The Mathematical Preparation of Secondary School Teachers

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Abstract

In the summer of 2007, a group of doctoral students at the University of Georgia gathered to discuss the mathematical preparation of secondary teachers. The group used Mathematics for High School Teachers: An Advanced Perspective by Usiskin, Peressini, Marchisotto, and Stanley (2003) as the catalyst for the discussion. Participants agreed that future teachers need opportunities to examine high school and college mathematics differently from the way they had as students, with specific emphasis on connections, representations, and history. Features of this text that were highlighted in the discussions were the attention topics with commonly held misconceptions, the historical rationales and development of mathematical topics, and the role of mathematical definitions. Group members felt that, depending on one's purpose for using the text and the backgrounds of the prospective teachers, this text could be used, in conjunction with supplemental materials, in a variety of capacities: for a capstone course, a connections course, or a set of replacement mathematics courses.

The Mathematical Preparation of Secondary School Teachers[1]

Introduction

At the meeting of the Association of Mathematics Teacher Educators in January 2007, a group of professors from across the nation gave a presentation on capstone courses. A number of issues brought up in the session were of interest to some of the University of Georgia doctoral students in attendance: the idea of a capstone course in mathematics education, the popularity of a particular text, and the content chosen by different institutions. Most of us were unfamiliar with the concept a of capstone course, having received our undergraduate education from universities with full mathematics education programs. But of even greater interest was the popularity of a book with which we were unfamiliar, *Mathematics for High School Teachers: An Advanced Perspective* by Zalman Usiskin, Anthony Peressini, Elena Marchisotto, and Dick Stanley (2003). Many of the presenters reported using this text for the majority of their class content.

As those present at the session discussed capstone courses with UGA professors and other doctoral students, the actual mathematics preparation of secondary teachers emerged as a primary point of interest. A growing group of doctoral students, who were actively involved in research about mathematical knowledge for teachers at the secondary level, decided to formalize our conversations about the role of capstone courses and the mathematics deemed essential for high school teachers. We developed a seminar to discuss the mathematical preparation of teachers, using the Usiskin et al. text as a guide for the discussion. This summer seminar was open to all doctoral students at UGA and was sponsored by Jeremy Kilpatrick, who had previously used the text in a course at UGA.

The Mathematical Preparation of Teachers

The Conference Board of the Mathematical Sciences (CBMS) released *The Mathematical Education of Teachers* (MET) in 2001. A major point of emphasis was that the preparation needed for teaching mathematics is different from the preparation needed to continue general graduate study in mathematics. This idea differs slightly from the historically held recommendation that those wishing to teach mathematics should earn the equivalent to a bachelor’s degree in mathematics. Rethinking teacher preparation was necessary due to the

[1] The author wishes to thank Bob Allen, Kathy Daymude, Sarah Donaldson, Ryan Fox, Gloria Jones, and Margaret Morgan for their thoughts, ideas, participation in the seminar, and ongoing conversations on this topic. I also wish to thank Jeremy Kilpatrick for his guidance and for sharing his personal experiences.

changing face of school mathematics and the expectations placed on teachers. Teachers need to be able to make informed decisions about curricula and instructional strategies that are qualitatively different from what they themselves experienced in school. To meet these needs, teachers should develop:

- Deep understanding of the fundamental mathematical ideas in grades 9–12 curricula and strong technical skills for application of those ideas.
- Knowledge of the mathematical understandings and skills that students acquire in their elementary and middle school experiences, and how they affect learning in high school.
- Knowledge of the mathematics that students are likely to encounter when they leave high school for collegiate study, vocational training or employment.
- Mathematical maturity and attitudes that will enable and encourage continued growth of knowledge in the subject and its teaching. (p. 122)

While some of these recommendations could be accomplished by redesigning present mathematics courses geared toward teachers, the CBMS (2001) advocated a year-long, 6-credit hour capstone course to connect upper-level mathematics courses to the mathematics studied in high school. The course would provide opportunities for future teachers “to look deeply at fundamental ideas of mathematics, to connect topics which students often see as unrelated, and to develop the important habits of mind” (p. 143). Additionally, the course should give prospective teachers “broad historical and cultural perspectives, insight into mathematics learning, and application of technology” (p. 123).

Requiring a capstone course for those intending to teach secondary mathematics is only one of the pathways for connecting undergraduate and high school mathematics proposed by Ferrini-Mundy and Findell (2001). They classify capstone courses under the heading of the “mathematical approach” to connecting content. Also included in this category are shadow courses that are taught alongside undergraduate mathematics courses. The authors caution, however, that the focus on mathematics in this approach may include insufficient pedagogical content knowledge for future teachers. Two other approaches they explicate are an integrative approach and an emergent approach. Integrative approaches weave together content and pedagogy, and emergent approaches begin with the practice, drawing on the mathematics that emerges from real situations. While these two approaches, especially the latter, are rare in the mathematics education programs of which I am familiar, the mathematical approach appears to be quite prevalent.

