

Virtual vs. Concrete Manipulatives in Mathematics Teacher Education: A Call for Research
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Abstract

Are virtual manipulatives as effective as concrete (hands-on) manipulatives to build conceptual understanding of number concepts and relationships in pre-service middle grades teachers? In the past, the use of concrete manipulatives in mathematics courses for Clayton State University's preservice middle grades teachers has proven to be a very effective way to build conceptual understanding of a variety of mathematical topics. This paper presents an argument for the need for research into the usefulness of virtual manipulatives for enhancing mathematics teacher education and their potential to supplement (or replace?) concrete manipulatives.

Virtual vs. Concrete Manipulatives in Mathematics Teacher Education: A Call for Research

Introduction

Since the inception of Clayton State University's (CSU) Middle Grades Teacher Education program in 1993, hands-on manipulatives have been used very successfully in the junior-level Number Concepts and Relationships (MATH 3010) course. In addition to a textbook, students were required to purchase a manipulative kit that included, among other things, Cuisenaire Rods, pattern blocks, fraction circles, and two-color counters. Students worked collaboratively to build conceptual understanding of the arithmetic of fractions and integers, using those manipulatives. That understanding was assessed by small group and whole class discussions and the use of performance tests.

Certain results occurred consistently, year after year, class after class. Invariably, students declared, "I wish I had been taught this way. This makes so much more sense." And "I always hated (*fill in different topics—fractions, negative integers, etc.*), but now I understand it." Some even admitted liking math for the first time! Those statements were usually made by the "less-mathematically-inclined" students. In fact, those students were consistently more adept at learning to use the manipulatives than the traditionally higher achievers; it was much harder for the latter to let go of the rules they had learned and to operate on the concrete level required for successful use of the manipulatives.

Because the junior-level mathematics courses in CSU's teacher education program combine content and pedagogy, students studied the NCTM *Standards* and reflected, both orally and in writing, on the importance of providing appropriate experiences and instructional materials to facilitate students' mathematics learning. Communication and reflection were emphasized throughout the course, contributing to the overall success of using manipulatives to build conceptual mathematics understanding.

The Problem

With the advent of the Georgia Performance Standards (GPS) reformation of the pre-K—12 curriculum, as well as the prevalence of websites providing free access to virtual manipulatives, the time is right for Georgia mathematics teacher educators to investigate the advantages and disadvantages of supplementing (or replacing?) concrete manipulatives with virtual ones. The GPS curriculum advocates the constructivist approach to mathematics learning and provides tasks to help teachers implement it. (<http://www.georgiastandards.org/math.aspx>) In addition, the new curriculum incorporates the TIMSS recommendation to teach fewer topics in more depth, thus allowing more time for the incorporation of manipulatives. (<http://nces.ed.gov/timss/results.asp>)

The ubiquitous use of various forms of technology by today's youth provides additional incentive to investigate the usefulness of virtual manipulatives. Might they be more motivating to students, especially those in middle and high school, than the more toy-like concrete manipulatives? On the other hand, should school funding be directed toward the more expensive technological resources, especially given the current economic slump? Considering the meaning of "conceptual understanding" and the often omitted component of bridging from the concrete to the abstract, is there a difference between concrete and virtual manipulatives in the ease of that transition? It is important for groups such as GAMTE to investigate these questions.

The teacher education classroom provides the perfect setting in which to examine the effectiveness of hands-on versus virtual manipulatives, while providing experiences in which pre-service teachers develop a deeper understanding not only of the mathematics but also of the resources themselves and their respective pedagogical impact. In other words, teacher educators have the opportunity to do research while facilitating not only the learning of mathematical concepts but also of pedagogy and pedagogical research methods.

Current Research

Conceptual Understanding

Because we are concerned with the enhancement of *conceptual understanding*, it is imperative that we define the term. Although definitions differ, we will adopt the statement from Hiebert and Lefevre (1996, pp. 3-4) as cited by Star (2005, p. 406), that conceptual knowledge is “knowledge that is rich in relationships... Relationships pervade the individual facts and propositions so that all pieces of information are linked to some network.” The relationships formed by the use of manipulatives incorporate visual, tactile, and kinesthetic experiences. Adding cooperative learning and reflective discussion further enhances the depth of understanding and the likelihood of retention. (Daniels et al., 1993; Garrity, 1998.)

