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Critical Thinking and the Languages of STEM

Connie H. Rickenbaker
Georgia College

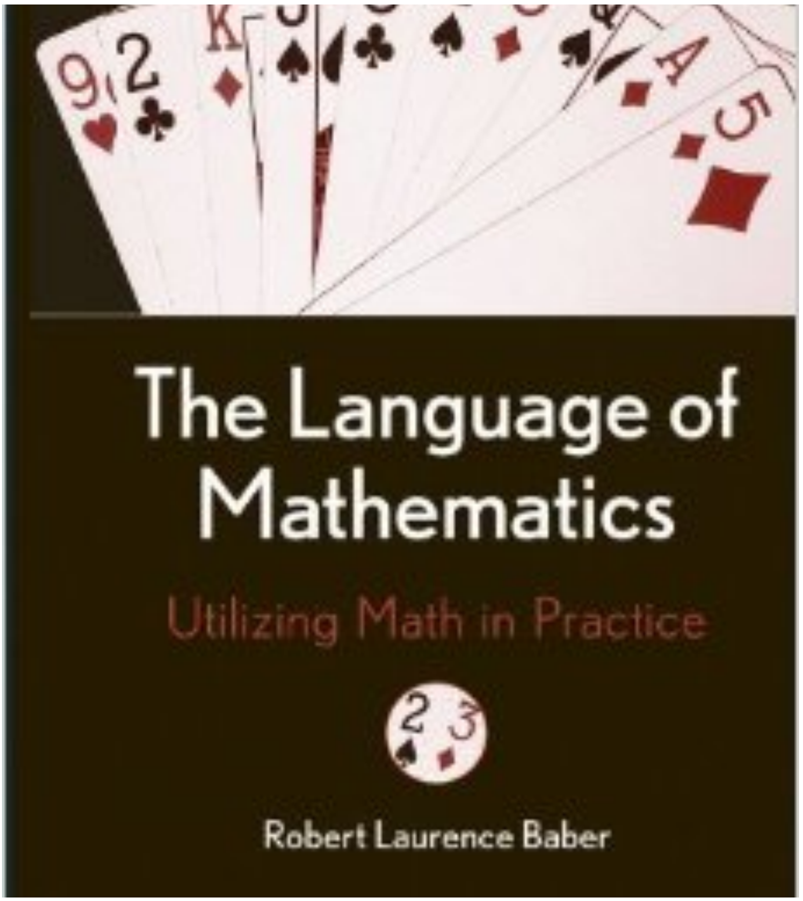
Sally Gilbreth
Georgia College

Critical Thinking and the Languages of STEM

Sally Gilbreth, Math Major with Teaching Concentration,
Graduate Student in Mathematics at Georgia Southern

