Cognitive Skills and Mathematics Problem-Solving Performance

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COGNITIVE SKILLS AND MATHEMATICS PROBLEM-SOLVING PERFORMANCE

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The Problem

- Importance of problem solving as a means to mathematics learning
- Mathematics learned through problem solving; mathematical ideas develop along with problem-solving capabilities
Interactions between Cognitive Skills and Problem-Solving Performance

- Spatial skills and production and use of drawings to solve word problems
- Verbal skills & mathematical problem-solving performance; associated with higher cognitive functions (critical thinking, sound reasoning, problem solving)
- Logical/Analytical component also related to problem solving
- Logical/Analytical skills tightly linked to verbal skills
Definition of Terms

Problem Solving – *activities that range from word problems to open-ended, exploratory types of problems, covering a variety of mathematical content; Polya’s (1945) idea of steps used by students during the problem-solving process*

Spatial Skills – *spatial visualization, spatial orientation, and visual imagery*
Definition of Terms (cont’d.)

► Verbal Skills – students’ understanding of vocabulary

► Analytical Skills – students’ ability to draw conclusions in syllogistic format
Significance of the Study

- Previous research not organized around a common framework
- Lack of consensus in researchers’ definitions of problem solving
- Definition and interpretation of verbal, spatial & analytical skills
- Difficulties analyzing & measuring spatial-orientation and spatial-visualization tasks separately
Significance of the Study (cont’d.)

- Inconclusive findings
- More systematic examination of relationships among each of the cognitive skills and problem-solving performance
- More precise assessment of verbal and analytical skills – measured using separate instruments
The Research Question

► To what extent are students’ spatial, verbal, and analytical skills related to their problem-solving performance?

- To what extent are students’ spatial, verbal, and analytical skills related to problem-solving performance on items that require verbal responses (e.g., explanation of a solution)?

- To what extent are students’ spatial, verbal, and analytical skills related to problem-solving performance on items for which a drawing or diagram is required for the main solution to the problem?
The Research Question (cont’d.)

► To what extent are students’ spatial, verbal, and analytical skills related to each other?
  - To what extent are students’ verbal and analytical skills related to each other?
  - To what extent are students’ spatial and analytical skills related to each other?
  - To what extent are students’ spatial and verbal skills related to each other?
Method

Research Design

► Correlational study

► Relationships between each of the cognitive skills and verbal-response items, spatial-response items, and overall problem-solving performance

► Pair-wise relationships among the cognitive skills
Subjects

► Ninety-eight (98) students
  - Private, Montessori-based school – 48 students;
    - 5th grade (11); 6th grade (10); 7th grade (15); 8th grade (12)
  - Public charter school – 50 students;
    - 7th grade (25); 8th grade (25)
Instruments

► Problem-Solving Test - two subtests

- PST-Spatial (20 spatial-response items)
  - Create a drawing/diagram represents relevant aspects of problem
  - Use the created drawing/diagram to solve problem

- PST-Verbal (20 verbal-response items)
  - Solve problem; describe solution to a friend
  - Give written step-by-step description of solution process
Instruments (cont’d.)

► Cognitive Tests

- Spatial skills – Factor Referenced Cognitive Test of Visualization (ETS, 1976)
- Measures of ability to manipulate or transform images of spatial patterns into other arrangements
  - Part 1 (Form Board Test) – figure completion
  - Part 2 (Paper Folding Test) – hole punches
Instruments (cont’d.)

- Verbal skills – Factor Referenced Cognitive Test of Verbal Comprehension – Vocabulary Test (ETS, 1976)

- Measure students’ ability to understand the English language

  - Vocabulary I
  - Vocabulary II
Instruments (cont’d.)

- Analytical skills – Factor Referenced Cognitive Test of Logical Reasoning (ETS, 1976)
  - Measure students’ ability to determine whether or not a conclusion is logically correct
    - Part I – Nonsense Syllogisms
    - Part III – Inference Test
Data Collection Procedure

Data collected in two phases

- Phase 1 – Problem-solving instrument
  - Two problems (one spatial-response; one verbal-response)/day over four weeks
  - Administered in order of increasing difficulty
  - Used as bell-ringer/warm-up

- Phase 2 – Cognitive skills assessments
  - One assessment per day (i.e., spatial, verbal, and analytical skills) over three days
  - 15-20 minutes/assessment
Data Analysis

Source of Scoring Guides/Obtaining Test Scores

- Problem-solving test
  - Task-specific rubrics & rules for applying the rubrics
  - Score of 1 (correct response) or 0 (incorrect response)
  - Dichotomous scoring for reliability measures (Hopkins, Stanley, & Hopkins; 1990)
  - Maximum PST-Spatial score of 20; PST-Verbal score of 20; PST-Overall score of 40
  - Converted to percentage scores
Cognitive skills assessments

- Based on scoring guides provided by test developers
- Responses scored “right” or “wrong”
- Scoring methods varied by test segments (e.g.: correct responses – incorrect responses; correct responses – 25% of incorrect responses)
- Students therefore had possibility of obtaining negative scores
- Maximum spatial score of 68; verbal score of 54; analytical score of 50
Frequency Analysis

