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Packing Densities of Colored and Non-Colored Patterns

Matthew R. Just
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Packing Densities of Colored and Non-Colored Permutations

Matthew Just

Georgia Southern University
Department of Mathematics

April 24, 2015

Why Study Patterns?

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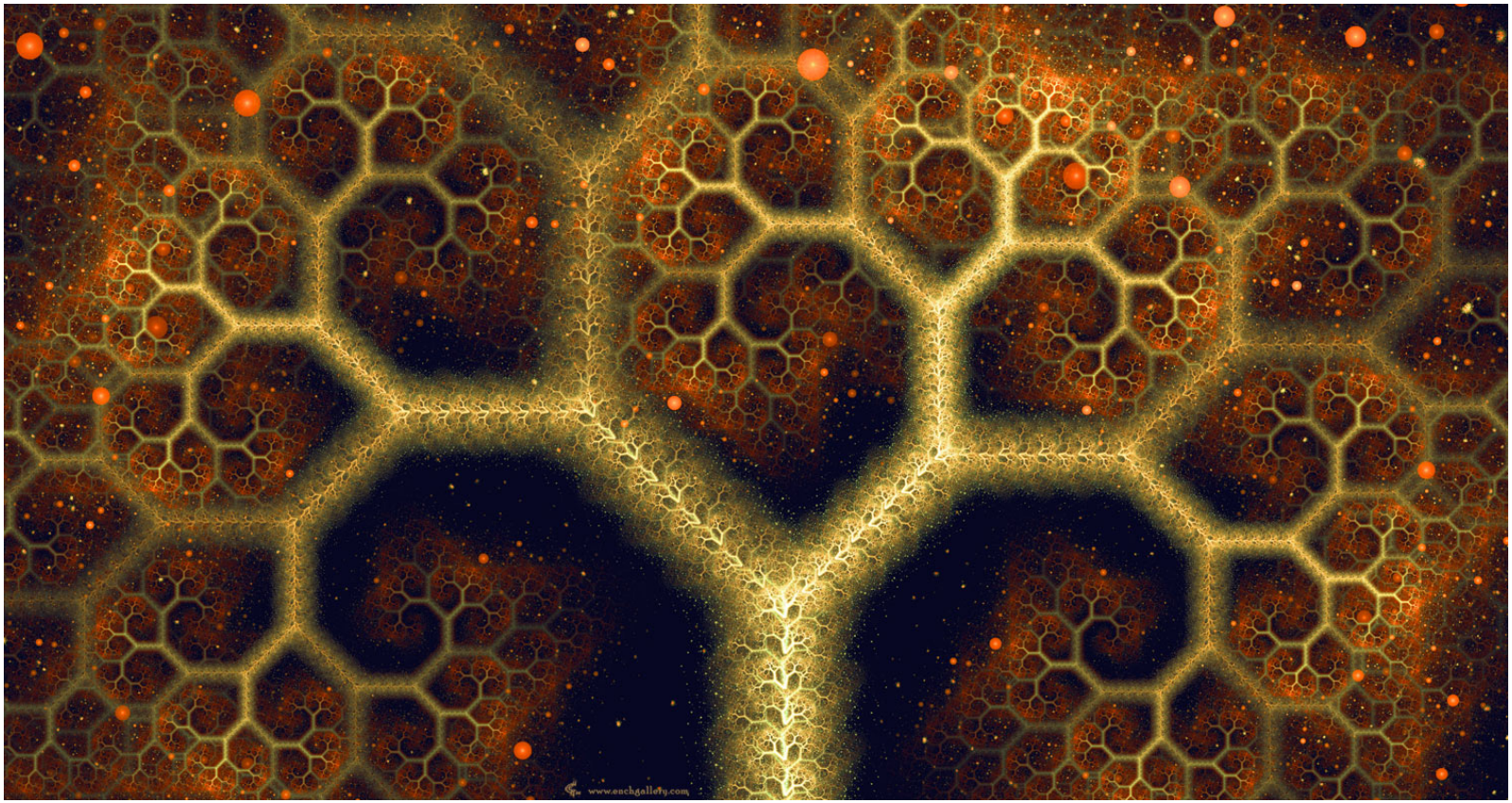