Mathematical Preparation at UGA

University of Georgia students intending to teach high school mathematics can major in mathematics education or pursue a dual degree in mathematics and mathematics education. Those earning the mathematics education degree alone are required to complete at least two calculus courses as part of their core program and nine upper level content courses, including introduction to higher mathematics, linear algebra, foundations of geometry, modern algebra, statistics, and instructional technology. Two of the three major electives must also be mathematics courses. Students complete education courses on curriculum and teaching methods and may opt to take courses in problem solving, historical and cultural foundations of mathematics, contemporary school topics (including discrete mathematics and modeling), and mathematics in context. Additionally, shadow seminars to accompany two courses, sequences and series (an elective) and modern algebra, are offered to help students see the connections between the college content and high school mathematics. In line with the CBMS recommendations, some of the mathematics courses, including statistics for teachers and the geometry course, have been revised or designed with the needs of future teachers in mind.

Doctoral Student Seminar[2]

Armed with our own personal experiences of being students and mathematics teachers, we began the seminar by reading selections from Ferrini-Mundy and Findell (2001) and Usiskin (2001). Participants were also encouraged to read selections from the MET report (2001). The goal for the readings was to acquaint seminar participants with a historical background for the discussions, with particular attention to rationale for the need for the seminar and understanding of Usiskin et al.'s purposes in writing their text.

As mathematics teachers, and as mathematics teacher educators, each participant brought with him unique perspectives and ideas as to what mathematical preparation was necessary for secondary school teachers. A commonly voiced belief was that teachers needed to complete the traditional mathematics courses, or at least two years beyond what would be taught in high school, followed later by pedagogy. Others, referring to Ma's (1999) work, questioned the needed

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Although this is a report of a group participation seminar, the author accepts responsibility for the ideas expressed in this paper. In many instances, disagreements were voiced. While it is the author's purpose to present an accurate view of the discussions, it is possible that participants' views were misunderstood. The author apologizes if any participant views are inaccurately presented.

for higher mathematics, advocating, rather, instruction of the deeper understandings of high school mathematics. Regardless of the overall beliefs expressed, participants agreed that future teachers need opportunities to examine high school and college mathematics differently from the way they had as students, paying particular attention to connections, representations, and historical development. As the introductory session drew to a close and we began discussing the content presented by Usiskin et al., a series of questions—which remained in our minds, but were never fully discussed—arose: Could undergraduate courses, designed solely for teachers, emphasizing both content and pedagogy, be developed to replace current undergraduate mathematics requirements? What would such classes contain? Could the Usiskin et al. text be used in such courses? In reflecting, I pose an additional question: What impact would a change in preparation such as this have on the view of teaching as a profession?

Background on Mathematics for High School Teachers: An Advanced Perspective

Usiskin (2001) outlines his conception of "teachers' mathematics" as comprising three broad areas: concept analysis, problem analysis, and connections and generalizations. More specifically, these three areas include:

(a) ways of explaining and representing ideas new to students, (b) alternate definitions and their consequences, (c) why concepts arose and how they have changed over time, (d) the wide range of applications of the mathematical ideas being taught, (e) alternate ways of approaching problems with and without calculator and computer technology, (f) extensions and generalizations of problems and proofs, (g) how ideas studied in school relate to ideas students may encounter in later mathematics study, and (h) responses to questions that learners have about what they are learning. (p. 3)

To address the needs of teachers' mathematics, the goals of the text were to provide more material than could be taught in one or two courses for upper level undergraduates or graduate students that focuses on mathematics, not methods, relevant to the classroom. One of the text authors' hopes was to move "towards a set of canonical courses in the field" (p. 4) that provides content and perspectives that are important for secondary teachers.

Although seminar participants tried to keep the authors' intentions and goals for *Mathematics for High School Teachers: An Advanced Perspective* in mind during our discussions, we also tried to see how important we deemed specific topics for secondary teachers. The text authors do provide suggestions for specific chapters to be taught, given different foci for possible courses that would use the text, but we were attempting to analyze the entire text as if we had unlimited time with our potential students. An obstacle we repeatedly faced, just as we face when planning any curriculum or reviewing any materials, was our conception of the mythical student who would be enrolled in a course that uses the text. Should we consider those who had completed the required mathematics courses or should we attempt to replace the required courses with this text? Which required courses should we assume to have been completed? Are all courses equal? That is, if a student completes modern algebra at one university, will the same content have been taught at a different university? My personal decision was to consider prospective teachers attending the University of Georgia, or any other university, who had completed up through college geometry and modern algebra. Further, my guiding question as I reviewed the text was, "What knowledge, or treatment of a topic, would be beneficial for prospective secondary teachers?"

