A Case Study of How Ninth Grade Mathematics Students Construct Knowledge during a Productive Failure Model

Amy F. Westbrook Dr.
Mercer University, awestbrook316@gmail.com

Follow this and additional works at: https://digitalcommons.georgiasouthern.edu/gera

Part of the Curriculum and Instruction Commons, Mathematics Commons, and the Science and Mathematics Education Commons

Recommended Citation
https://digitalcommons.georgiasouthern.edu/gera/2014/2014/63

This presentation (open access) is brought to you for free and open access by the Conferences & Events at Digital Commons@Georgia Southern. It has been accepted for inclusion in Georgia Educational Research Association Conference by an authorized administrator of Digital Commons@Georgia Southern. For more information, please contact digitalcommons@georgiasouthern.edu.
A CASE STUDY OF HOW NINTH GRADE MATHEMATICS STUDENTS CONSTRUCT KNOWLEDGE DURING A PRODUCTIVE FAILURE MODEL

Amy F. Westbrook
Mercer University
Ph.D. Candidate in Curriculum and Instruction
Statement of Problem

Mathematics educators continue to rely on a transmission model for teaching students.

• Results for students:
  o lack critical thinking skills – imagination and criticism
  o Not active participants in their learning
  o Do not acquire a deeper meaning of concepts
  o Beliefs – the “one right answer” – lacks multiple ways of knowing
  o does not support Common Core Process Standards - perseverance in problem solving
Productive Failure Model

• Grounded in constructivist theory
• Critical thinking and deeper understandings
  – greater capacity to understand novel mathematical concepts when they are initially afforded opportunities to problem solve even if they cannot produce correct answers
• Kapur (2010) found learning gains pre- post-test design
• Kapur (2011) found high variations of representations
• Limitation and Future Research – examining characteristics of the learner that relate to learning gains in group – explaining performance
**Kapur’s Productive Failure Model (Kapur, 2012)**

- **Verbal Thought and Concept Formation:**
  - Understanding the generalizations of prior knowledge and constructing new knowledge through talk (Vygotsky, 1934/1962)

- **Problem-Posing Education:**
  - Freedom of the Learner (Freire, 1970/2012)

- **Disequilibrium:**
  - Errors in thinking (Piaget, 1980)

- **Activates Prior Knowledge**

- **Initiates Multiple Representations**

- **Critical Analysis of Targeted Concepts**

- **Connection between failed attempts and successful endeavors**

- **Reflective Abstraction:**
  - Assimilation and accommodation through persistence (Piaget, 1971)

---

*Figure 1. Kapur’s Productive Failure Model Aligned with Theoretical Framework*
Purpose Statement

• The purpose of this case study is to understand how ninth-grade mathematics students at a rural high school in Georgia construct knowledge through student talk when problem solving using Kapur’s (2009) productive failure design.

• Benefit – education professionals, curriculum specialists, and policy makers
Central Research Question

How do ninth-grade mathematics students construct their own knowledge while problem solving using the productive failure model?
Subquestions

• What role does student talk have in making learning gains during the problem solving task for ninth-grade mathematics students?
• What role does student choice have in making gains during a productive failure modeled task?
• How do ninth-grade mathematics students utilize prior knowledge to make gains in solving the problem solving task?
• How do ninth-grade mathematics students persist during the problem solving task?
Limitations

• Productive Failure Model – bias towards the model
• transferability
• Use of videotaping – students may act differently – novelty
• I researched my students – researcher bias
Review of the Literature

Constructivism
Vygotskian Theory
Freirean Theory
Piagetian Theory

Problem-Solving Model

PFM
How do ninth-grade mathematics students construct knowledge during a PFM task?