- Measures of central tendency
- Range & distribution of scores

Multiple Regression Analysis

- Independent variables: spatial skills, verbal skills, analytical skills
- Dependent variables: PST-Spatial scores, PST-Verbal scores, PST-Overall scores
- Significance established at $p \leq 0.05$

Correlation Analysis

- Determine pair-wise relationships among the cognitive skills
- *a priori* decision: correlation of .70 or greater - a very strong relationship between variables
# Results
## Frequency Analysis: Problem-Solving Performance

<table>
<thead>
<tr>
<th></th>
<th>Distribution &amp; Range</th>
<th>Mean &amp; Standard Deviation</th>
<th>Percentage of Students at or above Mean</th>
</tr>
</thead>
</table>
| **PST-Verbal Subtest Scores** | Normal Distribution 5% - 95% | $m = 39\%^*$
$s.d. = 20.67$                     | 45%                       |
| **PST-Spatial Subtest Scores** | Normal Distribution 0% - 75% | $m = 38\%^*$
$s.d. = 18.48$                     | 54%                       |
| **PST-Overall Scores**    | Normal Distribution 5% - 84% | $m = 39\%$
$s.d. = 17.89$                     | 51%                       |
Frequency Analysis: Problem-Solving Performance (cont’d.)

► Difference between PST-Verbal & PST-Spatial means statistically significant \[ t(68) = 15.608; p = .000 \]

► Students performed better on PST-Verbal than on PST-Spatial subtests
  - 46.4% - higher PST-Verbal than PST-Spatial scores
  - 40.6% - higher PST-Spatial than PST-Verbal scores
  - 13% - equivalent PST-Verbal and PST-Spatial scores
# Frequency Analysis: Cognitive Test Performance

<table>
<thead>
<tr>
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<th>Distribution &amp; Range</th>
<th>Mean &amp; Standard Deviation</th>
<th>Percentage of Students at or above Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Verbal Skills Scores</strong></td>
<td>Normal Distribution</td>
<td>$m = 17.98$</td>
<td>49%</td>
</tr>
<tr>
<td></td>
<td>-3 to 44.5</td>
<td>$s.d. = 11.3$</td>
<td></td>
</tr>
<tr>
<td><strong>Spatial Skills Scores</strong></td>
<td>Normal Distribution</td>
<td>$m = -4.93$</td>
<td>58%</td>
</tr>
<tr>
<td></td>
<td>-54 to 32</td>
<td>$s.d. = 20.8$</td>
<td></td>
</tr>
<tr>
<td><strong>Analytical Skills Scores</strong></td>
<td>Normal Distribution</td>
<td>$m = 6.59$</td>
<td>43%</td>
</tr>
<tr>
<td></td>
<td>-11 to 33.25</td>
<td>$s.d. = 8.34$</td>
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</tbody>
</table>
Students tended to have higher verbal than spatial skills - 89.9% scored higher on verbal than spatial skills assessments.

Only 51% of those had higher PST-Verbal than PST-Spatial scores.
Results: Regression Analysis

- **Results of Regression Analysis**
  - **PST-Verbal Subtest scores**
    - Relationships with spatial and analytical skills stronger than relationship with verbal skills
  - **PST-Spatial Subtest scores**
    - Relationship with verbal skills stronger than relationship with spatial or analytical skills
  - **PST-Overall scores**
    - Relationships with verbal and spatial skills stronger than relationship with analytical skills
Results: Correlation Analysis

Results of Correlations

- Cognitive Skills & PST-Verbal Performance
  - Strongest relationship with spatial skills (.528)
  - Similar relationships with verbal and analytical skills (.486 and .484, respectively)

- Cognitive Skills & PST-Spatial Performance
  - Strongest relationship with verbal skills (.672)
  - Weaker relationships with spatial and analytical skills (.591 and .487, respectively)
Correlation Analysis (cont’d.)

- **Cognitive Skills and PST-Overall Performance**
  - Relatively similar relationships with verbal and spatial skills (.625 and .607, respectively)
  - Weakest relationship with analytical skills (.531)

- **Relationships among the Cognitive Skills**
  - Significant, but not very strong pair-wise relationships (below .70)
  - Verbal and spatial (.551); verbal and analytical (.636); spatial and analytical (.470)
Findings

Relationships between students’ verbal skills and performance on PST-Verbal problems

- Not clearly defined – not as strong as would be expected
- Stronger relationship with spatial skills
- Stronger verbal than spatial skills not necessarily related to higher PST-Verbal scores
- Similar relationship with analytical skills

Relationships between spatial skills and performance on PST-Spatial problems

- Not clearly defined – not as strong as would be expected
- Stronger relationship with verbal skills
- Higher PST-Spatial scores not necessarily achieved by students with stronger spatial than verbal skills
Students with equivalent PST-Verbal and PST-Spatial scores did not have similar verbal and spatial skills scores (i.e., within 5 points)
Limitations of Findings

► Spatial skills assessment instrument: Does it assess different skills than those required for producing/using a drawing to solve math problem?