Connie H. Rickenbaker, Ph.D., Project FOCUS Coordinator

Introduced/Used in All FOCUS Classes



Intended Readership: p. 13

Teachers – elementary through college of science, math or languages

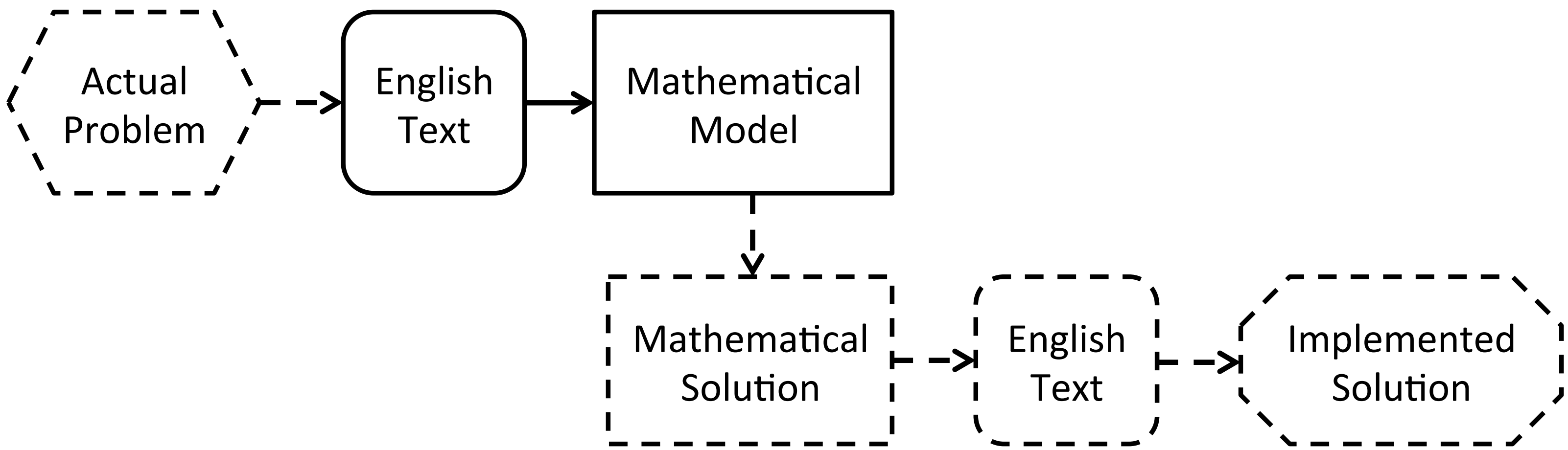
Students – interested in math, science, or languages

Educators – designing course content, and teaching materials at all levels

Engineers, consultants, scientists, technicians, and others – needing a greater ability to use and apply mathematics in their work

Baber’s Diagram Used in Project FOCUS Class

Steps in the Overall Process of Applying Mathematics to a Problem

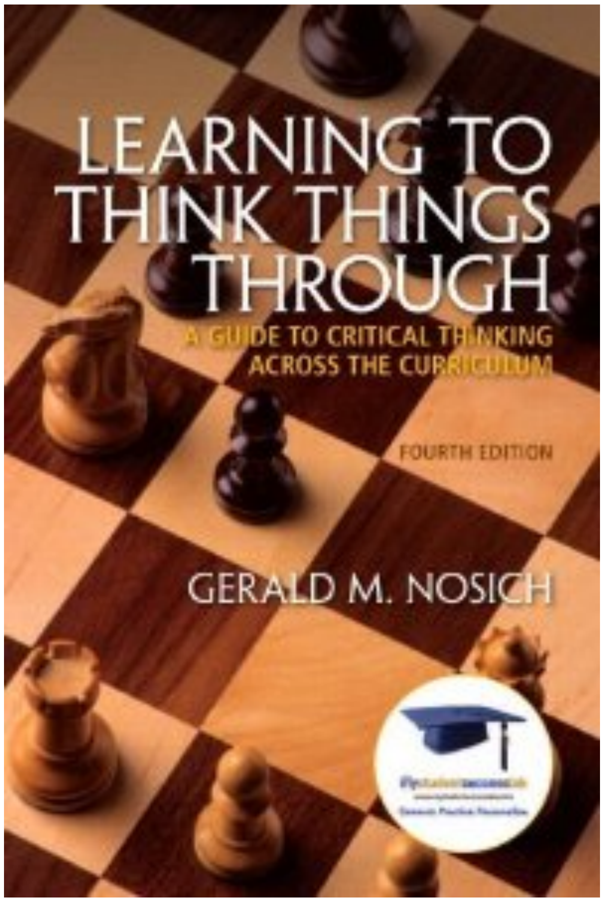


Great Question for Dr. R.

Does the SEE-I strategy relate to the Language of Math?

The collaboration begins!