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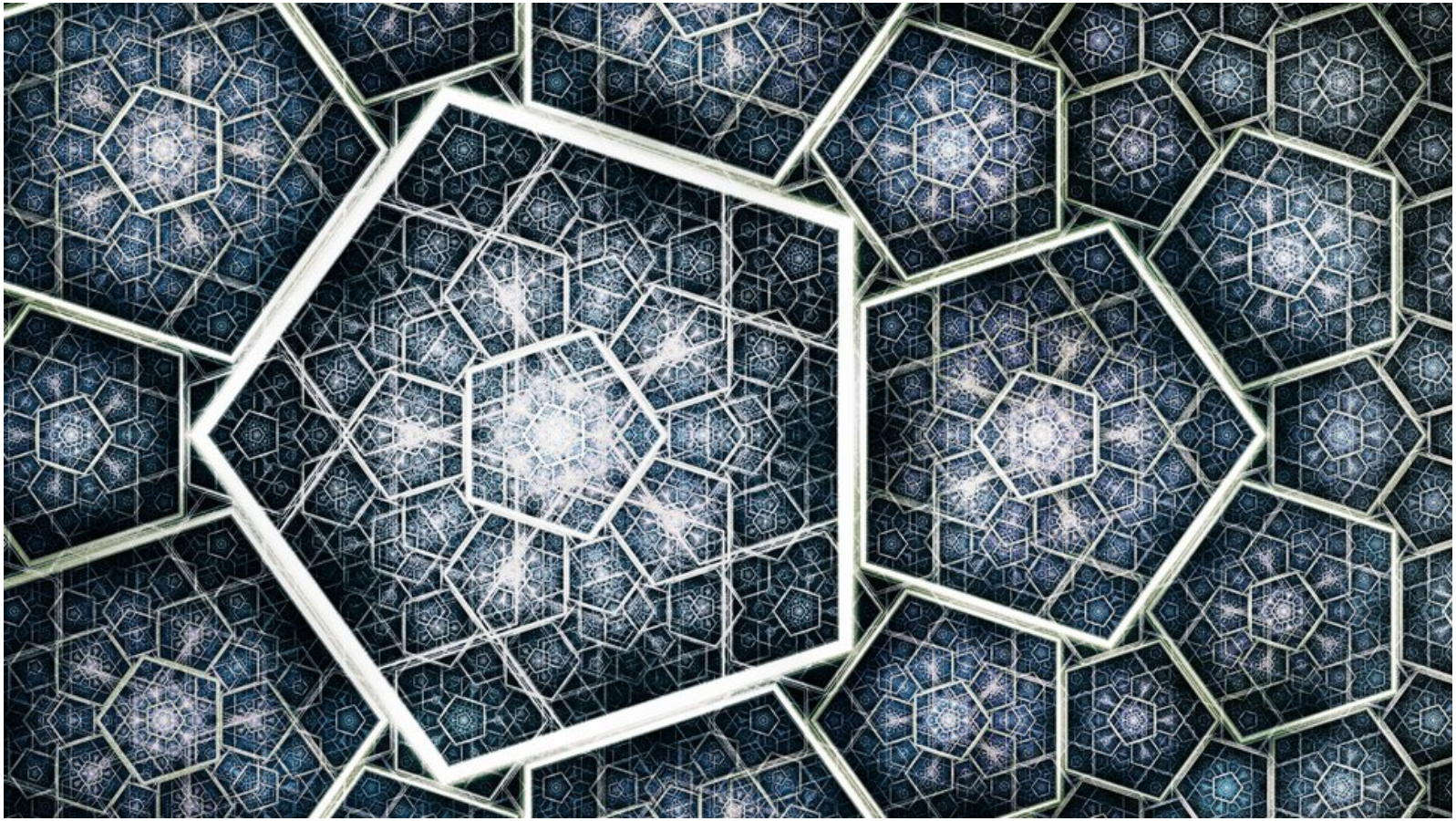

