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The PAAPS Strategy for Teaching Mathematics Content
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Abstract

In the fall of 2005, I started teaching the mathematics content course Algebra and Geometry for Teachers. The majority of students in the course are pre-service middle school teachers. Instead of teaching the course by demonstrating rigorous proofs, I wanted to use teaching strategies that would build the students' content knowledge and connect to their roles as future mathematics teachers. I chose to make problem solving a focal process standard by having students problem-solve for a majority of classroom time. In addition, the students complete a major project entitled "Provide, Attempt, and Assess Problem Solving" or PAAPS. For PAAPS, each student provides a non-routine algebra, geometry, or analytic geometry problem to five of their classmates. Each student then attempts the five problems received, and returns the attempted problems to be assessed by the student who provided them. In this article, I share the results of using PAAPS as evidenced by student surveys and student work (problems chosen, problems worked, and problems assessed). Included in the surveys are the mathematical and pedagogical ideas that the students reportedly learned.

Background

In the fall of 2005, I started teaching the mathematics content course Algebra and Geometry for Teachers, which is comprised almost entirely of preservice middle school teachers. I did not want to teach the course by only demonstrating rigorous proofs in the classroom, so I decided to redesign the course. I wanted to use teaching strategies that would build the students' content knowledge and connect to their roles as future mathematics teachers. I decided to have class meetings comprised of both traditional class lecture sessions and problem-solving sessions. The class lectures included direct instruction, which sometimes included rigorous proofs. In the problem-solving sessions, students worked on non-routine algebra and geometry problems in groups of four to five.

As I reflected upon my experiences using a problem-solving approach with preservice middle school mathematics teachers in another course, I realized that I needed to make some changes. While the problem-solving activities were well-grounded in terms of both "real-world" contexts and mathematical concepts that were appropriate to middle school mathematics, many students struggled with their attitudes toward problem-solving. Several students did not see why problem-solving was useful. They decided it was better for teachers to "just show students what they need to know." While I knew that the students learned a lot of mathematical content in the problem-solving sessions, several did not appreciate its usefulness. As a teacher, I considered myself a facilitator, but some students did not consider this "teaching." Many students did not see the problem-solving sessions as relevant to their roles as future mathematics teachers. Rather than watching me in the role of teacher in a problem-solving classroom, the students needed to be in that role themselves. The students needed to experience activities that make these connections explicit.

Development and Description of the PAAPS Teaching Strategy

While the students learned from the problem-solving sessions, they would have a more positive learning experience and they would be more motivated to problem solve if they could see the usefulness of the experience for their role as future teachers. As I thought of potential strategies to accomplish this, I remembered a strategy that a former colleague, Karen Cohen at the University of Florida, uses with one of her courses. Karen has students "prepare a problem to stump their peers." The students share their problem with other students and assess the work of the other students. Karen's course, however, is different from mine in two ways that concerned me. First, her course is populated with graduate students, while mine is populated with junior undergraduates. Second, her course is a methods course, while my course is a mathematical content course. The potential benefits of using this teaching strategy, however, alleviated my concerns.

Cohen's teaching strategy focuses on the foundation of facilitating problem solving: (a) finding good non-routine problems for the students to work, (b) working non-routine problems to ensure they are appropriate for the students, and (c) assessing students' problem solving. I developed a three-part at-home project for students, entitled "Provide, Attempt, and Assess Problem Solving," or PAAPS, to reflect this focus. PAAPS is a three-week project, with one week given for each of the three parts of PAAPS. To start PAAPS, students are randomly placed in groups of six. In part one of PAAPS, each student independently finds a non-routine algebra, geometry, or analytic geometry problem for the other students in the group to work. The student who found the problem successfully works their chosen non-routine problem, using as many different strategies as possible. On the due date, each student provides his/her problem, without

solutions, on a sheet of paper to the other members of his/her group. Also, the student provides the professor with a copy of his (or her) chosen problem along with a worked-out solution(s).

In part two of PAAPS, each student works the five problems that were furnished by the other group members. On the due date, the students return the five attempted problems to the group member that provided the problem. In addition, each student gives the professor a photocopy of each attempted problem.