Introduced/Used at Middle Georgia High School



Part of school’s mission is to prepare students for getting into their college of choice and to succeed in college

Student’s SAT and ACT scores were not where the school wanted them to be

Critical thinking has been shown to be effective in raising SAT and ACT scores

Therefore, SEE-I was implemented in the classroom as part of teaching critical thinking to students

SEE-I

- SEE-I is a very useful process for clarifying almost anything
- The letters stand for the four steps of the methodology

S: State it (usually the definition of a vocabulary word or concept; teacher or book definition)

E: Elaborate (explain it more fully, in your own words; “In other words …)

E: Exemplify (give a good example; “For example ….”)

I: Illustrate (give an illustration; maybe a metaphor, a simile, an analogy, a diagram, a concept map, and so forth; “It’s like ….”)

Slope Example

- **S:** Slope is rise over run. Expressed as a formula is $(y_2 - y_1)/(x_2 - x_1)$

- **E:** In other words, slope is the change in the vertical distance (rise) over the horizontal distance (run) between two points on a coordinate plane. It is a measure of the steepness of a line.

Slope Example (con't)

- **E:** For example, given the points (1,4) and (-2, 3), the slope is calculated as follows:

  (3-4)/(-2-1) = -1/3 = 1/3

  This means that from any point on a line, we can find another point by rising up 1 on a graph and going over 3 units to the right.

- **I:** It's like walking up the stairs. We have to lift our foot up first vertically and then extend it out horizontally. If we don't, we would fall flat on our face. The more slope a staircase has, the higher we have to lift our foot up first.

High School Student Example 1

Factoring and Solving

11/5/13

S: Factoring is writing a polynomial in a simpler form for solving.

E: In other words, it is breaking down a polynomial to its simplest terms in order to solve for the zeros.

Meaning what?

Once the polynomial is factored, you must set the equation equal to zero. According to the zero product property, the product of zero and any number is zero. Therefore, when the equation is set to zero, one of its components must equal zero.

In order to accomplish the task of factoring and solving, follow these steps:

1. look for a greatest common factor (GCF)
2. If there is no GCF, or after one is found, factor using:
   - the Box method
   - Sum of 2 Squares
   - grouping
   - as well as other factoring techniques
3. Set the fully-factored polynomial to zero.
4. Solve by setting each part of the equation to zero

E: Example one:

$$x^4 - 13x^2 + 42$$

$$(x^2 - 7)(x^2 - 6) = 0$$

$x^2 - 7 = 0$ $\quad x^2 - 6 = 0$

$x^2 = 7$ $\quad x^2 = 6$

$\boxed{x = \pm\sqrt{7}}$ $\quad \boxed{x = \pm\sqrt{6}}$

Example two: $3x^7 - 18x^5 = -27x^3$

$$3x^7 - 18x^5 + 27x^3 = 0$$

$$3x^3(x^4 - 6x^2 + 9) = 0$$

$$3x^3(x^2 - 3)(x^2 - 3) = 0$$

continued on next page

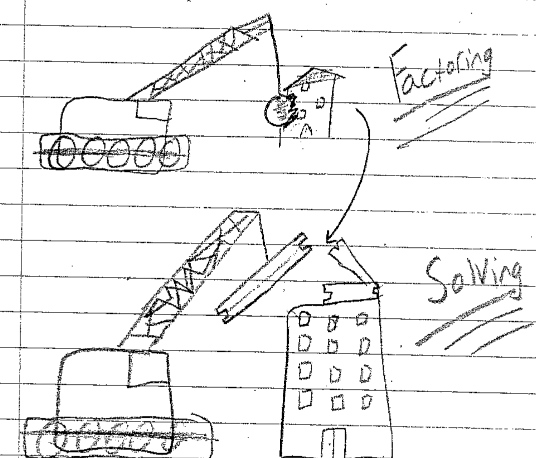
$$3x^3(x^2 - 3)(x^2 - 3) = 0$$

$3x^3 = 0$ $\quad x^2 - 3 = 0$ $\quad x^2 - 3 = 0$

$x^3 = 0$ $\quad \sqrt{x^2} = \sqrt{3}$ $\quad \sqrt{x^2} = \sqrt{3}$

$x = 0$ $\quad x = \pm\sqrt{3}$ $\quad x = \pm\sqrt{3}$

I: It is like a crane that breaks a building down to its simplest form in order to build it into something better.