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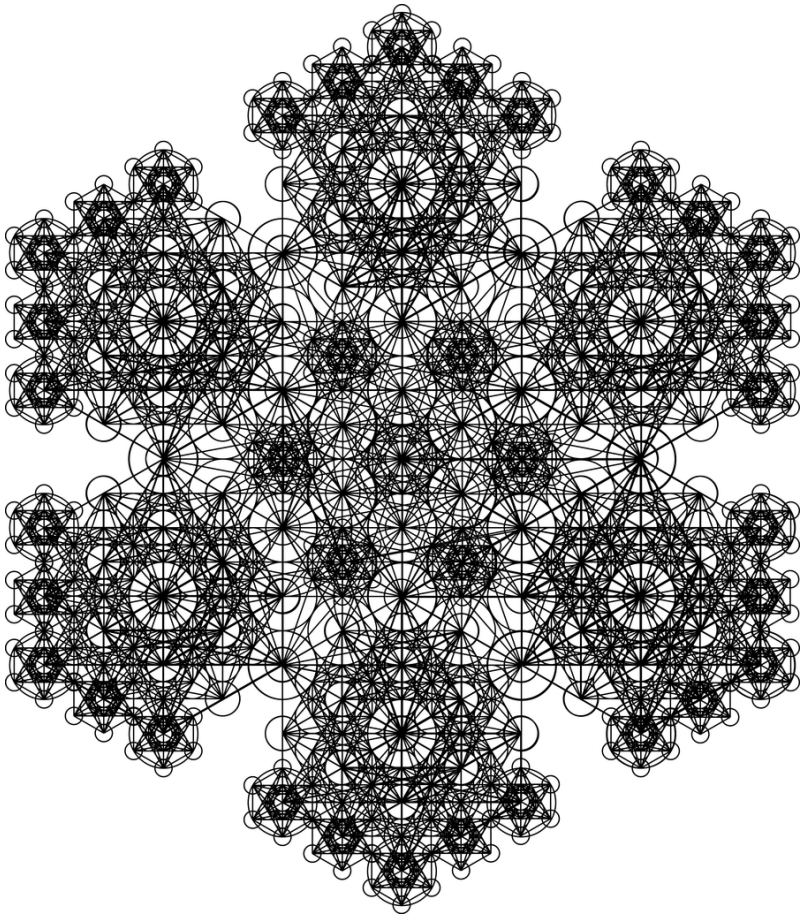

1. **What is a Permutation?**
   - Sets vs. Multisets
   - Permutations of Sets vs. Multisets
2. **Patterns in Permutations**
   - What is a pattern?
   - Pattern avoidance and pattern packing
   - Pattern packing in set permutations
   - Pattern packing in multiset permutations
3. **Colored Permutations**
   - Combining sets and multisets
   - Colored blocks
   - Concluding remarks

Sets

A **set** is a collection of objects that are all distinct.

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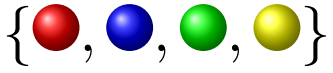

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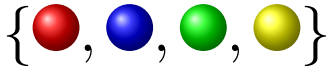

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Multisets

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"Hello World"

Permutations of Sets

A **permutation** is a rearrangement of the elements in a set.

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$$9! = 362,880$$

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How many total permutations of M?

$$\binom{9}{4,2,2,1} = 3,780$$

Patterns and Pattern Occurence

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A pattern **occurs** in a permutation if there is a subsequence of the permutation similar to the pattern.

Examples

Does the pattern $\{3, 2, 1\}$ occur in the following permutation?

$$\{5, 8, 2, 6, 1, 4, 3, 7, 9\}$$

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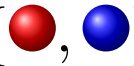
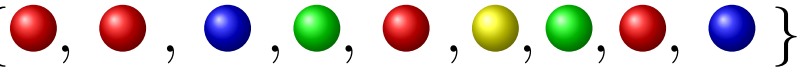
Does the pattern {  } occur in the following permutation?

{  }

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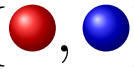
Does the pattern $\{$  $\}$ occur in the following permutation?