Pedagogical Content Knowledge

Broadening our net to include a focus on pre-service teachers’ construction of conceptual understanding in a course that incorporates content and pedagogy, it is also important to understand the term “pedagogical content knowledge.” In his review of Shuhua An’s book, *The Middle Path in Math Instruction: Solutions for Improving Math Education* (2004), Jeremy Kilpatrick notes An’s perception of pedagogical content knowledge as including a mathematics teacher’s ability for “addressing and correcting students’ misconceptions” (2005, p. 256). Kilpatrick further notes An’s observation of the teachers in her study having limited success “bridging from manipulative materials to mathematical ideas” (p. 258). For me, the useful implications of An’s findings are (1) the importance for teachers to build their own conceptual understanding to enable them to identify and correct their students’ misconceptions, and (2) the need to incorporate “bridging construction” into the course.

Research Involving Concrete Manipulatives

Regarding the pedagogical impact of *concrete manipulatives*, there is a plethora of available information. Based on their own experiences, authors of the NCTM *Standards* from 1989 and 2000 recommended giving students “experiences in using a wide range of visual representations” (e.g., *concrete manipulatives*) to solve mathematics problems (2000, p. 284). Many researchers (e.g., Van de Walle, 1973; Grouwns, 1992; Vinson et al., 1997) have found a connection between the use of manipulatives and a decrease in students’ math anxiety levels. In their study of pre-service mathematics teachers, Vinson et al. reported that the use of manipulatives served a “two-fold” purpose:

“First, the concrete experiences aided in preservice teachers having a better understanding of the mathematical concepts and purposes for procedures. Secondly, the use of manipulatives assisted the preservice teachers in learning how to teach with more than just modeling a procedure on the chalkboard...” (p. 8).

Research has also shown the use of manipulatives in the mathematics classroom to be motivating. In her study of high geometry students, Garrity (1998) found that using manipulatives and cooperative groups motivated her students. She concluded

“In order to give meaning to math teaching, students are best served by learning concepts by actual manipulation of physical materials. Motivation is best accomplished when there is an active involvement with physical objects” (p. 21).

Garrity cites other researchers who also support the “manipulation of “physical objects” as a deterrent to “pseudo-learning” (Carin & Sund, 1975, p. 338) and who recommend the inclusion of “opportunities for reflection” to balance effective learning practices (Daniels et al., 1993, p. 9). In a study using concrete manipulatives in two 8th-grade pre-algebra classes, Hinzman (1997) reports that students’ mathematics performance was enhanced and their attitudes were significantly more positive than those of students from previous years. (These results were well founded despite the fact that Hinzman’s data analysis appears to be flawed.)

Research Involving Virtual Manipulatives

Because the advent of *virtual manipulatives* is relatively recent, the research regarding them is less prevalent than that of concrete manipulatives. Moyer et al. (2002) define a *virtual manipulative* as “an interactive, Web-based, visual representation of a dynamic object that provides opportunities for constructing mathematical knowledge.”(p. 185) In addition, they identify one’s interaction with virtual manipulatives as an example of the process of representing mathematics recommended by the NCTM *Standards*.

“Because it is advantageous for students to internalize their own representations of mathematics concepts, interacting with a dynamic tool during mathematics experiences may be much more powerful for internalizing those abstractions.” (p. 5)

Noting that “students learn in different ways,” Schackow (2006-2007) describes several activities for which mathematics teachers can use virtual manipulatives to teach fraction concepts to middle school students. She recommends the National Library of Virtual Manipulatives website (<http://nlvm.usu.edu/en/nav/vlibrary.html>) as one that contains many worthwhile activities. Schackow, however, has expanded the list of concepts for which certain NLVM manipulatives can be used. For example, she illustrates how to use NLVM’s virtual Color Chips, Geoboards, and Pattern Blocks to model computations with fractions. In addition, she lists several advantages of virtual manipulatives. They are

- * Available—“Teachers may be limited in the quantities and types of concrete manipulatives available to them.” (p. 10)
- * Time-saving—“Teachers may not have time to make their own manipulatives.” (p. 10)
- * Motivating—“(Middle school) students may find working on a computer with virtual manipulatives more desirable than using concrete manipulatives that they might view as childish.” (p. 10)

Schackow concludes,

“Using the virtual manipulative activities discussed in this article can help students deepen their conceptual understanding of fraction computations and avoid such struggles and frustrations...(and) may lead to student exploration and classroom discussion that will enable students to make sense of fraction computations.” (p. 10)

Several studies compared the use of concrete and virtual manipulatives to teach mathematics. Brown (2007) conducted an experiment to determine if students who used virtual manipulatives would out-perform students who used concrete manipulatives. The subjects in her study were 48 6th-grade students in an urban public school. Her results indicated that students who received instruction with concrete manipulatives out performed students who used virtual manipulatives, but that both types of manipulatives enhanced the learning environment. Brown’s results are suspect, however, since there were differences in academic ability of the two groups. In addition, the types of virtual manipulatives used (fraction bars) were different from the types of hands-on manipulatives used (pattern blocks). Both differences could have influenced the results of the study. It would be interesting to determine whether the results would have been different if the lower-ability group had used the concrete manipulatives and the higher-ability group the virtual.

