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**Session 3pEA: Computational Methods in Transducer Design, Modeling, Simulation,
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3pEA6. Energy flux streamlines versus acoustic rays for modeling interaction with rigid boundaries: near field of sound from a circular loudspeaker

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Sound emitted by a circular loudspeaker can be treated as equivalent to a plane wave diffracted by a circular aperture in a rigid, sound absorbing screen. Axial symmetry leads one to expect constructive interference along the symmetry axis in the near field (the Poisson-Arago spot). An energy flux streamline model was developed to help visualize this and other features of the near sound field. The model is used to draw out similarities and differences between energy flux streamlines and acoustic rays, particularly in the transition to the far field.

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INTRODUCTION

The utility of energy flux streamlines in visualizing scattering phenomena has been demonstrated by David M. F. Chapman, who has also pointed out some of the differences between energy flux streamlines and ray analysis techniques.¹ Michel and Alexandre Gondran have shown how², in optics, energy flux streamlines “correspond to the diffracted rays of Newton’s *Opticks*.” A simple example, made somewhat numerically demanding by examining the near field, of single frequency sound from a circular loudspeaker in air, modeled as a plane wave diffracted by a circular aperture in an absorbing screen, is used to show how energy flux streamlines help in the visualization of various interference phenomena, especially along the symmetry axis of the diffraction problem.

THEORY

Using the Huyghens-Fresnel principle, we may solve for the sound pressure level in air in the near field due to a circular loudspeaker, treated as a circular aperture diffracting a normally incident monofrequency plane sound wave, by summing up the individual contributions of each differential area of the circular aperture dA wherein each differential area is taken to be an emitter of a spherical wavefront. See Fig. 1. The resulting sound pressure field is given by the real part of the following integral over the loudspeaker/circular aperture area:

$$p(\mathbf{r}, t) = e^{-i\omega t} \int_{\xi=-a}^{\xi=+a} \int_{\eta=-\sqrt{a^2-\xi^2}}^{\eta=+\sqrt{a^2-\xi^2}} \frac{p_0}{\Delta r} e^{ik\Delta r} d\xi d\eta, \quad (1)$$

where p is the complex representation of the pressure; t is time; \mathbf{r} gives the position of the field point P (the origin is at the center of the loudspeaker), and the plane of the loudspeaker is in the x, y – plane. Also a is the radius of the circular loudspeaker, i is the usual square root of minus one; ξ and η are dummy variables for integration over the area of the loudspeaker, treated as a circular diffracting aperture; p_0 is the overpressure amplitude of sound as measured at the loudspeaker (or, equivalently, the amplitude of the plane wave to be diffracted by the circular aperture). The angular frequency is $\omega = 2\pi f$, where f is the frequency of the sound. The wavenumber is $k = 2\pi/\lambda$, where λ is the wavelength of the sound. Finally, Δr is the separation distance between an individual source dA and the field point P (see Fig. 1):

$$\Delta r = [(x - \xi)^2 + (y - \eta)^2 + z^2]^{1/2}, \quad (2)$$

where the source dA is at $x = \xi, y = \eta$, and the field point P is at (x, y, z) . The relative displacement vector $\Delta \mathbf{r}$ points from the source dA to the field position at P , thus a unit relative displacement vector is given by

$$\mathbf{e}_{\Delta r} = \frac{x - \xi}{\Delta r} \mathbf{i} + \frac{y - \eta}{\Delta r} \mathbf{j} + \frac{z}{\Delta r} \mathbf{k}, \quad (3)$$

where \mathbf{i} , \mathbf{j} , and \mathbf{k} are the usual Cartesian unit vectors in the positive x , y , and z directions, respectively.

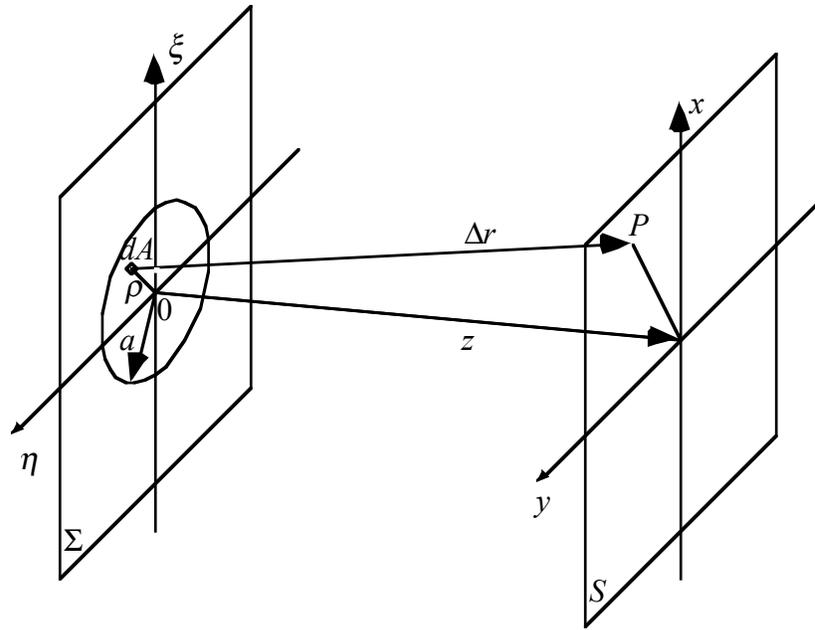


FIGURE 1. The geometry of the near-field scattering from a circular loudspeaker.