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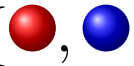
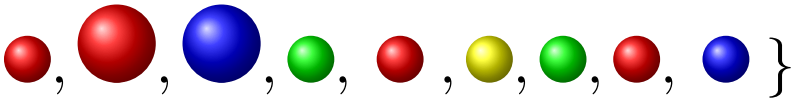
Does the pattern $\{$  $\}$ occur in the following permutation?

$\{$  $\}$

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Optimal Permutations of Patterns

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Every length-3 subsequence of S_3 is a $\{3, 2, 1\}$ pattern, thus no other permutation of length 9 has more occurrences.

Packing Densities of Patterns

When an optimal permutation is found, the **packing density** is defined

$$\delta(\{\text{pattern}\}) = \frac{\#\text{ of occurrences of }\{\text{pattern}\}\text{ in }\{\text{optimal permutation}\}}{\text{number of subsequences in }\{\text{optimal permutation}\}}$$

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If δ depends on the length of the permutation, supremum is taken.

Interpretation of the Packing Density

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Pattern packing is all about *efficiency*.

Increasing/Decreasing Patterns

An **increasing** pattern: $\{1, 2, \ldots, k\}$.
A **decreasing** pattern: $\{k, (k-1), \ldots, 1\}$.

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Every subsequence of an increasing/decreasing permutation is an increasing/decreasing pattern.

Layered Patterns

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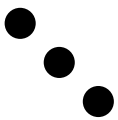

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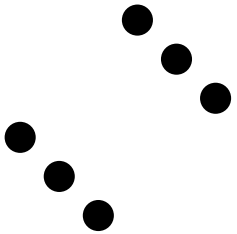

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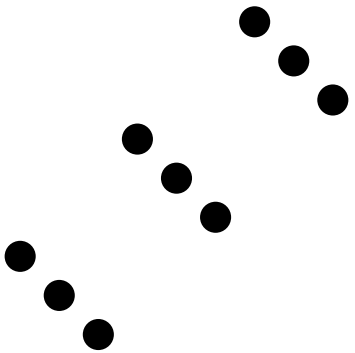

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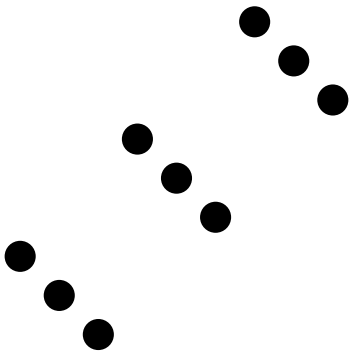

For any layered pattern, there exists an optimal permutation that is also layered [Albert, et al.].

Examples

As a consequence, finding the packing density of layered patterns reduces to numerical optimization.

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$\{\text{pattern}\}$	$\delta(\{\text{pattern}\})$	$\{\text{optimal permutation}\}$ (length 9)
$\{1,2,3\}$	1.000	$\{1,2,3,4,5,6,7,8,9\}$
$\{1,3,2\}$	0.464	$\{1,3,2,9,8,7,6,5,4\}$
$\{1,4,3,2\}$	0.424	$\{4,3,2,1,9,8,7,6,5\}$
$\{2,1,4,3\}$	0.375	$\{2,1,9,8,7,6,5,4,3\}$

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$\{2,1,4,3\}$	0.375	$\{2,1,9,8,7,6,5,4,3\}$

No current studies on **non-layered** patterns. Why?

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Consider the pattern $\{$ $\}$

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Consider the pattern $\{$ $\}$

An optimal permutation will *always* be colored in the same way.

$\{$ $\}$

Proof

Theorem

An optimal permutation of the pattern $\{$🔴, 🔴, 🔵$\}$ *exists that is colored in the same way.*

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{🔴, 🔴, 🔴, 🔴, 🔴, 🔵, 🔴, 🔵, 🔵}

...if we switch these...

