utilized in the present analysis can be seen in Figure 1.

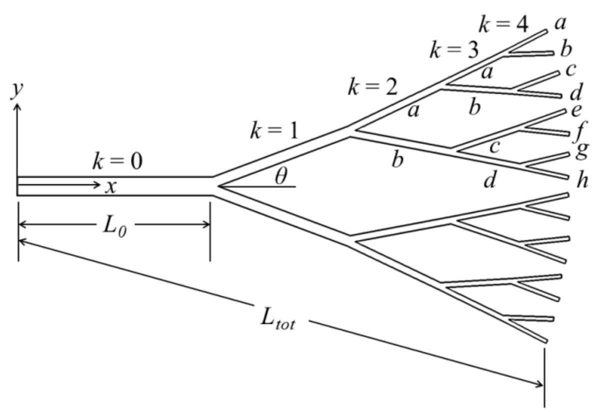

Figure 1. Nomenclature and Coordinate System for Fractal-Like Flow Network.

The first branch emanating from the inlet plenum is defined as the zeroth-order branch, where k = 0, and the last branch is defined as the fourth-order branch, where k = 4. The total length is defined as the radial distance between the entrance of the k = 0 branch and the exit of the k = 4 branch. The branch lengths are measured radially as seen in Figure 1. The flow network is asymmetric as the bifurcation angles are path-dependent. In order to characterize the asymmetric branching two different paths will be defined. Path 1 consists of the branch segments k = 0, 1, 2a, 3a, and 4a. Path 2 consists of the branch segments, k

= 0, 1, 2b, 3c, and 4f. The bifurcation angle is defined as the angle from which a higher level branch splits from a lower level branch. For example, the second order branch level, k = 2, splits from the first order branch level, k = 1, at an angle of 5.17° along path 1 and 30.89° along path 2. The following branch scale ratios were utilized,

$$\beta = \frac{d_{k+1}}{d_k} = n^{-1/3} \quad (1)$$

$$\gamma = \frac{L_{k+1}}{L_k} = n^{-1/2} \quad (2)$$

For this analysis, n = 2 as each parent channel splits into 2 daughter channels. The hydraulic diameters scale in accordance with Murray's law and are thus connected in a way that theoretically provides bulk fluid transport for the least amount of work. The fractal-like flow network employed in the present analysis is a scaled version similar to the microscale flow network employed by Alharbi et al. [4, 5]. The bifurcation angles chosen in the present analysis are identical to those employed by Alharbi et al. [4, 5].

The dimensions of the microscale fractal-like flow network employed in the present analysis is provided in Table 1. A scaling factor of ten was applied to the microscale network to achieve the mesoscale network. The scaling factor was chosen so that the channel width of the fourth-order branch level, where k = 4, would be on the order of microscale. The scaled network is thus denoted as "mesoscale" in the present analysis. Similarly, a scaling factor of ten was applied to the mesoscale network to achieve the macroscale network. The dimensions of the mesoscale flow network can thus be obtained by multiplying the dimensions of the microscale flow network by 10 and by 100 respectively.

Table 1. Channel Dimensions for Microscale Fractal-Like Flow Network

k	H_k (mm)	w_k (mm)	d_k (mm)	L_k (mm)	$\theta_{path\ 1}$ (°)	$\theta_{path\ 2}$ (°)
0	0.250	0.539	0.342	5.80	-	-
1	0.250	0.296	0.271	4.10	20.36	20.36
2	0.250	0.189	0.215	2.90	30.89	5.17
3	0.250	0.130	0.171	2.05	28.95	0.38
4	0.250	0.093	0.136	1.45	24.15	1.42

Computational Fluid Dynamics Model

The computational fluid dynamic simulations were performed using commercially available computational fluid dynamics software, employing the finite volume method. The steady, incompressible form of the three-dimensional continuity and momentum equation are as follows:

$$\frac{\partial u_i}{\partial x_i} = 0 \tag{3}$$

$$\rho \frac{\partial (u_i u_j)}{\partial x_i} = \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_i}\left(\mu \frac{\partial u_j}{\partial x_i}\right) \tag{4}$$

The governing equations are solved with the finite volume method on a spatially rectangular computational mesh with the planes orthogonal to the Cartesian coordinate system axes and refined locally at the solid/fluid interface and in the fluid region. The governing equations are discretized in a conservative form and the numerical model is second-order accurate in space.