Excellent!

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High School Student Example 2

Relationship Between Factoring & Solving Polynomials

S: Factoring means to break numbers or terms up into numbers/terms that can be multiplied together to get the original number.

Not tied to solving like the S I gave you in class

E: After factoring the equation completely, set the equation equal to zero and solve for the variable.

1. Set the original equation equal to zero.
2. Factor completely
3. Set the completed factorization equal to zero
4. Solve for the variable

E: $x^2 + 7x + 6 = 0$

1 6

$(x^2 + 1x)(6x + 6) = 0$

$x(x+1) + 6(x+1) = 0$ ← $(x+6)(x+1) = 0$

$x + 6 = 0$ $x + 1 = 0$

$x = -6$ $x = -1$

$x^4 - 625 = 0$

$(x^2 - 25)(x^2 + 25) = 0$

$(x-5)(x+5) = 0$

$x - 5 = 0$ $x + 5 = 0$

$x = 5$ $x = -5$

Good, but need to explain what → you are doing.

I: It's like a preview for a movie. In the preview, they give you the big picture of what is going on. When you get to the theater to see the movie, the big picture is broken down further to better understand the movie. Factoring is the same idea. You are given the big equation to solve. To make it less complicated, you break it down further.

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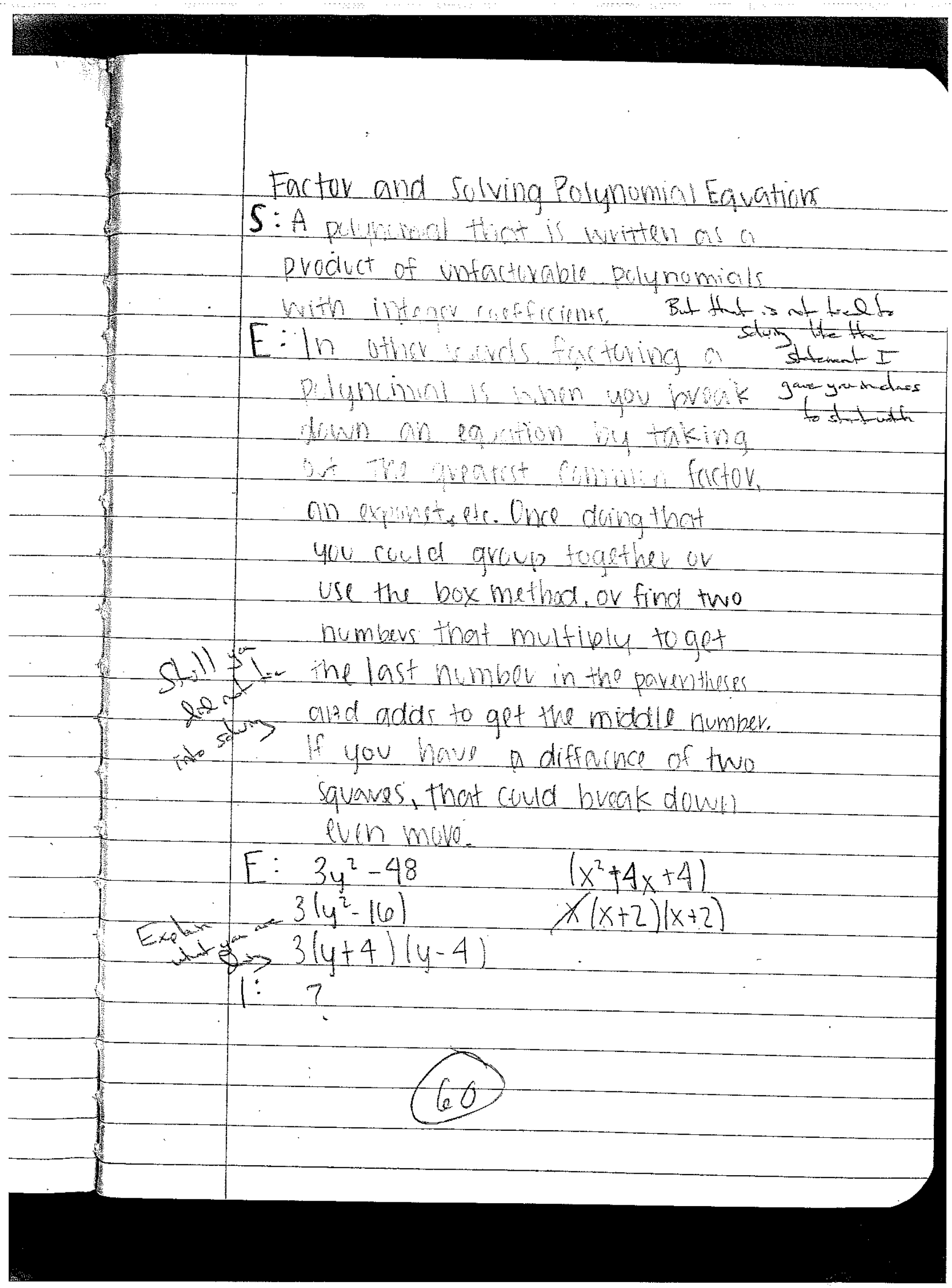
High School Student Example 3



