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Optimization of Costs and Service Level by Choosing the Best Customer Order Decoupling Point

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Abstract

The customer order decoupling point (CODP) as the link in the supply chain between processes based on uncertain information, such as sales forecasts, and certain information in the form of customer orders is crucial for production efficiency, storage costs, and the quality of logistics. This paper focuses on developing an approach for identifying a CODP that provides the highest potential for achieving business objectives. Within the context of the above-mentioned topics, this paper focuses on the interrelations and the tradeoffs that have to be made when positioning the COPD. The goal of the paper is to present an effective and chronological sequence of tasks, analyses and methods, criteria, and indicators that can help a production planner determine the CODP. In the first part of the paper, the factors affecting the positioning of the CODP (internal and external to the company) and their characteristics are identified. Based on extensive literature research, these factors are then mapped to manufacturing concepts, such as make-to-stock, assemble-to-order, and make-to-order. The factors that lead to moving the CODP are also identified and used as the foundation for the development of an iterative procedural method with four stages. In stage 1, the as-is state and current goals are captured and suitable CODPs are identified for the above-mentioned logistic factors. In stage 2, the products are combined into groups to reduce the complexity of the analysis and a procedure for accomplishing this is proposed. The number of suitable CODPs is reduced in stage 3 and the factors that influence the positioning of CODPs are taken into consideration. In stage 4, a financial and qualitative evaluation is performed on the various CODPs.
1 Introduction

Today, production planning and control (PPC) still forms the core of any industrial enterprise [1] and is an essential component of a production system [2]. The principles of PPC are well documented in the literature. The same is true for inventory management. Both production planning and control and inventory management are strongly influenced by the position of the customer order decoupling point (CODP). These play an essential role for a manufacturing company but also for entire supply chain networks. The objectives of controlling the production system before and after the CODP are universally known from the dilemma of materials planning [3], which depicts the incompatibility of the goals of a high logistic performance (short throughput times, fast delivery service) and low logistic costs (high utilization/performance, small inventories). The CODP becomes an important factor when weighting the sub goals because a strategic positioning can support the desired prioritization.

Despite the strategic importance of the CODP positioning, the CODP is often determined by growing cost structures in the respective industries. Many companies use flat rates for specific items and order types. These flat rates are based on long-time experience or qualitative criteria. Organizational structural problems such as the positioning of the CODP do not resolve themselves on their own. Changes to existing cost structures are usually only carried out when it is clear that a situation is causing an economic inefficiency. Only in a few cases are model calculations performed. Not even the literature contains a lot of information on where it makes the most sense for a company to position their CODP. The available literature limits itself primarily to a description of the general principles of the CODP and the influences and effects of positioning the CODP (see [4], [5], [6], [7], [8]). The literature does not contain any models that could be used to concretely identify the CODP.

A procedure for selecting a suitable CODP is outlined in the following sections. This procedure takes into account the various internal and external influencing variables and focuses on the manufacturing of multipart goods of simpler and more complex cost structures as well as on simple goods. The objective is to depict a goal-oriented logical system in the form of a model that can be applied universally. The model takes into consideration market-related requirements along with the requirements that arise from the products, the manufacturing organization, and the business objectives for costs and service quality. The model is comprised of (i) the identification of the influencing factors, problems, and activity, (ii) the explanation of methods and tools for finding solutions, (iii) the presentation of qualitative evaluation rules and indicators, and (iv) the placement in a goal-oriented logical order.

This paper is structured as follows: the theoretical concept of the customer order decoupling point is presented first and then the procedure is introduced on the basis of the theoretical concept.
2 Theory
A manufacturing company must verify on a regular basis which item is being manufactured for a specific customer at which production stage and what material, parts, and finished goods are anonymously being manufactured or acquired in stock. They also have to determine if and for which items stock is being held [9]. These facts circumscribe the decoupling problem. Decoupling points help to reduce the complexity in systems by lowering susceptibilities to interferences, increasing forecast reliability and controllability, and increasing utilization of capacity. These advantages come at the expense of higher stock costs.