**Constructivism**
- What role does student talk have in making learning gains?
  - Verbal Thought & Concept Formation (Vygotsky, 1934/1962)
  - Authentic Problems (Howard & Aleman, 2008)
  - Solving the task in a group setting (Cotic & Zulian, 2009)

**Problem-Solving Model**
- How do ninth-grade mathematics students utilize prior knowledge?
  - Reflective Abstraction (Piaget, 1971)
  - Initiates Prior Knowledge (Hansen, 2008)
  - Activates Prior Knowledge (Kazan & Bixler, 2011)

**Productive Failure Model**
- What role does student choice have in making learning gains?
  - Open Problems (Borasi, 2008)
  - Promotes Critical Thinking (Dewey, 1934/2008a)
  - Initiates Multiple Representations (Kazan, 2012)
  - Critical Analysis of Targeted Concepts (Kazan, 2008)

- How do ninth-grade students persist during problem solving?
  - Disequilibrium (Piaget, 1980)
  - Allowing for Errors (Borasi, 1996)
  - Connection between failure and success (Kazan, 2010)

**Review of the Literature on Resistance**
- Behaviorism & Maturationalism (Rosnov & Perry, 2005)
- Transmission Model (Bingolhali, 2011)
- Worked Examples (Remnowski et al., 2010)

*Figure 4. Synthesis of the Review of the Literature*
Research Design

• Case Study – used to answer “how” questions
  – Deep understanding of how students construct knowledge – in context
  – Representative Single-case embedded design

Merriam (1998), Stake (2010), and Yin (2009)
Participants and Setting

- Setting: rural high school in Georgia
- Case – group of four ninth-grade students enrolled in a coordinate algebra course (subgroup of a convenience sample)
- Embedded Units – the individual students
- Heterogeneous group- more collaborative reasoning vs. Homogenous group – learning linked to collaboration (Saleh et al., 2005)
- Selection Process
  - 25/32 returned the IRB
  - 21/32 minus two students repeating and two for absenteeism
  - Mean scale score for 8th grade: 827.04 (Georgia Department of Education, 2013) – 13/32 – had available scores “met” 819-842
  - Gender – 2 boys and 2 girls (Zinicola, 2009)
  - Former grades/current average – similar – mainly a “B” student
  - Ethnicity
  - Socio-economic status
  - Behavior reports – not an issue
  - Special services - gifted
The Case Group

- (All names are pseudonyms)
  Sara: Hispanic female; 819 CRCT; A-C’s; 82 class average
  Isaac: African American male; 833 CRCT; B’s; 82
  Katie: Caucasian female; 827 CRCT; A-B’s; 82
  Trey: Caucasian male; 833 on CRCT; A-B’s; 82
Data Collection and Instrumentation

- Pre-assessment – 10 open-ended questions
- Learning-log for each student (Robotti, 2012)
  - Work pages
  - Note-taking pages
  - Questionnaires (Stephens and Winterbottom, 2010) vs. Zinicola (2009)
- Posters
- Student Interviews and confessionals
- Video-tapings – main camera and student cameras (Shreyar et al., 2010)
- Researcher log – instructional plan and field notes
- Final Assessment – same as pre-assessment
Three Cycles of Data Collection

• Cycle 1 – Reasoning with Equations and Inequalities
• Cycle 2 – Interpreting One Variable Data
• Cycle 3 – Centers of Triangles
The Task:
The Game Place

You were just hired as the store manager of "The Game Place". The Game Place sells used video games. As the store manager, you are responsible for purchasing games from other used video game distributors. You will need to know some basic math skills to solve problems involving the budget and purchasing. For example, two new types of games are available to offer in our store. The first type of game costs $30 dollars and the second one is $20. You want to stock at least $600 worth of games to be competitive with the surrounding stores, but your store’s purchasing budget cannot exceed $1200 worth of games.

Question: How many possible combinations of orders can be made that will satisfy the minimum and maximum requirements?

