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Visual Modeling of Fractions and Ratio

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Once it has been demonstrated that algebra can be taught three or even thirty ways, it will be malpractice to declare “Johnny could not learn algebra my way- bring me another child.

Howard Gardner

Gardner, H. (2006). *Multiple intelligences: New horizons*. New York, NY: Basic Books.

We view the study of fractions as foundational to the study of algebra in particular because it offers students the opportunity to grapple with the fundamental mathematical relationships that constitute the core of algebra. These relationships govern how addition, subtraction, multiplication, and division work in algebra as well as arithmetic; allowing a tightly integrated understanding of number and operation that prepares them to understand algebra and at the same time, develops computational fluency.

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Empson, S. B., & Levi, L. (2011). *Extending children's mathematics: fractions and decimals*. Portsmouth, NH: Heinemann.

All too often, children’s disenchantment with mathematics begins late in elementary school or early in middle school when, even after years of practice, they cannot remember how to “do” fractions after summer vacation, or when they can perform steps, but are totally bored because they do not know what the steps mean or why they are doing them.

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Lamon, S. J., & Levi, L. (2006). *Teaching fractions and ratios for understanding: essential content knowledge and instructional strategies for teachers*. Mahwah, NJ: Lawrence Erlbaum.

Standards for Mathematical Practice

Make sense of problems and persevere in solving them

Reason abstractly and quantitatively

Construct viable arguments and critique the reasoning of others

Model with mathematics

Use appropriate tools strategically

Attend to precision

Look for and make use of structure

Look for and express regularity in repeated reasoning

Common Core State Standards Initiative (CCSSI). 2010. *Common Core State Standards for Mathematics*. Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers.

My Teaching Philosophy

Mathematics should be taught for genuine conceptual *understanding*. Students should find the mathematical concepts meaningful, and they should be able to *flexibly* use these concepts in *novel situations*. *Problem solving* and *reasoning* should be the dominant theme of every mathematics class every single day. Students should be given ample opportunities to *communicate* their mathematical ideas to their peers and teacher. Classrooms should be more *student-centered* and less teacher-centered.

I would rather teach a few important topics in *depth*, than teach numerous topics superficially.

I believe rote learning (that is, learning with minimal or no understanding) should never be the focus of any mathematics class at any time. Rote learning of mathematics should be avoided whenever possible.

Levels of Learning

1. Rote (with little or no understanding)

2. Meaningful
   - understand
   - paraphrase
   - translate

3. Integration
   - compare
   - contrast

4. Critical Thinking
   - analyze
   - create
   - evaluate
   - interpret

Dr. Harvey Brightman

Often teachers (at all levels) need to understand how their students think! Consequently, it is the job of the teachers to make sense of their student’s understanding of a solution to a particular mathematical problem.

Unfortunately this important aspect of teaching is very inadequately addressed in most mathematics (content or method) courses designed for teachers.

What we are going to see is to precede the teaching of formal algebra, not replace it.

The local zoo has two more new friends, a baby panda and a baby tiger. The baby panda weighs 3/5 of his weight plus 1/3 of the weight of the baby tiger. If the baby tiger weighs 15 pounds more than the baby panda, how much does the baby panda weigh?

http://youtu.be/FsAoFiBPrhY

The local zoo has two more new friends, a baby panda and a baby tiger. The baby panda weighs 3/5 of his weight plus 1/3 of the weight of the baby tiger. If the baby tiger weighs 15 pounds more than the baby panda, how much does the baby panda weigh?

Suppose the weight of the panda is p lbs and
the weight of the tiger is t lbs.

Then the following must be true:

t-p = 15

$$p = \frac{3}{5}p + \frac{1}{3}t$$

Ans: t = 90 lbs and p = 75 lbs

The local zoo has two more new friends, a baby panda and a baby tiger. The baby panda weighs 3/5 of his weight plus 1/4 of the weight of the baby tiger. If the baby tiger weighs 75 pounds more than the baby panda, how much does the baby panda weigh?