Assuming a harmonic time variation, the particle velocity of sound in air is proportional to a gradient of the sound pressure³. In complex representation it is

$$\mathbf{u}(\mathbf{r}, t) = \frac{1}{i\omega\rho_0} \nabla p(\mathbf{r}, t), \quad (4)$$

where ρ_0 is the mass density of air. Then the time-averaged energy flux vector for the sound field is⁴

$$\mathbf{I}(\mathbf{r}) = \text{Re}\left\{\frac{1}{2} p \mathbf{u}^*\right\}, \quad (5)$$

where the asterisk denotes complex conjugation.

By taking the gradient operator inside the integral for the complex sound pressure, the particle velocity can be shown to be

$$\mathbf{u}(\mathbf{r}, t) = \frac{p_0 e^{-i\omega t}}{i\omega\rho_0} \int_{\xi=-a}^{\xi=+a} \int_{\eta=-\sqrt{a^2-\xi^2}}^{\eta=+\sqrt{a^2-\xi^2}} \left(\frac{1}{\Delta\mathbf{r}} - ik\right) \frac{\mathbf{e}_{\Delta\mathbf{r}}}{\Delta r} e^{-ik\Delta r} d\xi d\eta. \quad (6)$$

Using Eqs. (6) and (1) in Eq. (5) results in

$$\mathbf{I}(\mathbf{r}) = \frac{p_0^2}{2\omega\rho_0} \text{Re}\left\{\int_{\xi=-a}^{\xi=+a} \int_{\eta=-\sqrt{a^2-\xi^2}}^{\eta=+\sqrt{a^2-\xi^2}} \frac{e^{ik\Delta r}}{\Delta r} d\xi d\eta \int_{\xi=-a}^{\xi=+a} \int_{\eta=-\sqrt{a^2-\xi^2}}^{\eta=+\sqrt{a^2-\xi^2}} \left(\frac{1}{\Delta\mathbf{r}} - ik\right) \frac{\mathbf{e}_{\Delta\mathbf{r}}}{\Delta r} d\xi d\eta\right\}. \quad (7)$$

PRELIMINARY RESULTS

Using values typical of recent experimental work at Georgia Southern University⁵, specifically $\lambda = 0.03447$ m, $f = 10\,000$ Hz, $a = 0.0525$ m, $p_0 = 0.2882$ Pa, and $\rho_0 = 1.205$ kg/m³, streamlines for the energy flux vector are plotted using MATHEMATICA in the near-field region with particular attention to the field in and around the central symmetry axis. Using far-field approximations results in a sound pressure proportional to a so-called “jinc” function, a J_1 Bessel function divided by its argument (the angular displacement from the symmetry axis).⁶ The

particle velocity is also proportional to a jinc function of like argument, so that the time-averaged energy flux vector is proportional to the square of a jinc function. This asymptotic representation leads to a very simple field as shown in Fig. 2 as a three-dimensional grid of energy flux vectors using the Macintosh GRAPHER program, or as a set of streamlines in a plane containing the symmetry as axis as plotted using MATHEMATICA as in Fig. 3. The correct near-field solution is much more intricate and requires significantly more computation time since the integrals involve rapidly oscillating arguments for both the pressure field and the particle velocity field. Final results will be presented at the Montréal ICA/ASA Meeting in 2013.

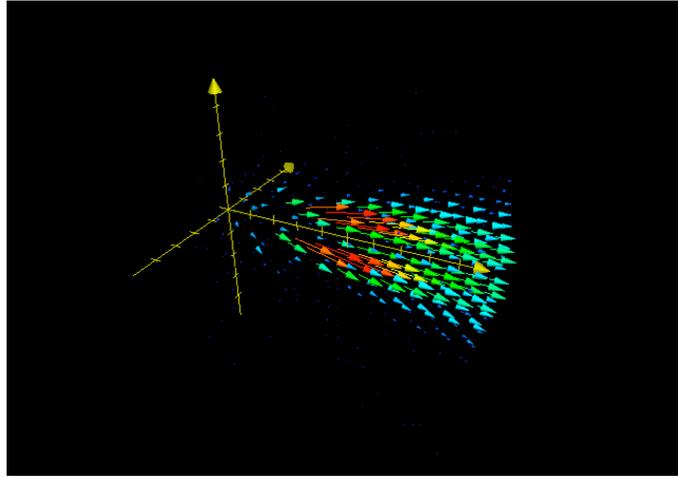


FIGURE 2. Preliminary asymptotic results as a three dimensional grid of intensity vectors. The far-field “jinc” function approximation is being used.

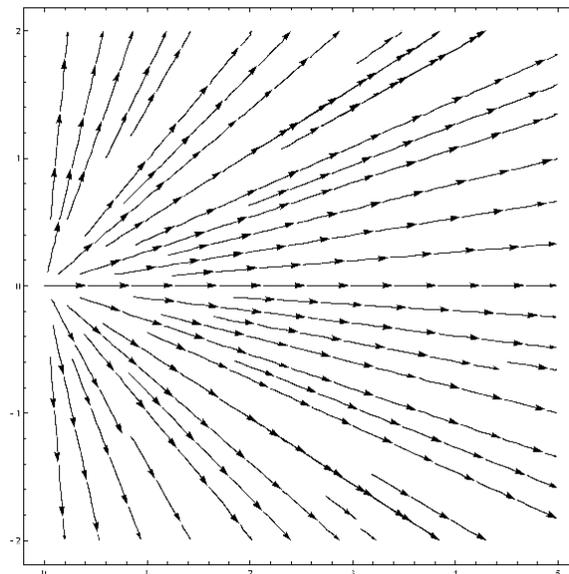


FIGURE 3. Preliminary asymptotic results as intensity streamlines. The far-field “jinc” function approximation is being used.

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