(adapted from a word problem from the Jordan and Granite online textbook, p. 82)
<table>
<thead>
<tr>
<th>DAY</th>
<th>ACTIVITY</th>
<th>DURATION</th>
<th>QUALITATIVE DATA COLLECTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Day 1</td>
<td>Pretest: open-ended questions</td>
<td>25 minutes</td>
<td>pretest recorded in learning log</td>
</tr>
</tbody>
</table>
| Days 2 and 3 | Problem-Solving Task Cycle 1: Systems of Equations | 45 minutes | 1. Student work is recorded each day in the learning log.  
2. Students are videotaped while they work in a group on the task.  
3. At the end of the task each day, each student records in the learning log a questionnaire page (Student Questionnaire 1 and Student Questionnaire 2).  
4. At the end of the task each day, students record what they thought about the task on video, a “student confessional”  
5. Researcher Log—records general observations that were noticed during the task before the video-tape is viewed. |
| Day 4  | Poster Presentation: students make a poster and present their findings to the class | 90 minutes | 1. Students are videotaped during their presentation.  
2. Teacher collects the poster used during the presentation.  
3. Researcher Log—records general observations after the consolidation lesson before the video-taping is viewed.                                                                 |
| Day 5  | Consolidation Lesson: teacher models ways to solve the task; uses their presentations to make connections | 45 minutes | 1. Students are asked to take notes in the learning log. Students can ask questions/make comments.  
2. Researcher Log—records general observations after the lesson occurred.                                                                                     |
| Day 6  | Assessment (re-administer pretest)                 | 45 minutes | 1. Students take the post assessment which identical to the pretest.  
3. Students are interviewed as a group.  
4. Researcher Log—records general observations after the group interview.                                                                                     |
Data Analysis

• Constant Comparative Analysis
  – Coding for the themes – group roles, problem-solving approaches, and group processes
  – Panel of peers- feedback
  – Outlines
  – Continual
  – Sorted into categories – begin with clusters
  – Look for patterns
  – Answer my research questions
  – Exhaustive
  – Captures essence of theme

• Database – document ID – Excel Spreadsheet
• Word Count Analysis
• Interactive Model- synthesis of the three components
Codes for Group Processes (GP)

I. Making a Plan (MP)
   A. New Idea (NI)
   B. Explain Initial Idea (EI)
   C. Idea Extension (IE)
   D. Provoking Question (PQ)

II. Working a Plan (WP)
   A. Work Dynamics (WD)
      1. Individual Work (IW)
      2. Paired Work (PW)
      3. Tri-ad Work (TW)
      4. Whole Group Work (WW)
   B. Tools (TL)
   C. Give Instructions (GI)
   D. Show Work (SW)
   E. Shared Thinking (ST)
   F. Adopted Thoughts (AT)
   G. Asking Questions (AQ)

III. Understanding Concepts (UC)
   A. Solving Steps (SS)
   B. Types of Solutions (TS)
   C. Define Vocabulary (DV)
   D. Set-up Equations (SU)
   E. Solving Methods (SM)
   F. Define Variables (DR)

IV. Reporting Results (RR)
   A. Explain Steps (ES)
   B. Compare Work (CW)
   C. Give an Answer (GA)
   D. Ask a Question (AQ)
   E. Explain Why (EW)

V. Evaluating a Plan (EP)
   A. Check Answer against Task (CT)
   B. Answer in Context (AC)
   C. Decision Making (DM)
      1. Yes
      2. No
      3. Maybe
      4. Stuck (SK)
Dependability and Credibility

• Construct Validity – instructional plan reviewed, triangulation through multiple sources of data, chain of evidence

• Internal Validity – member and peer checks, disclose bias, participant feedback, instructional plan feedback

• External Validity – rich descriptions, typical ninth-grade students, different mathematical concepts