As the last warehouse position of a product in the manufacturing process or supply chain, the CODP represents interfaces between two control loops. Chronologically, the positions before the CODP are dominated by forecast-driven processes. Demand is determined based on forecasted numbers and calculations based on historical data – a push control is implemented. The focus of production planning and control is on efficiency. High utilization and a low number of Works in Progress (WIP) are strived for while delivery reliability plays a secondary role. The processes after the CODP are usually determined by customer demand. Instead of an efficient execution, agility ends up taking a backseat. The dominating parameters are fast delivery times and a high level of readiness for delivery. Customer orders are the cycle generators of the processes (pull control).

The influencing factors for positioning the CODP can be divided into three basic areas: market, product, and production and/or processes.

As the environment of the manufacturing company, the (sales) market is essentially responsible for the decision for or against a CODP. The market-related influences include order-specific variables and expectations of the customer. Order-specific variables are demand quantities, frequencies, and volatility. Customer expectation variables are price, quality, adherence to delivery dates, product diversity, product customization possibilities, and delivery times. It has to be verified if these market requirements are from the category Order Winner or Order Enabler. The selected CODP must be in the position to support Order Winner and thereby make a contribution to the overall goal of the company (to make money). The definitive product-specific variables are product spectrum and
product structure, including the degree of standardization, influence on the design by the customer and the construction-related design of a product.

In the area of production, the central position where the services are rendered, the influencing factors are identified as production type and manufacturing and assembly processes. As well, the interruptibility of a process step and/or a process sequence, the flexibility of the production process (including set-up times in particular), and the ratio between production throughput times and delivery times all need to be taken into consideration.

According to [10], the different values can be mapped to the parameter order trigger type and thereby to the manufacturing concepts\(^1\) make to order (mto), assemble to order (ato), and make to stock (mts), as shown in Table 1. This represents an ideal mapping of the parameters and values.

2.1 Impact on customer order decoupling points
Inventory in companies is usually the consequence of steady and unsteady stochastic processes [9]. It results in random disruptions and failures and the independence of the market participant and the company regarding planning and the issuance of (production) orders. The level of the inventory is determined by the replenishment lead time, the diversification of the consumption during the replenishment lead time, and the availability to deliver as specified by the company. Goods inventory ties up capital, costs interest, requires storage space, and has associated risks (loss of stock or an item becoming obsolete). No or low stock levels lower the planning reliability and controllability within the production system and increase the risk of the non-fulfillment of customer orders. Accordingly, as the last warehouse position of a product, the CODP is associated with the non-fulfillment risk and the investment risk. If the CODP is moved, it is always to the disadvantage of one of the risks.

There are two reasons for moving the CODP downstream. By increasing the number of customer-anonymous processes, the ability to plan the system increases and the number of customer-specific processes decreases. This leads to (i) a reduction in delivery times and (ii) an increase in production efficiency. Moving the CODP upstream can result in a reduction in inventory or in an increase in the influence that the customer has on the design of the product.

2.2 Evaluation of a CODP
The suitability of a CODP must be measured against its ability to contribute to the achievement of the business goal. To do this, financial and qualitative values are compared with each other. Cost changes in production and inventory are offset against each other. It is difficult to quantify the risk of non-fulfillment. Generally, this risk increases

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\(^1\) The concept production concept captures an entire feature bundle [10]. In the context of this paper, the concept is always used to mean the creation of one of the following concepts: engineer to order (eto), make to order, assemble to order, or make to stock.
with each additional customer-specific process that has to be carried out. How high it goes up depends on the resources that are used. If all of the resources and processes are 100% controllable, then the risk of non-fulfillment is 0% [6].

3 Phase Model
A procedure model can be viewed as an instrument that provides knowledge and activities for a specific process. It is a type of reference model because it also contains recommended actions. In addition to the procedure steps (activities), the model also contains several methods and tools for each phase as well as a description of the relationships between the activities. The basic thought behind the development of the model being presented here was the determination that the more or less complex process of finding a solution – in comparison with numerous phase diagrams – can be broken down into several sub steps. The result was a procedure model with four sequential phases.