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Proof: Assume not! (Reductio ad absurdum)

{🔴, 🔴, 🔴, 🔴, 🔴, 🔵, 🔴, 🔵, 🔵}

{🔴, 🔴, 🔴, 🔴, 🔴, 🔴, 🔵, 🔵, 🔵}

We have gained a {🔴, 🔴, 🔵} pattern, a contradiction! □

Examples

$\{$pattern$\}$	$\delta(\{$pattern$\})$	$\{$optimal permutation$\}$ (length 9)
$\{\bullet, \bullet, \bullet\}$	1.000	$\{\bullet, \bullet, \bullet, \bullet, \bullet, \bullet, \bullet, \bullet, \bullet\}$
$\{\bullet, \bullet, \bullet\}$	0.444	$\{\bullet, \bullet, \bullet, \bullet, \bullet, \bullet, \bullet, \bullet, \bullet\}$
$\{\bullet, \bullet, \bullet, \}$	0.222	$\{\bullet, \bullet, \bullet, \bullet, \bullet, \bullet, \bullet, \bullet, \bullet\}$
$\{\bullet, \bullet, \bullet, \bullet\}$	0.094	$\{\bullet, \bullet, \bullet, \bullet, \bullet, \bullet, \bullet, \bullet, \bullet\}$

Colored Permutations

A **colored permutation** is a combination of a set and a multiset.

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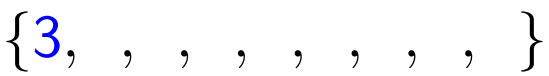

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Colored permutations can be thought of as a deck of cards.

Colored Blocks

Every colored permutation can be split into **colored blocks**:

$$312|6|54|879$$

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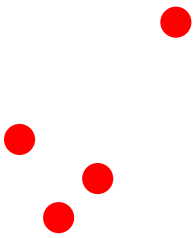

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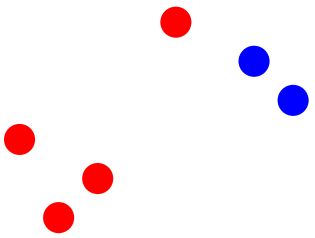

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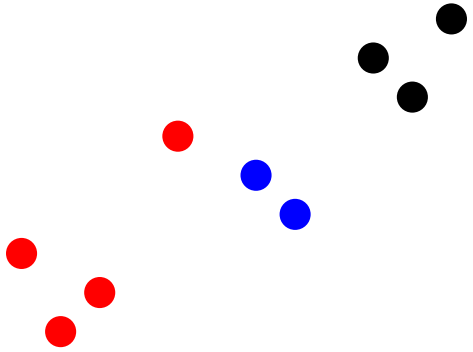

Optimal Permutations of Colored Patterns

Theorem (Just, Wang)

For every colored pattern consisting of two or three colored blocks there exists an optimal permutation that has the same number of colored blocks, each colored in the same way as the pattern.

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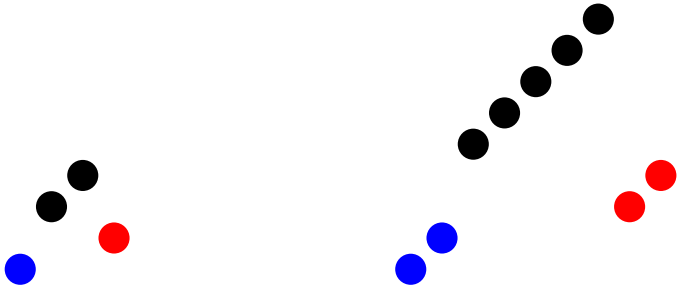
For every colored pattern consisting of two or three colored blocks there exists an optimal permutation that has the same number of colored blocks, each colored in the same way as the pattern.

$\{1, 3, 4, 2\}$ $\{1, 2, 5, 6, 7, 8, 9, 4, 5\}$



Examples

$\{\text{pattern}\}$	$\delta(\{\text{pattern}\})$	$\{\text{optimal permutation}\}$ (length 9)
$\{1, 3, 4, 2\}$	0.188	$\{1, 2, 5, 6, 7, 8, 9, 4, 5\}$
$\{1, 4, 2, 3\}$	0.198	$\{1, 2, 3, 5, 4, 9, 8, 7, 6\}$

Further Research

Any better way to tackle non-layered patterns?

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Studying more than two or three blocks in colored permutations?

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Combining pattern avoidance with pattern packing?

Thank You![1]

[1] This work was partially supported by grants from the Simons Foundation (#245307).