In her report, Brown cites other studies that investigated virtual manipulatives. Enderson’s study (1997) showed that using virtual manipulatives to explore volume of a box expanded students views of the mathematics involved (p. 10). Reimer and Moyer (2005) reported increased success teaching fractions with virtual manipulatives over paper-and-pencil instruction. Brown quotes Reimer and Moyer as indicating that virtual manipulatives’ ability to “connect dynamic images with abstract symbols” (pp. 10-11) is an advantage over concrete manipulatives. Brown (2007) adds the advantage that virtual manipulatives take less time to manipulate than concrete ones. Studies by Olkun (2003) and Dorward & Heal (1999) indicate that virtual manipulatives are as engaging and provide equally as strong an effect on mathematical understanding as do concrete manipulatives.

Implications for Teacher Education

A survey conducted in Australia by Howard et al. (1997) to determine the use of manipulatives among primary and secondary mathematics teachers raised questions regarding the issue of whether teachers’ acceptance of the usefulness of manipulatives has “a solid conceptual base” (p.9). The researchers also indicated that

“There is a clearly expressed need...for further training in the use of manipulatives in mathematics teaching”, a fact that “has implications for both pre-service teacher education programs and teacher development sessions” (p. 9).

Surely the same need exists in the United States. Certainly, the NCATE/NCTM Program Standards (2003) for Middle Level Mathematics Teachers and the GPS Process Standards also support investigation of the role of virtual manipulatives in the teaching and learning of mathematics.

Perhaps the most compelling charge regarding the role of virtual manipulatives in mathematics education comes from two Turkish educators, Durmus & Karakirik (2006). They define a *concrete experience in a mathematics context*

“not by its physical or real-world characteristics but rather by how meaningful (are the) connections it could make with other mathematical ideas and

situations...Hence, it is very important to encourage learners to reflect on actions they make in order to be able to perceive mathematical processes as objects.”(p. 3)

They further advocate,

“Every student should be given an opportunity to play with manipulatives. Just a demonstration by a teacher is not sufficient to realize their full potential and not in line with the theoretical rationale of their usage since they are meaningful to the extent they involve interactive activities.”(p. 4)

Citing Suydam & Higgins (1976), Durmus & Karakirik concur that

“Manipulative materials should be used in conjunction with exploratory and inductive approaches” (p. 4)

and conclude that

“Most manipulatives in mathematics simply implement the ‘learning *with*’ model’ approach. However, educators also need to consider the possibility of designing manipulatives employing the ‘learning *to*’ model’ approach since full potential of any technological device could be achieved through its usage as a communication tool to model the concepts and relations at hand.” (p. 6) (Italics and bolding added.)

Clearly, more research is needed to determine the appropriate role of virtual manipulatives, in both mathematics teacher preparation and pre-K-12 mathematics instruction. The educational impact of the ever-burgeoning accessibility and advancement of technology demands the attention of mathematics educators. With the Georgia Performance Standards as our springboard, Georgia teacher educators are in a position to influence the preparation of internationally competitive mathematics students. Will the use of virtual manipulatives contribute to our progress? It’s an important question that begs an answer.

Final Comments: Plans for Future Research

Faculty at Clayton State University plan an investigation of the effectiveness of both concrete and virtual manipulatives will take place in CSU’s Number Concepts and Relationships class fall 2008. Approximately 25 juniors in CSU’s Middle Level Teacher Education program will participate in the investigations. They will have access to concrete manipulatives, including fraction circles, pattern blocks, decimal mods, and two-color counters. They will also have laptop computers and Internet connections, giving them access to virtual manipulatives websites and applets. Lessons will address Georgia Performance Standards number concepts for grades 6 – 8, including the properties and arithmetic of rational numbers, integers, and prime and composite numbers. Students will work in groups of 2 or 3, using concrete manipulatives to build conceptual understanding of number concepts. Each of these experiences will be followed by similar activities, replacing the concrete manipulatives with virtual manipulatives. Communication and reflection will be incorporated throughout and feedback will be gathered from the students.

Assessment of the effectiveness of the various manipulatives will include observation, written feedback, performance tests, and tasks similar to those advocated by Ball et al. (2008). Students will also study the research on the effectiveness of manipulatives. Statistical methods

will be used to investigate possible correlations among students' characteristics and the effectiveness of the different types of manipulatives, etc. Details of this work will be presented at the GAMTE conference in October.

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