3.1 Objective definition and as-is analysis
The first step of the procedure model is to define the objective that will lead to the initiation of the CODP evaluation and to define the limits of the evaluation. These come from the long-term objective of increasing the profit of the company. Both (service) quality objectives and performance and cost objectives will be addressed through the positioning

Table 1: Influencing factors and their CODP mapping

<table>
<thead>
<tr>
<th>marked-related factors</th>
<th>make to order</th>
<th>assemble to order</th>
<th>make to stock</th>
</tr>
</thead>
<tbody>
<tr>
<td>Customization opportuni</td>
<td>Important</td>
<td>Less important</td>
<td>Unimportant</td>
</tr>
<tr>
<td>Delivery</td>
<td>Long</td>
<td>Medium - less important</td>
<td>Short - important</td>
</tr>
<tr>
<td>Sales volume and demand volatility</td>
<td>Low to medium volumes, High to medium volatility</td>
<td>Limited suitability for high volumes</td>
<td>High volumes, Low volatility</td>
</tr>
<tr>
<td>Product range</td>
<td>Products according to customer specification, Products with customer-specific variations</td>
<td>Products with customer-specific variations, Standardized products with customer variations</td>
<td>Standardized products</td>
</tr>
<tr>
<td>Product structure</td>
<td>Multipart products with complex or simple structure</td>
<td>Multipart products with complex or simple structure</td>
<td>Simple Products</td>
</tr>
<tr>
<td>Production lead-time/delivery lead-time ratio</td>
<td>P / D &lt; 1</td>
<td>P / D &gt; 1</td>
<td>P / D &gt; 1</td>
</tr>
<tr>
<td>Production type</td>
<td>Non-repetitive production, Single item and small scale production</td>
<td>Single item and small scale production, Volume production</td>
<td>Volume production, Mass production</td>
</tr>
<tr>
<td>Production process</td>
<td>Job shop production, Cellular manufacturing</td>
<td>Job shop production, Cellular manufacturing, Mass production</td>
<td>Mass production, Flow production</td>
</tr>
<tr>
<td>Assembling process</td>
<td>Field assembly, Group assembly</td>
<td>Group assembly, Sequential assembly</td>
<td>Sequential assembly, Continuous assembly</td>
</tr>
</tbody>
</table>
of the CODP. The CODP will either be positioned upstream or downstream depending on the differently weighted values of the above-mentioned factors.

The second part of this step is the capturing of the as-is state. This neutral capturing of the actual state is usually carried out using both primary and secondary methods of data collection. These methods include capturing the current manufacturing concept and the respective values of the above-mentioned criteria for production and the product as well as the market-specific importance of product customization and the importance and length of delivery times. The as-is analysis has to be performed for all items that need to be verified. The evaluation limits of the model have to be taken into consideration at this point. This step does not involve evaluating the structure of manufacturing, for example the materials planning strategy, or reengineering production or assembly processes. Possible modifications in the area of technical equipment, available space, or strategic or economic considerations such as make or buy are also not part of this step. Each item is mapped to one of the manufacturing concepts according to the current CODP: mto, ato, or mts. The results of this mapping should be displayed in an aggregated form such as item groups.

At this early point in the planning, it is possible that some of the products could be excluded from the evaluation under certain circumstances (for example, due to delivery time requirements).

3.2 Grouping

As a general rule, companies group their products into product families or product lines. These groups include a number of products with similar features or functions, a similar product structure, and a high percentage of similar manufacturing processes. These groups usually contain different sub families. To analyze these products in the model, it is necessary to disband these continually growing groups formed often under the influence of marketing and to form new groups based on product and manufacturing aspects. After the groups have been formed, the following is proposed: examine the entire range of products and identify different product features. These can be the fundamental characteristics, functionalities, or areas of application of a product or a single feature that is common to several different products. To help with this step, it might be useful to use the existing product groups as a reference. These general groups are then divided into smaller groups based on other aspects of the products and the manufacturing process. The next step involves evaluating the groups based on the manufacturing process. Products with the same or similar manufacturing cycles (processes, process times) are put into one group. The groups are then divided again based on product components. The result of all of this grouping and regrouping is homogenous groups of items with (i) the same function that have (ii) numerous multiuse components, which means components that are used in multiple finished products, and (iii) are manufactured using many production processes that are similar or the same. These groups form the basis for the further analysis required to determine the CODP.
The product groups determined using the procedure described in the previous section can be further combined using the fundamental characteristics of the products, production, and the market mentioned above. Performing this step will take into account any clear indications for or against one of the different manufacturing concepts and will combine the products into product groups accordingly. The goal of this step is to reduce the work required in the following steps as much as possible.