• Reliability – case study protocol, case study database
Results – Group Talk

<table>
<thead>
<tr>
<th>Student Pseudonym</th>
<th>Number of Words</th>
<th>Percentage of Words Used During Group Work</th>
<th>Number of Lines of Text</th>
<th>Percentage of Lines Spoken During Group Work</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sara</td>
<td>1449</td>
<td>57%</td>
<td>97</td>
<td>44%</td>
</tr>
<tr>
<td>Isaac</td>
<td>282</td>
<td>11%</td>
<td>30</td>
<td>14%</td>
</tr>
<tr>
<td>Katie</td>
<td>563</td>
<td>22%</td>
<td>65</td>
<td>29%</td>
</tr>
<tr>
<td>Trey</td>
<td>260</td>
<td>10%</td>
<td>28</td>
<td>13%</td>
</tr>
<tr>
<td>Total</td>
<td>2554</td>
<td>100%</td>
<td>220</td>
<td>100%</td>
</tr>
</tbody>
</table>

Vygotsky (1934/1962)
Results - Roles For Verbal Thought

Roles Coded for Instances of Verbal Thought: Cycles 1 and 2

<table>
<thead>
<tr>
<th>Role</th>
<th>Sara</th>
<th>Isaac</th>
<th>Katie</th>
<th>Trey</th>
<th>Group Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Responder</td>
<td>5%</td>
<td>2%</td>
<td>6%</td>
<td>2%</td>
<td>15%</td>
</tr>
<tr>
<td>Explainer</td>
<td>20%</td>
<td>6%</td>
<td>11%</td>
<td>7%</td>
<td>44%</td>
</tr>
<tr>
<td>Questioner</td>
<td>11%</td>
<td>4%</td>
<td>12%</td>
<td>4%</td>
<td>31%</td>
</tr>
<tr>
<td>Hypothesizer</td>
<td>4%</td>
<td>&lt;1%</td>
<td>1%</td>
<td>&lt;1%</td>
<td>6%</td>
</tr>
</tbody>
</table>

Vygotsky (1934/1962)
Results - The Approaches for Problem Solving

1. Freedom of the Learner: generate ideas, choose tools

2. Reflective Thinking: write ideas, read for understanding, admit being stuck (5%), define variables

3. Student Voice: algebraic, guess-and-check, graphing, and statistical

Freire (1970/2012)
Assimilation/ Accommodation And Disequilibrium Piaget (1971)
Findings

• “Stuck” – Kapur’s studies: experiencing puzzlement, using the explainer role - Zinicola (2009): 2/12 no explainer roles

• Multiple Representations – cyclic patterns of persistent problem solving- precipitated by moments of confusion

• Delaying Supports to Understand Concepts at a Deeper Level – struggle precipitated deeper evaluation
Kapur’s Productive Failure Model (Kapur, 2012)

1. Solves the task in a group setting
2. Activates Prior Knowledge
3. Initiates Multiple Representations
4. Critically Analyzes Targeted Concepts
5. Makes Connections between failed attempts and successful endeavors

Making a Plan
Hypothesizer
Generate new ideas

Working the Plan:
*write/use tools
Explainer:
methods/steps
Recorder

Reporting Results:
*state work
Explainer: steps/methods/solutions HOW?

Understanding Concepts:
Explainer:
*confusion/ set-up/steps/ solutions/methods generate ideas/define variables HOW/WHY?

Evaluating the Plan:
*generate new ideas/define variables
Explainer: solutions/task/
*confusion HOW?

STUCK

Figure 1.5: Synthesis Model: Conceptual/Theoretical Framework and Persistency Model
Implications for Teaching

1. Analyze Verbal Thought: Assessment through Group Talk
2. Celebrate Errors: Creating a Need for Accommodations through Persistence
3. Student Voice: Allowing Students to Become Critical Thinkers
Implications for Future Research

• Use different tasks that build upon one another
• Determine how the PFM aligns with Common Core Process Standards and its usefulness to meeting the standards
• Use a different group of students
• Role of Gender
• Use a different course of study
References


References continued


References continued


References continued


References continued