Factor and Solving Polynomial Equations

S: A polynomial that is written as a product of unfactorable polynomials with integer coefficients.

E: In other words factoring a polynomial is when you break down an equation by taking out the greatest common factor, an exponent, etc. Once doing that you could group together or use the box method, or find two numbers that multiply to get the last number in the parentheses and adds to get the middle number. If you have a difference of two squares, that could break down even more.

E: $3y^2-48$ $\quad (x^2+4x+4)$

$3(y^2-16)$ $\quad (x+2)(x+2)$

$3(y+4)(y-4)$

I: ?

60

Experience with student submissions is that at first they have a hard time with the first E (Elaborate) and with the I (Illustrate)

Many will try the first E, but skip the I

University student SEE-I for lesson on solving a system of linear equations by elimination

- **S**<u>tate it</u>: Elimination is a method that can be used to solve a system of linear equations by eliminating one of the variables and substituting the result into one of the original equations to find the remaining variable.
- **E**<u>laborate</u>: In other words, first you identify the additive inverse in the system; that is, two like terms that when added together result in zero. You may have to manipulate one or both of the equations in order to make an additive inverse. Once you have the zero pair you then add the two equations of the system together thus eliminating one of the variables. Solve for the other variable then plug in the result into one of the original equations to solve for the variable that was first eliminated.
- **E**<u>xemplify</u>: For example, take the system $2x-3y=10$; $4x+y=6$. Let's choose to make our y terms into an additive inverse by multiplying the equation $4x+y=6$ by 3. The resulting system will be

  $2x-3y=10$; $12x+3y=18$. Adding the two equations together we get, $14x=28 \rightarrow \mathbf{x=2}$. Plugging 2 back into one of the original equations we get $42+y=6 \rightarrow 8+y=6 \rightarrow y=6-8 \rightarrow \mathbf{y=-2}$. Therefore the solution written as an ordered pair is (2,-2).
- **I**<u>llustrate</u>: Using elimination to solve a system of linear equations is like finding a lid for a Tupperware container. For example, some containers have a lid that fits perfectly right off the bat and sometimes a container may not have a lid so you have to use tin foil to cover the top. The container that has a lid is similar to a system of equations that already has an additive inverse so all that needs to be done is add the two equations together and find the solution. Whereas the bowl that requires the manipulation of tin foil to make a temporary lid is similar to the system of equations that needs to be manipulated in order to make an additive inverse, like we did in the example above.