Reviewing the Text

As we read and worked through bits of each chapter, we generally decided that, for each chapter used, the entire chapter should be read by the prospective teachers; however, some content was definitely more essential than other. We attempted to point to topics we felt were treated well, areas we felt were lacking, and, if possible, identify sources that could be used as supplemental to this text. We also suggested exercises for discussion, class presentations, or homework; however, we do believe that such decisions should be based on one's particular students[3]. The remainder of this section highlights the discussions that took place. Note that, although almost the entire text was addressed in the seminar, only aspects that sparked conversation or disagreements are detailed here.

Chapter 1: What is meant by "an advanced perspective"? As the goal of this chapter was to introduce prospective teachers to the ideas of concept analysis, problem analysis, and mathematical connections, the seminar participants felt that this chapter was essential to helping students understand the perspective to be taken in this text. Not only did we see the value in the approach taken in the chapter, but we were also impressed with the content and the chapter problems. We believed that secondary teachers could benefit from discussing or investigating the majority of the chapter problems.

Chapter 2: Real numbers and complex numbers. Seminar participants believed that prospective teachers should understand how the number systems are built up; however, it may not be essential for teachers to be able to prove that development. The historical connections, especially if students do not take other history courses, were especially helpful for both

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[3] A list of suggested problems is available from the author by email.

prospective teachers and their future students. We specifically saw value in chapter problems that required teachers to dig deeply into high school mathematics. An activity we found particularly thought-provoking was developing an accurate Venn diagram of the partitions of the real numbers (cf. Usiskin et al., 2003, p. 43).

Chapter 3: Functions. Teachers should understand how functions permeate school mathematics. The attention to representations, connections, and technology, as well as to conceptual understanding of what it means to be a function, was felt to be important. This chapter included a multitude of problems that could also be used with high school students, particularly exercises on discontinuities, common functions (including piece-wise and general step functions), and composition and inverses of functions. We did feel that the topics of fitting linear, exponential, and polynomial functions to data could be better addressed with other methods. Incorporating the use of the TI-83, Fathom, or Excel would better connect the mathematics to high school mathematics. An additional resource with an in-depth treatment of fitting polynomials to data is *Mathematical Connections* by Al Cuoco (2005).

Chapter 4: Equations. We felt that the majority of this chapter could serve as a good refresher of high school mathematics topics as well as address misconceptions about equations that high school students may hold. Of interest was the authors' discussion of the equality, the treatment of the equal sign in school mathematics, and notational issues for equality versus geometric congruence.

Chapter 5: Integers and polynomials. Much of the content found in this chapter may be found in an introductory number theory course or perhaps in an abstract algebra course, particularly if the Shifrin (1996) text is used. One point of division within our group was how we would treat Unit 5.1: Natural numbers, induction, and recursion. Some participants suggested reviewing the first three sections of the unit on the basics of recursion and induction and skip the "extended analysis" of an induction situation, determining the number of regions created by n non- concurrent and non-parallel lines. Others felt that mathematics majors or education majors who had completed a few high level mathematics courses would have sufficient knowledge and understanding of induction and would, therefore, garner greater benefits from the problem solving required of the task.

The remainder of the chapter is highly proof-based. Because students may have completed mathematics courses that address divisibility of integers and polynomials, instructors must assess their students' needs in determining what is important to address. Those of us who completed

Shifrin's (1996) abstract algebra found little from units 5.2 and 5.3 that we deemed essential to teach in an additional course. The one exception is section 5.2.5, the base representation of positive integers. This algebraic approach to place value could aid students in deepening their understanding of place value and of the decimal number system, in particular. An enhanced discussion could include historical approaches to place value, counting methods in different cultures, and the use of different bases in technology (cf. Eves, 1990).

Chapter 6: Number system structures. As with chapter 5, much of this content appears to reinforce abstract algebra topics, with few additional connections to the actual mathematics found in high schools. However, the applications of modular arithmetic to cryptology and calendars may not have been seen by teachers in traditional mathematics courses. As these are topics that may engage their future students, exploring them may be beneficial.