The fluid enters the fractal-like flow network at the entrance of the k = 0 branch level and discharges at the exit of the k = 4 branch level. At the inlet of the k = 0 branch the fluid temperature is fixed and a uniform velocity profile is assumed. The velocity profile was assumed to be uniform as in Alharbi et al. [4, 5]. The volumetric flow rate is 0.9 ml/s for the microscale flow network, 9.0 ml/s for the mesoscale flow network, and 90 ml/s for the macroscale flow network. The flow is assumed to be laminar only as the inlet volumetric flow rates correspond to an inlet Reynolds number on the order of 2000. The fluid exits the k = 4 branch level at environmental pressure. The boundary conditions for the flow networks of varying scale are identical to the boundary conditions employed by Alharbi et al. [4, 5] with the exception that a uniform heat flux was not applied at channel walls. The pressure drop across the tree-like flow network is defined as the static pressure difference between the fluid inlet at the entrance of the k = 0 branch and the fluid discharge at the exit of the k = 4a branch.

The computational model was previously validated by Alharbi et al. [5] with experimental data. The number of cells in the three-dimensional computational model of the fractal-like flow network was successfully doubled until grid independence was achieved. The three-dimensional computational mesh consisted of approximately 1,500,000 fluid cells and 500,000 partial cells. The use of approximately 2,000,000 cells proved to be sufficient for the analysis.

Results and Discussion

The results were cast in dimensionless form for ease of scaling comparison. Results are presented for an inlet Reynolds number of 2000. The Reynolds number is the ratio of inertia to viscous forces and is defined as,

$$\mathrm{Re} = \frac{ud}{\nu} \tag{5}$$

The non-dimensional axial distance is defined as,

$$X = \frac{x}{L_{tot}} \tag{6}$$

The Euler number is the ratio of pressure to inertia forces and is defined as,

$$\text{Eu} = \frac{\Delta p}{\rho u^2} \tag{7}$$

The pressure difference term is defined as the difference between the static pressure along the centerline and environmental pressure at the exit. Similarly, the velocity term is the average fluid velocity along the centerline. The non-dimensional mid-depth centerline pressure distribution along paths 1 and 2 for the microscale and mesoscale flow networks can be seen in Figure 2 and 3.