These analyses are of the qualitative type and serve to exclude any of the product groups that can already be mapped to a manufacturing concept on this basis and put the focus on specific concepts as much as is possible at this point in time. The results of this analysis are combinations of product groups for which (i) a specific concept (mto, ato or mts) can be mapped, (ii) one of the concepts can already be excluded and (iii) still none of the manufacturing concepts can be or should be excluded for the further analyses.

3.3 Identification

The processing of the data to identify a suitable CODP occurs in two steps: one forming the basis for the other. In the first step, the positioning of the CODP is examined from a logistical perspective. After the positioning possibilities have been limited by this first step, the remaining possible positions are examined in detail from a manufacturing and process perspective respectively.

Products differ not only in their design or manufacturing process but also in their importance for the company and the sales market. These differences can have a direct impact on the CODP. For this reason, it is necessary to classify and prioritize the products according to different criteria. The best method for doing this is an ABC/XYZ analysis, which is a combination of an ABC and an XYZ analysis.

The manufacturing concept make to stock or make to order or a combination of the two is alluded to in connection with the position of the CODP. Customer orders are not just of great importance according to the definition of the CODP: they also play a role in determining the position of the CODP. An ABC analysis of the number of order items per product can be used to illustrate the influence that customer orders have on the manufacturing concept. In particular, this analysis is necessary to answer the question if it is possible to move the CODP upstream. The increased influence of the customer orders (frequency and quantity) results in a limitation of the possibilities for planning and controlling production. In particular, this can lead to unmanageable costs for machines with significant set-up times and with bottlenecks: machines that are usually controlled by fixed parameters. This problem does not exist for a downstream movement of the CODP. Instead, the question here is if moving the CODP downstream for product A will improve the usage of the production capacity.

Other not insignificant factors are the size of the production lot and the quantity that has to be offset from the current products. An ABC analysis of the sales volume should be used to take into account these factors when considering an upstream move of the CODP. The average lot size can be derived from the quotient of the items and the sales volume.
Additional information is acquired by categorizing the items into XYZ classes, through which the product range is classified according to the predictability of the goods. X items are usually easy to predict because they are consumed on a regular basis. Thus, a late CODP is usually considered for these types of goods. Make to stock and a late CODP respectively are suitable for A items because the inventory risk is low and a high level of readiness for delivery can be achieved. Contrary considerations have to be taken into account for Z items. Sales forecasts for items with a Z classification are hard or even impossible to predict. An early CODP should be considered for these items as most of them are C items, possibly with a low degree of readiness for delivery. The results of the XYZ analysis have to be plotted against the combined ABC analyses. This creates a three dimensional matrix with 27 fields. These fields are subsequently referred to as sales classes. The sales classes are named based on their respective equivalents in the analyses, according to the sequence of the execution.

When trying to make a decision, companies often base their decisions on indicators. Indicators are used to bundle the abundance of available data in a company into meaningful quantities. This should be done to the 27 sales classes at this point in the procedure.

Within the scope of determining the CODP, the following indicators can be used for decision-making:

i. Number of items per order item (according to item and sales class)
ii. Order items per product and period (according to item and sales class)
iii. CODP-required change in lot size \[= \text{new production lot size} / (\text{ii})\] (according to item and sales class)

The significance of the indicators emerges especially when they are compared with each other. For example, the division of indicators (ii) by indicators (iii) represents the factor by which the number of production lots has to changes for the CODP. The following general statements can be made about the indicators:

- The larger the number of order items, the smaller the difference in set-up costs with a downstream movement of the CODP.
- The lower the order frequency of a product and period, the higher the storage costs (in relation to the value of the product) because of higher levels of safety stock and lower inventory turnover.
- The lower the order frequency, the higher the possible savings by moving the CODP upstream.
- The closer indicators (iii) lie to value 1, the lower the savings/additional costs in production by changing the lot size for the CODP.