VS.

My SEE-I for MCA3. Students will demonstrate knowledge of differentiation using algebraic functions.

<u>State:</u> Optimization: method of finding the minimum or maximum value of a problem using the first derivative

<u>Elaborate:</u> It is the process of finding the best solution. When optimizing a solution you use the first derivative to find the critical points of your function. Test these values by plugging them into the original function. Find the maximum or minimum solution (depending on what was asked for).

<u>Exemplify:</u> Use optimization to find what length and width of a pool provide the maximum area if you only have 60 feet of lining to use.

Constraints: 2L+2W=60 (where L = length and W = width)

Maximize: A=LW

Solving:

$$2L = 60 - 2W \Rightarrow L = 30 - W$$

$$A = (30 - W)W \Rightarrow 30W - W^2$$

$$\Rightarrow \frac{dA}{dw} = 30 - 2W\,dw \Rightarrow 0 = 30 - 2W \Rightarrow -30 = -2W \Rightarrow W = 15 \text{ feet}$$

$$2L + 2(15) = 60 \Rightarrow 2L = 30 \Rightarrow L = 15 \text{ feet}$$

The biggest (largest area) of a pool can be obtained by making it 15 feet long and 15 feet wide.

<u>Illustrate:</u> Optimization is like being given several routes on a GPS and choosing the best one according to whether or not you want the shortest time, the most direct route, or the most scenic route.

Two Physics & One Engineering Professors' SEE-I

STATE **(usually the definition of a vocabulary word or concept; teacher or book definition)**

Finding Total resistors in ~~Series~~ parallel

ELABORATE **(explain it more fully, in your own words; "In other words ...)**

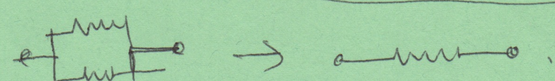
- Resistor: oppose the flow of electrical charge (current)
  In situation ~~in~~ which resistors are connected in parallel (both leads are connected together) the current will be divided among the resistor depending on their values. The lowest value resistor gets the highest current. * This ~~the~~ is governed by Ohm's law.

EXEMPLIFY **(give a good example; "For example ....")**

Consider two resistors $R_1 = 10\Omega$, ~~$R_2 = 10$~~ $R_2 = 20\Omega$.

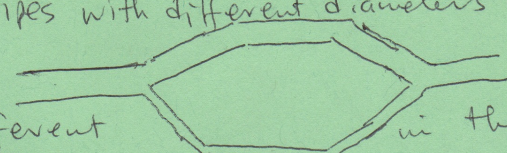
$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \rightarrow R_t = \frac{R_1 R_2}{R_1 + R_2}$$