Ans: 125 pounds

A pizza was cut into two pieces, Piece X and Piece Y. The two pieces are not necessarily the same size. Suppose 2/5 of Piece X is the same size as 3/4 of Piece Y.

a. What part of the whole pizza is Piece X?

b. What part of the whole pizza is Piece Y?

c. Piece Y is how many times as large as Piece X?

http://youtu.be/f4rtNLqHGLQ

A pizza was cut into two pieces, Piece X and Piece Y. The two pieces are not necessarily the same size. Suppose 2/5 of Piece X is the same size as 3/4 of Piece Y.

a. What part of the whole pizza is Piece X?

b. What part of the whole pizza is Piece Y?

c. Piece Y is how many times as large as Piece X?

Let piece X represent x parts of the original pizza, and piece Y represent y parts of the original pizza.

Then the following must be true:

$x+y = 1$

$$\frac{2}{5}x = \frac{3}{4}y$$

Ans: $x = 15/23$ and $y = 8/23$

A pizza was cut into two pieces, Piece X and Piece Y. The two pieces are not necessarily the same size. Suppose 2/3 of Piece X is the same size as 3/4 of Piece Y.

a. What part of the whole pizza is Piece X?

b. What part of the whole pizza is Piece Y?

c. Piece Y is how many times as large as Piece X?

Ans: a. 9/17; b. 8/17; c. 8/9

Together Geraldine and Josephine baked 46 cookies. Geraldine took 3/7 of her cookies to a family party and saved the rest of the cookies for her grandkids. Josephine took 2/5 of her cookies to the same family party and saved the rest of the cookies for her grandkids. Altogether Geraldine and Josephine took 19 cookies to the family party. How many cookies did each bake?

http://youtu.be/TffVbaGXNTI

Together Geraldine and Josephine baked 46 cookies. Geraldine took 3/7 of her cookies to a family party and saved the rest of the cookies for her grandkids. Josephine took 2/5 of her cookies to the same family party and saved the rest of the cookies for her grandkids. Altogether Geraldine and Josephine took 19 cookies to the family party. How many cookies did each bake?

Suppose Gerladine baked *a* cookies altogether and Josephine baked *b* cookies altogether.

Then the following must be true:

$a+b = 46$

$$\frac{3}{7}a + \frac{2}{5}b = 19$$

Ans: $a = 21$, and $b = 25$

Together Geraldine and Josephine baked 66 cookies. Geraldine took 3/7 of her cookies to a family party and saved the rest of the cookies for her grandkids. Josephine took 2/5 of her cookies to the same family party and saved the rest of the cookies for her grandkids. Altogether Geraldine and Josephine took 27 cookies to the family party. How many cookies did each bake? Do not use trial and error. Credit will be given for algebraic or visual method only.

Ans: Geraldine: 21 and Josephine: 45

Suppose one painter can paint the entire house in 4 hours, and the second painter takes 5 hours. How long would it take the two painters together to paint the house?

http://youtu.be/l-dw2lju5QM

Suppose one painter can paint the entire house in 4 hours, and the second painter takes 5 hours. How long would it take the two painters together to paint the house?

Suppose together the two painters take *t* hours to finish the job.

Then the following must be true:

$$\frac{t}{4}+\frac{t}{5}=1$$

Ans: t = 20/9 = 2.22 hours.

Suppose one painter can paint the entire house in 2 hours, and the second painter takes 3 hours. How long would it take the two painters together to paint the house?

Ans: 6/5 = 1 and 1/5 hours

Al received some money from his grandmother. Al spent $50, and gave 1/3 of what was left to Bob. Bob spent $2, and gave 3/4 of what was left to Carl. Carl spent $6, and gave 3/5 of what was left to Dan. If Dan received $27 from Carl, how much money did Al receive from his grandmother?

http://youtu.be/z-jmQqLNeQM

Al received some money from his grandmother. Al spent \$50, and gave 1/3 of what was left to Bob. Bob spent \$2, and gave 3/4 of what was left to Carl. Carl spent \$6, and gave 3/5 of what was left to Dan. If Dan received \$27 from Carl, how much money did Al receive from his grandmother?

Suppose Al received \$$x$ originally from his grandmother.