In part three of PAAPS, each student assesses his/her five classmates' work on the problem provided. When assessing the work, they must: (a) provide a numerical score on the paper, giving partial credit for the work; and (b) provide written feedback on the paper, including proper mathematical content and process language in the writing. In addition, the students write a cover sheet to turn in to the professor. On the cover sheet, they describe the: (a) technique used to assign a numerical score to the papers, (b) mathematical ideas learned from the project, and (c) teaching ideas learned from the project. On the due date, each student returns a copy of the assessed papers to their group member that worked the problem. In addition, he or she gives the professor a copy of the papers assessed with a cover sheet stapled on top.

When I assigned a grade to the students' work on the PAAPS project, I created a scoring rubric based on what I requested the students do on the project description handout. I scored them based on whether or not they completed what they were asked to do as well as their effort on the tasks. See Appendix A for a sample scoring rubric.

Analysis of Student Work: Provide, Attempt, Assess Problem Solving

I used the PAAPS teaching strategy during the fall of 2005, and devised techniques to document its use for the fall of 2006. During the fall of 2006, there were 19 students in the class, who provided 18 unique problems (one problem was duplicated). Of the 19 students in the class,

63% $\left(\frac{12}{19}\right)$ successfully worked their chosen problem, 21% $\left(\frac{4}{19}\right)$ attempted but did not

successfully work their chosen problem, and 16% $\left(\frac{3}{19}\right)$ turned in a copy of a solution from an external source (internet or text) and did not attempt to work their chosen problem themselves. Appendix B includes a selection of the problems chosen by students.

For part 2 of PAAPS (attempt problem solving), I only assess whether the students make a good effort on the five problems that were provided to them by their classmates. Eighty-nine percent $\left(\frac{17}{19}\right)$ of the students made a good effort on all (five) of the problems provided to them.

Of the remaining two students, one student did not attempt one of the problems, and the other student did not attempt four of the five problems.

There were several commonly used comments in the students' written feedback when they assessed the other students' problems. Virtually all of the students used comments such as "Great job," "You got it right," "This is a good start," and "Great strategy." Overall, the students did a very good job of promoting confidence with positive feedback. Other commonly used comments, such as "Start the problem by..." and "Show how you got your answers" gave direction on how to start the problem-solving process and how to properly communicate that process. Several students noted in their feedback that the problem-solver used a different strategy from their own, with comments such as "This is a good idea. I did not think of it."

Analyzing the quality of student feedback revealed 21% $\left(\frac{4}{19}\right)$ of the students wrote “good” to “excellent” feedback on the papers that they assessed. For example, Melanie’s feedback “You found the area of one rectangle, now find the dimensions” was categorized as excellent feedback. Max’s feedback “Find the five main ways to get 11 points, then use those ways to get other combinations. There are 19 combinations.” was categorized as good. Twenty-six percent $\left(\frac{5}{19}\right)$ of the students wrote “average” feedback comments. For example, Brandon did an excellent job of writing feedback on papers with strategies that were different from his, as he properly assessed Caleb’s solution, which used the Law of Cosines. However, for the incorrect solutions, his feedback was poor since he did not give feedback to help students toward a correct solution. In particular, for one student with an incorrect strategy, Brandon did not write any comments. For another student with an incorrect strategy, Brandon wrote “Great deduction!” Thirty-seven percent $\left(\frac{7}{19}\right)$ of the students wrote feedback that was “fair” or “poor.” Sixteen percent $\left(\frac{3}{19}\right)$ of the students only wrote feedback such as “good job” since all students worked their problems correctly. These students’ feedback was categorized as “none.”

Of particular note are the students whose feedback was “poor” $\left(\frac{6}{19} \approx 32\%\right)$. All three of the

students who copied their solutions (from the internet or a text) in Part One of PAAPS rated “poor” in the quality of their feedback. For example, in Part One, Carl turned in a copy that described the box as being inscribed in a circle of radius r with all 8 corners of the box touching the surface of the sphere. However, he left this information out when he provided the problem to his classmates to work, so there was not enough information for students to obtain a solution. In his feedback, he never realized that his stated problem was incomplete. He simply wrote the text solution that he copied, no matter what the problem-solvers wrote on their papers. Similarly, Jay simply wrote “wrong answer” and “no work” for his feedback, while Leigh wrote feedback that was centered on her copied solution, and did not build well on what her classmates wrote. For example, Caleb wrote in his solution to Leigh’s problem (See Appendix B for problems), $d = \text{outer edge} - \text{inner edge} = 2\pi(r + 5) - 2\pi r = 10\pi \text{ meters}$. Caleb then stopped working the problem. Leigh wrote the feedback $d = \frac{2\pi r}{\pi} = \frac{2\pi 5}{\pi} = 10 \text{ m/s}$, which was a formula in her π solution and did not connect well to Caleb’s work.