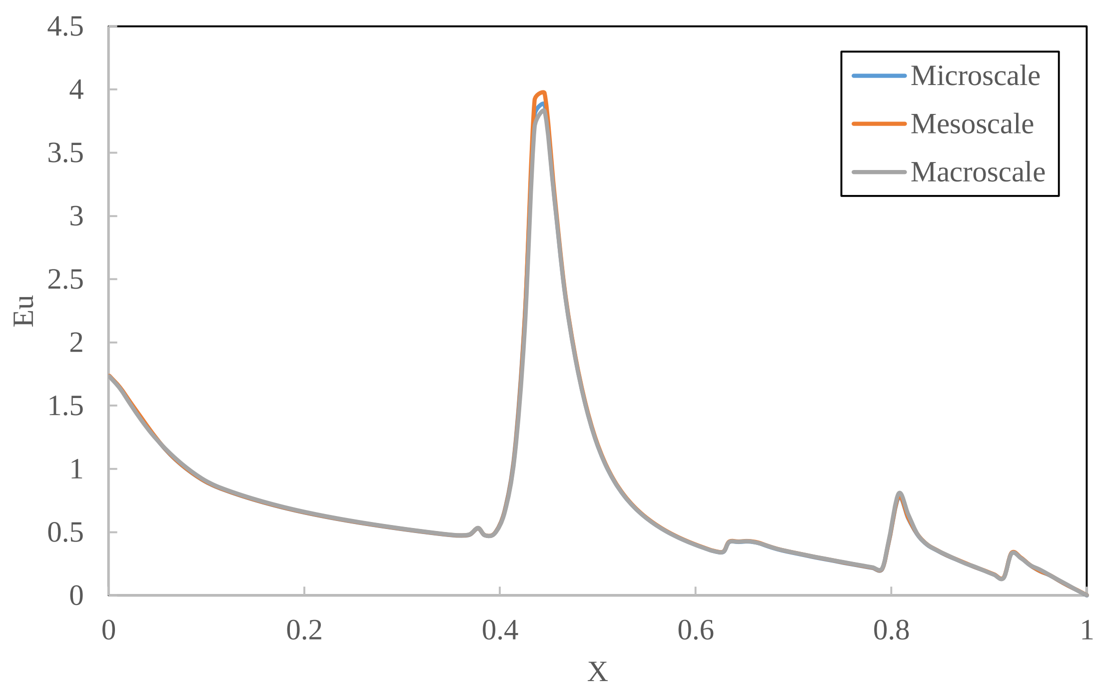

Figure 2. Centerline Euler Number Distribution along Path 1

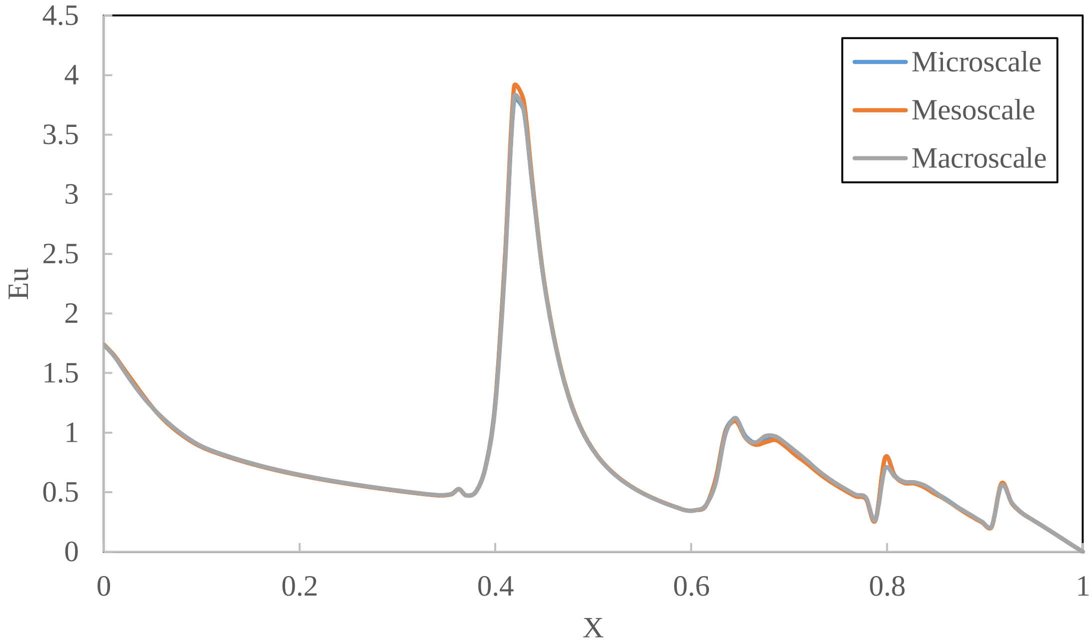

Figure 3. Centerline Euler Number Distribution along Path 2

As evidenced in Figures 2 and 3 the micro-, meso-, and macroscale flow networks all exhibit the greatest local pressure recovery at the first bifurcation. The local pressure recovery is path, and thus bifurcation angle, dependent. For example, there is a larger local pressure recovery at the second bifurcation along path 2 when compared with path 1. The local pressure recovery is a consequence of secondary flow conditions that result from increased cross-sectional areas at bifurcations and subsequent flow deceleration. The importance of these local pressure recoveries is a reduction in total pressure drop and thus lower required pumping power for the system that implements these channels.

As previously mentioned, flow behavior is highly dependent on the path taken due to the asymmetric nature of the flow networks. The flow separates symmetrically at the first bifurcation between the zeroth-order branch and the first-order branch. The flow divides asymmetrically at all subsequent bifurcations. Mid-depth velocity magnitude contours at all bifurcations for the microscale, mesoscale, and macroscale flow networks can be seen in Figures 4-6 respectively. Following each bifurcation there is a significant decrease in velocity. This is expected due to the conservation of mass requirement. As seen in Figures 4-6, the flow travels closer to the inner walls due to the inertia of the flowing fluid and the flow separation associated with the bifurcation angles.