Then the following must be true:

$$\frac{3}{5}\left(\frac{3}{4}\left(\frac{1}{3}(x-50)-2\right)-6\right)=27$$

Ans: x = \$260

Al received some money from his grandmother. Al spent $50, and gave 2/3 of what was left to Bob. Bob spent $4, and gave 3/4 of what was left to Carl. Carl spent $6, and gave 3/4 of what was left to Dan. If Dan received $54 from Carl, how much money did Al receive from his grandmother?

Ans: $212

Elizabeth saved $105 more than Jenny. 2/3 of Jenny's savings is same as 3/8 of Elizabeth's savings. How much money did the two girls save altogether?

http://youtu.be/g2EjOdkdlZ8

Elizabeth saved \$105 more than Jenny. 2/3 of Jenny's savings is same as 3/8 of Elizabeth's savings. How much money did the two girls save altogether?

Suppose Elizabeth had \$$a$, and Jenny saved \$$b$.

Then the following must be true:

a-b = 105,

$$\frac{3}{8}a = \frac{2}{3}b$$

Ans: a = \$240, and b = \$135. Hence a+b = \$370

Jenny saved $10 more than Elizabeth. 2/3 of Jenny's savings is same as 3/4 of Elizabeth's savings. How much money did the two girls save altogether?

Ans: $170

Originally the ratio of the number of German coins to the number of French coins in Sandra's collection was 2:3. After she sold 35 of the French coins, the new ratio of the number of German coins to the number of French coins in her collection turned into 3:1. How many German coins did Sandra have in her collection?

http://youtu.be/4D-r_C4mi4E

Originally the ratio of the number of German coins to the number of French coins in Sandra's collection was 2:3. After she sold 35 of the French coins, the new ratio of the number of German coins to the number of French coins in her collection turned into 3:1. How many German coins did Sandra have in her collection?

Let the original numbers of German and French coins be g and *f*, respectively.

Then the following must be true:

$\frac{g}{f} = \frac{2}{3}$ and $\frac{g}{f-35} = \frac{3}{1}$.

Ans: $g = 30$ and $f = 45$

Originally the ratio of the number of German coins to the number of French coins in Sandra's collection was 3:7. After she sold 90 of the French coins, the new ratio of the number of German coins to the number of French coins in her collection turned into 3:1. How many German coins did Sandra have in her collection?

Ans: 45

James can wax his car 3 times as fast as his son can. Together they can do the job in 4 hours. How long does it take James to wax the car all by himself?

http://youtu.be/9UbNQ0MDo-Y

James can wax his car 3 times as fast as his son can. Together they can do the job in 4 hours. How long does it take James to wax the car all by himself?

Suppose James can complete the job all by himself in *t* hours, then his son must take 3*t* hours to do the same job all by himself.

Then the following must be true:

$$\frac{1}{t}+\frac{1}{3t}=\frac{1}{4}$$

Ans: *t* is 5 and 1/3 hours

James can wax the family car 5 times as fast as his son can. Together they can do the job in 2 hours. How long does it take James to wax the car all by himself? Assume they are both working on the same car at the same time.

Ans: 2 & 2/5 = 2.4 hours = 2 hours and 24 minutes

The local zoo has two more new friends, a baby panda and a baby tiger. The baby panda weighs 5/7 of his weight plus 1/3 of the weight of the baby tiger. If the baby panda weighs 30 pounds more than the baby tiger, how much does the baby panda weigh?

http://youtu.be/x9c0cIfhc9M

The local zoo has two more new friends, a baby panda and a baby tiger. The baby panda weighs 5/7 of his weight plus 1/3 of the weight of the baby tiger. If the baby panda weighs 30 pounds more than the baby tiger, how much does the baby panda weigh?
Suppose the weight of the panda is p lbs and
the weight of the tiger is t lbs.

Then the following must be true:

p-t = 30

$$p = \frac{5}{7}p + \frac{1}{3}t$$

Ans: t = 180 lbs and p = 210 lbs

The local zoo has two more new friends, a baby panda and a baby tiger. The baby panda weighs 3/7 of his weight plus 1/4 of the weight of the baby tiger. If the baby tiger weighs 108 pounds more than the baby panda, how much does the baby panda weigh?

Ans: 84 pounds