The described analyses are based on historical data. Thus, the statements derived from this data have only limited significance for future manufacturing concepts. As the selection of the CODP is a strategic business decision (see [5]) that will be applied over the long term, future developments in this area should be anticipated and taken into consideration as much as possible. Modifications to the materials planning structure, the
sales principle, or the product range can result in important changes in the planning data. Therefore, it is advisable to anticipate any upcoming changes that have an impact on the sales performance of the company (changes in the quantity of sales, number of shipments, or the materials planning frequency) and to perform the analyses on the modified data, which means the data calculated based on historical figures and on forecasted change. The use of this method will result in the revelation of CODP tendencies, which then need to be substantiated and evaluated.

Substantiating the tendencies requires the consideration of the (i) production lead time/delivery time ratio, (ii) structure levels and low-level codes, (iii) variant emergence points, (iv) lot size changes, (v) added value, and (vi) bottleneck considerations. The evaluation encompasses the changes in storage costs and production costs. The following procedure is the recommended course of action for this evaluation:

**Production lead time/delivery time ratio**  Rough scheduling of the workflows at the (sub) group level (see the section Grouping). *Level of detail:* Changes made to the resources, workshops, work stations, and equipment must be easy to recognize.

**Structure levels and low-level codes, variant emergence points**  The convergence point of the parts list is a good location for positioning the CODP (for example, see [7]). The biggest potential for reducing inventory costs is by the so-called pooling or centralization effect (see the well-known correlation of the *Square Root Law* from the centralization considerations of distribution systems [11]). The convergence point of the parts list is the low-level code that is on the lowest number of items. Parts usage lists can be used to identify variant emergence points.

**Lot size changes**  These changes can be extracted from the indicators (iii).

**Added value**  The added value is the value added to each production step. The added value has to be taken into consideration during the evaluation.

**Bottlenecks**  A bottleneck analysis of downstream processes has to be carried out if the CODP is moved upstream. There are some recommended methods for performing this analysis (for example, see [12]).

### 3.3.1 Changes in the lot-size dependent unit costs of production

The following calculation can be used for the initial estimate of the development of the costs in the planning phase of the identification:

\[
\begin{align*}
    c_{\text{unit, actual}} &= \frac{\sum_{i=1}^{n} C_{s_i} + (c_{M_i} + c_{P_i}) \times t_{M_i}}{x_{\text{actual}}} \\
    c_{\text{unit, new}} &= \frac{\sum_{i=1}^{n} C_{s_i} + (c_{M_i} + c_{P_i}) \times t_{M_i}}{x_{\text{new}}},
\end{align*}
\]
where

- \( i = 1, \ldots, n \); All of the process affected by the movement of the CODP.
- \( x_{\text{actual}} \) and \( x_{\text{new}} \): The actual and the new average production lot sizes.
- \( C_{Si} \): The average set-up costs for process \( i \).
- \( c_{Mi} \): The machine hourly rate for process \( i \).
- \( c_{Pi} \): The personnel hourly rate for process \( i \).
- \( t_{Mi} \): The average working time for process \( i \).

If the processing time, in a certain frame, is independent from the size of the production lot, the following calculation can be used to determine the difference of the unit costs:

\[
\begin{align*}
\Delta c_{\text{diff},i} &= c_{\text{unit,new}} - c_{\text{unit,actual}} \\
&= \frac{C_R}{x_{\text{new}}} + \frac{C_M}{x_{\text{new}}} - \frac{C_R}{x_{\text{actual}}} - \frac{C_M}{x_{\text{actual}}}
\end{align*}
\]

where

- \( C_R \): The total of the set-up costs.
- \( C_M \): The total of the machine costs.