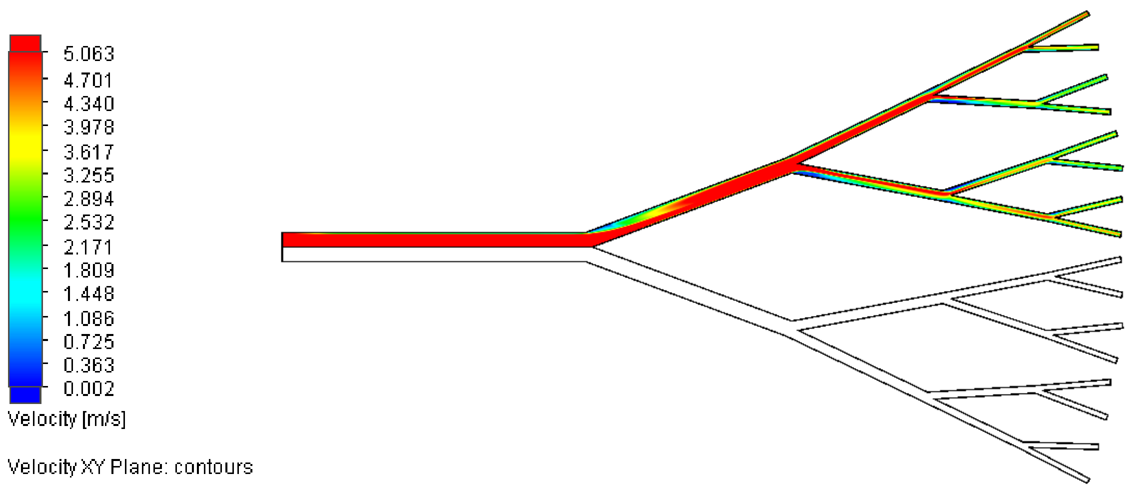

Figure 4. Mid-Depth Velocity Contours for the Microscale Fractal-Like Flow Network

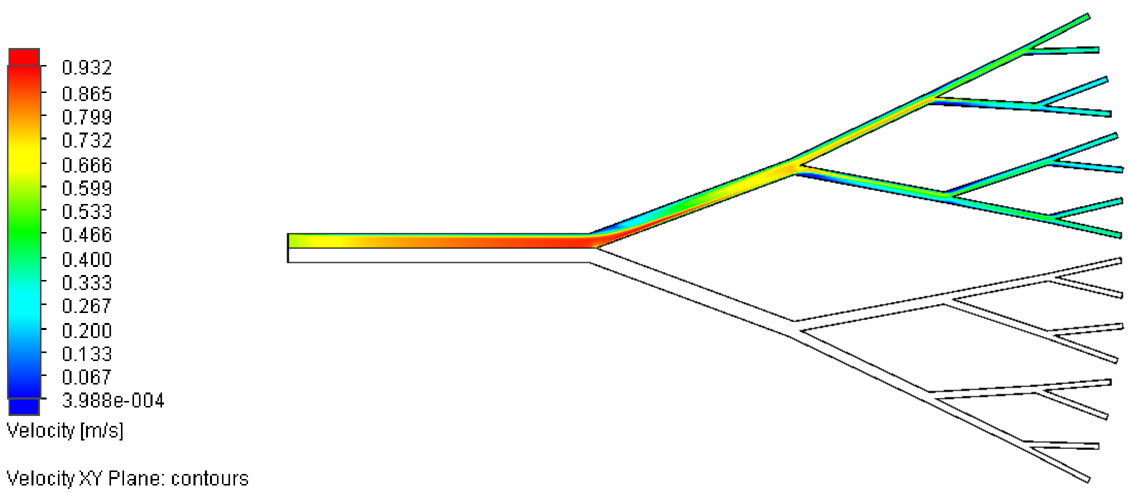

Figure 5. Mid-Depth Velocity Contours for the Mesoscale Fractal-Like Flow Network