Lisa’s feedback was also rated “poor.” In Part One of PAAPS, her solution to the problem that she chose was incorrect since she based all of her work on the assumption that $\overline{AF} = \overline{FC} = 5$ ft. This gave her the incorrect height $\overline{EF} = 5.15$ ft. In Lisa’s assessment of six classmates’ work, two of the six, Holly and Ashley, had the correct height $\overline{EF} \approx 6.8$ ft. Holly used strategies similar to Lisa, first the Pythagorean theorem to find $\overline{AB}$ and $\overline{CD}$, then noted that triangles AEB and CED are similar to set up the ratio $\frac{\overline{AE}}{15 - \overline{AE}} = \frac{\overline{AB}}{\overline{CD}}$ and find $\overline{AE}$. Holly then used the ratio

for the similar right triangles $\frac{\overline{EF}}{\overline{CD}} = \frac{\overline{AE}}{15}$ to find the desired height $\overline{EF} \approx 6.8$ ft. Lisa wrote on

Holly's paper "your work is clear and your answer is right," so perhaps Lisa caught her own mistake from her previous solution of $\overline{EF} = 5.15$ ft. Ashley started the problem in the same way as Lisa, using the Pythagorean theorem to find $\overline{AB}$ and $\overline{CD}$. After this, Ashley used a different strategy. She treated point A as the origin on the coordinate system. She then found the equations of the lines $\overline{AD}$ and $\overline{BC}$ in slope-intercept form $y = mx + b$. Lastly, she set the two equations equal to determine their point of intersection, with the y-coordinate of this intersection giving her the height $\overline{EF}$. Lisa, however, did not see this as an acceptable strategy, writing the comment on Ashley's paper "AEF and ADC are similar triangles. See that $\frac{EF}{CD} = \frac{AE}{AD}$, continue with this and

$CD \quad AD$

you will get the answer."

Analysis of Cover Sheet Survey

When asked to identify the mathematical ideas that they learned from the PAAPS project, the major theme in student responses was that there are many different ways to solve a mathematics problem. In addition, some students focused on specific mathematical ideas that they learned, such as Ellen's statement, "I learned geometry. Within geometry, I became more familiar with topics such as area, volume, perimeter, 2-D and 3-D figures, and induction and deduction." Other students stated that they "refreshed their memory" on topics, such as the Law of Cosines. Some students focused on the process of doing mathematics. For example, Heather stated, "I learned many mathematical ideas from this assignment. One idea was how to change from one type of problem to another. Each problem was very different and required different ways of approaching it. It was very helpful to learn how to change mentally to look at different problems."

There were also several themes identified in the teaching ideas students reported learning from PAAPS. In particular, the students reported they learned: (a) to provide problems that include clear and concise directions; (b) to provide fun, challenging problems to children that are centered on a certain concept so they will be more likely to want to learn how to solve [problems] and understand [concepts]; (c) to ensure students see different ways to solve a problem; (d) teachers need to be knowledgeable of different strategies because students work problems in different ways; (e) assigning points (grades) can be difficult; (f) how to assess someone else's work and how to better understand their thinking; and (g) feedback to the student is important. In relation to feedback, one student wrote, "If they are never told what they did wrong and what they can do to improve, then they are not being given the proper chance to improve."

Conclusion

According to *The Mathematical Education of Teachers* (CBMS, 2001), preservice teachers need to: (a) acquire a deep understanding of mathematics, (b) learn to pose good mathematical questions, and (c) look at problems from different perspectives. A deep "mathematical knowledge for teaching...allows teachers to assess their students' work, recognizing sources of student errors and the students' understanding of the mathematics being taught" (CBMS, 2001, p. 13). According to *The Professional Standards for Teaching Mathematics* (NCTM, 1991), mathematics teachers' education should enhance knowledge of: (a) ways to reason and communicate mathematically, (b) multiple representations of mathematical concepts, and (c) means for assessing student understanding of mathematics. In addition, it should help one "develop a sense of self" as a teacher of mathematics (NCTM, 1991, p. 161). The PAAPS teaching strategy is designed to facilitate these aspects of the preservice teachers' preparation.