► Sample size
## Discussion: Relationships between Cognitive Skills and PST-Verbal Performance

<table>
<thead>
<tr>
<th>Findings</th>
<th>Prior Research Supported</th>
<th>Prior Research Not Supported</th>
<th>Not Addressed by Prior Research</th>
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<tbody>
<tr>
<td>Verbal skills not necessarily related to performance on verbal-response items</td>
<td></td>
<td>Lean &amp; Clements (1981) – Students using verbal-logical means outperform others</td>
<td></td>
</tr>
<tr>
<td>Stronger relationship with spatial skills</td>
<td></td>
<td></td>
<td>No such relationship indicated by previous research</td>
</tr>
</tbody>
</table>
# Relationships between Cognitive Skills and PST-Spatial Performance

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<tr>
<td>Spatial skills not necessarily related to performance on spatial-response items</td>
<td><strong>Fennema &amp; Tartre</strong> (1985) – no difference in accuracy of mathematical solutions between students with high/low spatial or verbal abilities;  <strong>Landau</strong> (1984) – pictorial representations not helpful;  <strong>Presmeg</strong> (1986a) – standard vs. nonstandard diagrams; produce inflexible thinking &amp; inability to recognize concepts</td>
<td><strong>Battista</strong> (1990) – spatial visualization &amp; logical reasoning significantly related to geometrical problem solving</td>
</tr>
<tr>
<td>Weak relationship with analytical skills</td>
<td><strong>Eisenberg &amp; Dreyfus</strong> (1986) – visual or analytical approaches used by “expert” mathematicians;  <strong>Lean &amp; Clements</strong> (1981) – verbal-logical vs. visual approach</td>
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# Relationships between Cognitive Skills and PST-Overall Performance

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<td>Verbal &amp; spatial skills have relatively strong, relatively equal relationships to overall problem-solving performance</td>
<td><strong>Fennema &amp; Tartre</strong> (1985) – high spatial/low verbal, or high verbal/low spatial and accurate mathematical solutions; <strong>Landau</strong> (1984) – strong relationship with spatial skills</td>
<td><strong>Lean &amp; Clements</strong> (1981) – spatial conventions have small influence; <strong>Fennema &amp; Tartre</strong> (1985) – emphasis on spatial skills not effective</td>
</tr>
<tr>
<td>Practically significant relationship with analytical skills</td>
<td><strong>Krutetskii</strong> (1976) – logical/analytical component &amp; overall problem-solving performance</td>
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## Pair-Wise Relationships among Spatial, Visual, and Analytical Skills

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<tr>
<td><strong>Spatial &amp; analytical skills</strong>: statistically significant relationship</td>
<td>Battista (1990); Lean &amp; Clements (1981) – viewed as existing at opposite ends of a scale</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Spatial &amp; verbal skills</strong>: Statistically significant relationship</td>
<td>Fennema &amp; Tartre (1985) – viewed as discrepant</td>
<td></td>
<td>Any relationship between the two ignored in the literature</td>
</tr>
</tbody>
</table>
Implications

► Cognitive skill strength vs. mathematical conceptual knowledge
  ▪ Future Research: Investigate & unravel complexities
  ▪ **Teaching Practice**: Interpret student performance with respect to math understanding and skill strength

► Effective use of cognitive skills for expression of mathematical ideas may be a learned skill
  ▪ Future Research: Will teaching students effective strategies influence problem-solving performance?
  ▪ **Teaching Practice**: Instruction on strategies for effective use of skills; practice on both skill types
  ▪ **Teacher Education**: Pre-service training & examples of effective instructional strategies
Implications (cont’d.)

► Possible mismatch between assessed spatial skills & those needed to solve given problems
  - Future Research: Does the skill assessment format play a role in the observed discrepancies?
  - Teaching Practice: Practice in producing drawings
  - Teacher Education: Strategies for developing/utilizing spatial interpretation skills & skills in producing drawings

► Relationship between verbal and logical/analytical skills
  - Future Research: Measured separately (varying effects on problem-solving performance)
Implications (cont’d)

► Interpretation of students’ responses on problem-solving assessments

- **Teaching Practice**: Analysis and interpretation of student responses (interviews); develop multiple ways of expressing math ideas
Implications for Differentiated Instruction

► Effective differentiated instruction requires a clear understanding of where each student is

► Assessment-driven data is of utmost importance in the design of effective differentiated instruction

► Cognitive styles affect how individuals process information, and

► They also affect the types of tasks that they find difficult/easy (Sternberg & Zhang, 2001)
Regardless of cognitive style, individuals can use either mode of representation if they make a conscious choice (Sternberg & Zhang, 2001).

Educational focus should not necessarily be on who has/does not have a particular ability, but on how to capitalize on individual strengths, and how to develop learning potential.
Questions?