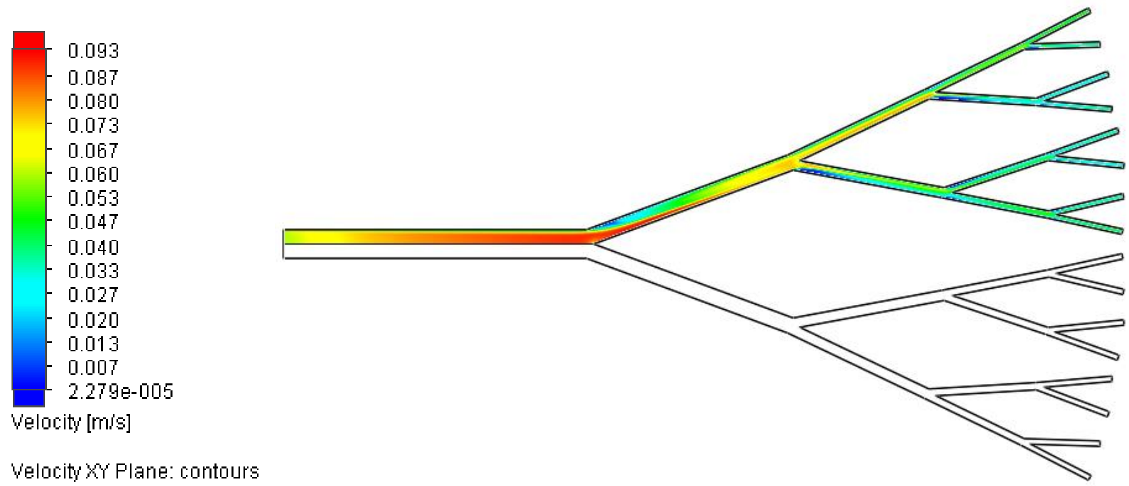

Figure 6. Mid-Depth Velocity Contours for the Macroscale Fractal-Like Flow Network

It can also be observed that the fluid stagnates at the inner wall of bifurcations which results in a drop in velocity and thus an increase in pressure. While the velocities within the flow networks are quantitatively different due to the scale of the networks the trends are qualitatively similar. There is a clear deceleration in fluid velocity at bifurcations. Following bifurcations, there is flow separation as the inertia of the fluid carries the fluid towards the inner wall. This phenomena is less pronounced for smaller bifurcation angles. Regardless, flow behavior at microscale is qualitatively similar to flow behavior at mesoscale and macroscale for a laminar inlet Reynolds number. This analysis thus validates the scalability of the fractal-like branching channels under the conditions investigated. Results obtained by studying microscale flow networks can thus be extended to flow networks of larger scale. This could result in the implementation of fractal-like flow networks in heat exchanger and flat plate collectors. It is thus hypothesized that in the future these network designs could result in increased efficiency of various heat exchangers and solar flat plate collectors.

Conclusions and Recommendations

A microscale fractal-like flow network as presented in prior literature was scaled up twice to produce a mesoscale and macroscale flow network. The purpose of the scaling was to assess if results obtained by studying microscale fractal-like flow networks could be applied to flow networks of larger scales. Computational fluid dynamics simulations were performed for laminar inlet Reynolds numbers. While the pressure drop, and thus pumping power, across the micro-, meso-, and macroscale flow networks was quantitatively different when the results were non-dimensionalized with the Euler number it was apparent that local flow behavior present at microscale was also present at macroscale. From the results, there is no noticeable difference between the non-dimensional local pressure recoveries experienced by various flow networks. While the flow behavior for laminar flow is similar it may not be if the flow was turbulent. In the future, it is recommended to study a range of inlet Reynolds numbers. In addition, a rectangular cross-section was utilized in this analysis. It may be appropriate to investigate the effect of cross-sectional geometry on local flow behavior and pressure drop. For example, the veins in tree leaves have circular cross-section. It is concluded that scaling may be appropriate and fractal-like branching channel networks have potential in applications beyond microscale heat sinks. The main obstacle will be cost effectiveness between the reduction of pumping power and the increase in manufacturing resources associated with the complex network design.

Nomenclature

d	hydraulic diameter
H	channel height
k	branch level
L	channel length
n	number of daughter branches for each parent branch
p	pressure
Re	Reynolds number
u	velocity
w	channel width
X	non-dimensional channel length
x	local distance along channel
β	diameter scale ratio
γ	length scale ratio
θ	bifurcation angle
μ	dynamic viscosity
ν	kinematic viscosity
ρ	density

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