The PAAPS process allows students to find problems that interest them and work the problems furnished by their peers. In addition, they see their chosen problem worked by five other students, so most of the students are challenged to think about their chosen problem more deeply. These six problems provide a rich experience. Many students turn in excellent attempts at problem-solving both on their own problem and the problems of others (see the strategies of Holly and Ashley on Lisa's problem that is previously discussed). The students wrote numerous comments concerning teaching ideas learned. This demonstrates that PAAPS was successful at highlighting the relevance of problem solving to their roles as future teachers.

The PAAPS teaching strategy facilitates mathematical learning by promoting problem solving as a valid mathematical endeavor. In addition, students were able to refresh their memory on some mathematical ideas, and learn about other mathematical ideas. The PAAPS teaching strategy clearly demonstrates to students that there are many ways to solve problems, and exposes them to multiple representations.

The PAAPS process allows students to make mistakes and learn from them. Some students made a mistake when they worked their own chosen problem, but then found their mistake when they assessed the work of their classmates. Brian chose his problem "simply because it fell under the geometry category on the *Figure This* web site. When asked what teaching ideas he learned from PAAPS, Brian stated that he needed to be more careful in his problem selection.

In addition to making discoveries on their own, I critique each student's work. This provides the opportunity to further guide their thinking. Students receive comments such as, "Guide your students on what to do next," "Provide written feedback," and "Carefully consider solutions that are different from yours." For students who simply copy the solution of their chosen problem from a text or the internet, I write, "You need to work the problem yourself so that you can properly assess the work of others." I comment on how this affects assessment of their classmates.

After the students turned in their work for part three of PAAPS (assess problem solving), the project was concluded, other than receiving their grades for the project. In the future I would recommend three additions to PAAPS to enrich the mathematical content knowledge of the students. First, have the students meet with their group when they return the assessed problems. Ask each student to mathematically discuss and justify their solution to each problem and have each student justify the scores given for each problem. Second, share all of the problems that fit within the targeted category (algebra, geometry, analytic geometry) with the class. This will allow all students to work all of the problems provided by the class, thereby increasing the learning experience for them as well as providing a means to assess their mathematical understanding of the problems at a later date. Third, conduct the project more than once. In this way, one can assess improvement in students' ability to provide, attempt, and assess problem solving.

References

- Conference Board of the Mathematical Sciences. (2001). *The mathematical education of teachers*. Providence RI and Washington DC: American Mathematical Society and Mathematical Association of America.
- National Council of Teachers of Mathematics (NCTM). (1991). *Professional standards for teaching mathematics*. Reston, Va.: NCTM.

Appendix A Sample
Scoring Rubric

Part 2 - Attempt 5 Problems from 5 Classmates (20 points total)

Problem Solving Effort (10 points possible)

0, 1, or 2 points assigned for each of the 5 problems as follows:

- 0 The student did not attempt the problem or did not work the problem himself or herself.
- 1 The student attempted the problem (him/herself), but there is not a clear process.
- 2 The student attempted the problem (him/herself), and there is a clear process. If the problem is a multi-step problem, then the “clear process” may only be the first step of the multi-step problem. The strategy may or may not be a strategy that leads to a correct answer.

Assignment Requirements Met (10 points possible)

Assign the points below if the listed requirement was met. If it was not met, then assign 0 points.

- 4 Turned in two copies of each problem attempted.
- 4 Returned all of the worked problems to the group members who provided them.
- 2 Collected problem (chosen in Part 1) from each group member who worked it.

Appendix B
Problems Chosen by u
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1. Brandon: In the picture is an equilateral triangle inscribed in a circle. Given that the radius of the circle is 1, what is the length of a and b ?

Stated source: <http://mathproblems.info>

2. Melanie: Helix and Polygon both used the same number of identical concrete pieces to make their patios. The area of each patio is the same:

180 square meters. What are the dimensions of a single piece of concrete?

Stated source: www.figurethis.org

3. Leigh: If a circular track is 5 meters wide and it takes a horse, traveling at its fastest speed, π

more seconds to travel the outer edge than the inner edge, what is the horse's speed? Stated source: www.hawaii.edu/suremath/jhorseRace.html

4. Carl: Find the width of the box w .

Stated source: none

5. Lisa: Two walls are 10 ft. apart. Two ladders, one 15 ft. long and one 20 ft. long are placed at the bottom of the walls, leaning against the opposite walls. How far from the ground is the point of intersection? Stated source:

www.mathforum.org