If the processing time is proportional to the production lot size, the following is true:

\[
\Delta c_{\text{diff},ii} = c_{\text{unit,new}} - c_{\text{unit,actual}} = \frac{C_R}{x_{\text{new}}} - \frac{C_M}{x_{\text{new}}}
\]

When the CODP is moved, the production costs of an item changes by a value between \( \Delta c_{\text{diff},i} \) and \( \Delta c_{\text{diff},ii} \). If it is assumed that production lot sizes increase as a result of a downstream movement of the CODP, then the following is valid: \( x_{\text{new}} \leq x_{\text{actual}} \). Thus, it follows that \( \Delta c_{\text{diff},i} \geq \Delta c_{\text{diff},ii} \). Accordingly, the following is valid for an upstream movement of the CODP: \( \Delta c_{\text{diff},i} \leq \Delta c_{\text{diff},ii} \). These costs have to be offset against any potential savings or additional costs that arise in the area of inventory as a result of moving the CODP. Under the assumption that transportation and replenishment costs are not dependent on CODP changes and by disregarding storage bin costs, the following mathematical relation, based on the Square Root Law, is valid for the inventory costs per unit if the CODP is moved from P1 to P2:

\[
\Delta c_{\text{P1-P2}} = \pm \sqrt{\text{Var}_{\text{P1-P2}}} \cdot (I_c + I_r) \cdot \text{Value}_{\text{P1-P2}}
\]

where

- \( \text{Var}_{\text{P1-P2}} \): Change in the number of multiple uses of the item from the original CODP P1 to the possible new CODP P2.
- \( I_c + I_r \): Interest rates for tying up capital in stocks and the risk of keeping stocks.
- \( \text{Value}_{\text{P1-P2}} \): Change in the value of the goods from the original P1 to the possible new CODP P2.

### 3.4 Evaluation

The lot-size related costs of production have to be taken into consideration when estimating the cost impact of a new CODP. These include the production costs per piece...
(personnel costs and set-up costs) as well as the hourly rates for machines, which include the fixed machine-related costs for production. The results of the estimation of the warehousing and production costs are added to the estimated lot-size related costs. The total can be compared to the results from other CODP scenarios and with the current situation.

Quality standards are the benchmark for achieving quality objectives. These standards are specified by company management, customers, lawmakers, or the market. Quality standards are numerical values that determine the permissible values of individual goals. With regard to the CODP-dependent influences on the quality of service (not on the product quality!), the following can be concluded:

The risk of non-fulfillment of customer orders increases with an upward movement and decreases with a downward movement of the CODP because more or fewer processes are executed specifically for customer orders and, thus, tend to need to be executed quickly.

The fastest delivery times are possible with mts. The longest delivery times are for products that are manufactured using the mto or eto manufacturing concepts.

The manufacturing concept mts can only be used to produce standardized products. The possibilities for influencing the product features increase with the upstream movement of the CODP.

While the service qualities of delivery time and product customization are relatively easy to determine, the risk of non-fulfillment in connection with the CODP is difficult to determine. The service level can be used for non-fulfillment. The shipping quality can be considered as independent from the CODP. Thus, the commitment to performance and the adherence to delivery dates are crucial factors in evaluating the qualitative influence of the CODP. The risk of these influencing factors is usually determined by management – the planning and control of the processes. If all processes can be 100% controlled, the risk decreases to 0%. The process quality, which means the quality of the manufacturing process for a product, can be calculated using the First Pass Yield indicator. The First Pass Yield is the number of defect-free results from a process cycle. Indicators for the reliability and availability of production resources as well as the breakeven performance of production and the anticipated capacity utilization are all factors that need to be taken into consideration.

4 Conclusion

A procedure model for determining the CODP was developed and presented in this paper. In the first part of the paper, the theoretical concept was presented and then, on this basis, the procedure model for determining the CODP was developed. Like other procedure models, the model was based on the fact that most complex problems can be broken down into sub problems and solved successively. The procedure model developed in this paper is comprised of four phases with different work packages for each phase. These phases are to be carried out one after the other and partly even iteratively.
A possible follow-up to this paper is the further examination of the developed procedure model. A quantification of the decision rules, for example the determination of the correlation between the developed indicators and the selected manufacturing concept, would be useful for this further examination. As well, an inclusion of a production lot size optimization and the identification of decoupling points for production optimization or the inclusion of postponement considerations (process reengineering) should be considered for the model. Follow-up research could be performed on the firm establishment of the evaluation, in particular qualitative, as well as the comparison with the cost-related influences. Finally, it should be examined if the procedure model could be converted into a completely mathematical model. Even an exact mathematical solution for the problem has advantages over the approach used in this paper.

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