Chapter 7: Congruence. Three major ideas in chapter 7 proved fertile ground for discussion. One would hope that teachers who complete a college geometry course would have experience with and understand the implications of Euclid's *Elements*, specifically the 5 postulates. However, teachers may have not fully explored the affordances of different definitions and assumptions. Our discussions repeatedly called into question our own definitions for geometric figures, such as cylinder, trapezoid, and rhombus. In examining high school texts, we found that they, too, sometimes differed in their definitions and in what they classified as theorems and postulates. Investigating different texts' definitions of common terms could lead to a discussion of the benefits and strengths of different characterizations.

Another point of interest was the transformational approach to congruence presented by the text. High school texts also differed in their approaches. Some discussed congruence in terms of transformations whereas others conceptualized it in terms of equal measures. Teachers must be aware of how texts treat high school concepts and know where to look for materials that may provide a different approach. This type of awareness, or curricular knowledge (Shulman, 1986), is not likely inherent in prospective teachers; it must be developed.

A final note on this chapter, which also applies to others, is the role of geometry programs such as GSP. Seminar participants differed in their beliefs about the degree to which these programs should be used. All agreed that dynamic software programs are valuable for investigations and testing conjectures; disagreement arose in determining if theorems could be proved with the software and in how much reliance should be placed on such programs. It was suggested that prospective teachers also need experience with a compass and straightedge, and they should know how the software constructions work. Further, prospective teachers should have opportunities to explore a variety of dynamic software programs.

Chapters 8–11. The discussion of this text occurred during summer semester, and as is the case with many courses, we found ourselves with too little time to adequately accomplish our full agenda. Therefore, the discussion on chapters 8–11 was limited but reinforced our previous assessments of the text in terms of a deeper awareness of importance of definitions, providing links to high school mathematics that prospective teachers may have not visited since they were high school students, and situating mathematical topics in historical contexts.

Next Steps

Seminar participants expressed interest in continuing the discussion of *Mathematics for High School Teachers: An Advanced Perspective* in future semesters. In addition to engaging in deeper conversations about the last chapters in the text, we would like to design courses with specific emphases, supposing specific prerequisite knowledge, for prospective teachers. One potential design could be that of a capstone course to serve the needs of mathematics majors who seek certification. Another design could characterize a “connections” course, with the goals of connecting college mathematics to high school mathematics and refreshing prospective teachers’ knowledge of high school mathematics content. A third design could include a number of courses, namely a set of courses designed to replace present mathematics requirements with courses that not only address the high level of mathematics taught in the traditional courses but also the pedagogical connections needed by teachers. All three of these design paradigms seem to be aligned with the assertion that teachers need to understand qualitatively different mathematics from that needed by mathematics majors.

Closing Thoughts

High school mathematics teachers need a deep understanding of mathematics. Most teacher educators would likely agree with this statement. *Mathematics for High School Teachers: An Advanced Perspective* by Usiskin et. al (2003) is one text that may prove helpful in deepening prospective teachers’ mathematical knowledge. The authors hold high expectations for users’ previous mathematical knowledge and ability to engage in high level mathematics. A few of the features highlighted in the doctoral student seminar were the attention topics that may aid teachers in correcting or avoiding the formation of student misconceptions, the historical rationales and development of mathematical topics that help build connections within—and to topics outside—mathematics, and the importance of one’s mathematical definitions to their ability to extend ideas.

No one text can meet all the mathematical preparation needs of high school mathematics teachers. Topics that we deemed important for teachers that received little-to-no attention in this text are logic, truth tables, statistics, and probability. Additionally, we believe that teachers need greater knowledge of historical and cultural topics in mathematics than what is provided in the text; a history course would benefit teachers as well as their future students. Technology should be used, where possible, in teachers' preparations; teachers should be aware of uses, advantages, and limitations of various instructional technology. Integrating this type of knowledge with their deepening understanding of mathematics may provide prospective teachers with ideas of how to use technology to engage their students and facilitate student mathematical understanding. Finally, we felt that mathematics teacher educators could include pedagogical ideas, not a focus of the text, into classroom discussions, developing more of an integrative approach to mathematical preparation, rather than a purely mathematical approach as is the focus of the text.

Just as teachers need curricular knowledge, including an awareness of how to find information or alternative treatments of mathematics topics, teacher educators should also develop an arsenal of instructional materials. Decisions about the specific content to be addressed in a given course with a particular group of students should be based on the goals of each course, the instructor's specific goals, and the needs of the students. However, the UGA doctoral students who participated in this seminar on the mathematical preparation of high school teachers feel that the Usiskin et. al (2003) text could be a valuable addition to the libraries of mathematics teacher educators. The text, and others written with similar intentions, can serve as catalysts for conversations about the mathematical preparation of high school teachers: how it should be accomplished and what should be taught.

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