$$R = \frac{200}{30} = \frac{20}{3}\ \Omega.$$



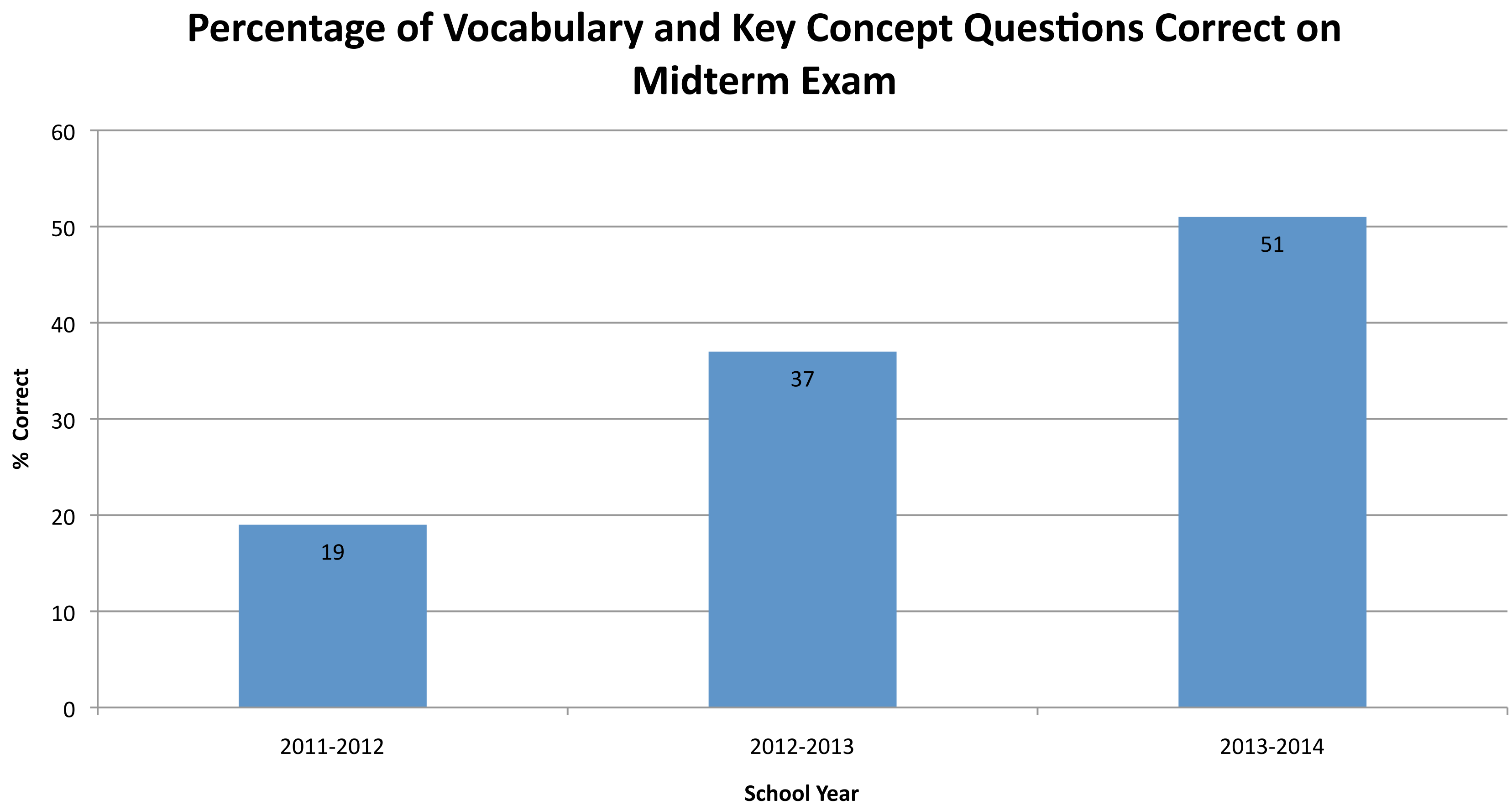
ILLUSTRATE **(give an illustration; maybe a metaphor, a simile, an analogy, a diagram, a concept map, and so forth; "It's like ....")**

Similar to pipes with different diameters connected in parallel (together),



the flow rate will be different in the two pipes. In larger ~~[illegible]~~ pipe will have the less flow rate compared to the smaller pipe ~~with~~ (small diameter). The larger pipe represents the higher resistance while ~~the~~ the smaller pipe represents the one with the lower resistance.

Results of Using SEE-I



SEE-I Activity

- It is your turn to try clarifying something using SEE-I
- Pair up with someone and pick a word or concept to use
- Share ideas and share what you are thinking

References

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Cavey, L.& W. Mahavier. (2010). Seeing the potential in students' questions. *Mathematics Teacher*, 104 (2), 133-137.

Nosich, G. (2011). *Learning to think things through: A guide to critical thinking across the curriculum (4th ed.).* Upper Saddle River, NJ:Prentice-Hall, Inc.