13th IMHRC Proceedings (Cincinnati, Ohio. USA – 2014)

Progress in Material Handling Research

2014

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Recommended Citation

Sari, Zaki; Ghomri, Latefa; Ekren, Banu Y.; and Lerher, Tone, "Experimental Validation of Travel Time Models for Shuttle-based Automated Storage and Retrieval System" (2014). *13th IMHRC Proceedings (Cincinnati, Ohio. USA – 2014)*. 24.
https://digitalcommons.georgiasouthern.edu/pmhr_2014/24

XXII. EXPERIMENTAL VALIDATION OF TRAVEL TIME MODELS FOR SHUTTLE-BASED AUTOMATED STORAGE AND RETRIEVAL SYSTEM

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Abstract

In this paper, we aim to validate travel time models for single and dual command cycle displacements of lifts and shuttles in a shuttle-based automated storage and retrieval system (SBS/RS) by using experimental computer simulation. The models under consideration take into account acceleration and deceleration delays. We use ARENA 12 software for the simulation modeling. By simulation, we emulate the real functioning of the system. Therefore, we assume that the results from the ARENA simulation are equivalent to the onsite experimentation. Simulation results are very close to those obtained by analytical travel time models. This shows the high precision of these models to predict operations of SBS/RS.These models can be used at design or operation phases to calculate throughput of the system, to compare between different topologies of SBS/RS or with other types of AS/RS to help decision makers to choose among different alternatives of automated storage systems.

1 Introduction

Automated Storage and Retrieval Systems (AS/RSs) are widely used in modern distribution centers (DCs) because they can provide continuously fast, accurate and efficient service. An AS/RS consists of storage racks, storage and retrieval (S/R) devices where products are stored and retrieved automatically. Typically there are two types of AS/RSs: traditional crane-based AS/RS (CBAS/RS) and autonomous vehicle (AV) based AS/RS (AVS/RS). This study deals with a kind of AVS/RS. An AVS/RS is composed of AVs, storage racks and lifts that are mounted along the periphery of the reserve storage area to provide vertical movement for loads [1]-[2]. AVs function as S/R device in an AVS/RS.

An aisle and tier-captive AVS/RS is also known as shuttle-based automated storage retrieval system (SBS/RS) which is highly efficient AVS/RS designed for expedited handling of cartons, totes, and trays in high transaction environments. This system is also known as one-level shuttle based AS/RS (OL-SBS/RS) by the practitioners [4]-[7]. SBS/RS is relatively a new technology in automated storage and retrieval system and is ideal in high transaction environments in which a Mini-Load AS/RS Crane may not be able to keep pace with the transaction rate needed over a given number of storage locations. Loads are stored and removed from the shelves by the SBS/RS at high speed, and the shuttle's load handling equipment is designed for short handover times (http://www.dematic.com/multishuttle).

Typically, an SBS/RS is comprised of multiple tiers of storage with dedicated shuttles for each level. In this paper, we aim to validate travel time models for single and dual command cycle displacements of lift and shuttle in an OL-SBS/RS via simulation.

1 Literature Review

There is very limited number of studies on SBS/RS in the literature. We provide all of them in this section.

Marchet et al., [5] present main design trade-offs for SBS/RS using simulation. They complete their study for several warehouse design scenarios for OL-SBS/RS. They present several performance measures from the system – utilizations of lifts and shuttles, average flow time, waiting times as well as cost – for the pre-defined rack designs.

Recently, Lerher [6] and Lerher et al., [7] have studied SBS/RS by considering energy efficiency concept in the system design. The proposed models provide several designs and their performances. Designs are considered in terms of velocity profiles of elevator and shuttle-carriers while performances are considered as energy (electricity) consumption, amount of CO_2 oscillation and throughput capacity. These studies provide significant contribution to environment friendly automated warehouse planning by taking into consideration the energy efficiency in the system design.

Another related study is completed by Carlo and Vis [4]. They study a type of SBS/RS developed by Vanderlande Industries where two non-passing lifting systems are mounted along the rack. In this paper, they focus on the scheduling problem of lifts where two (piece-wise linear) functions are introduced to evaluate candidate solutions. They develop an integrated look-ahead heuristic for the solution procedure.

Ekren and Heragu[1] have studied simulation based performance evaluation of AVS/RS. They study near optimum rack configuration design under pre-defined scenarios of number of vehicles and lifts in the system using simulation-based regression analysis. Ekren et al. [2] also implemented design of experiment on AVS/RS to identify factors affecting performance of this system.

Later, Ekren and Heragu[3] studied simulation based performance analysis of AVS/RS for several rack design and number of vehicles and lifts in the system. They came up with when the number aisles increases in the system, the average flow time decreases. This is valid when the policy of there is one lift per zone is considered.

2 System Description

The studied SBS/RSis composed of elevator with lifting tables that are attached on a mast, shuttle carriers, buffer positions and storage racks (see Figure 1& 2).

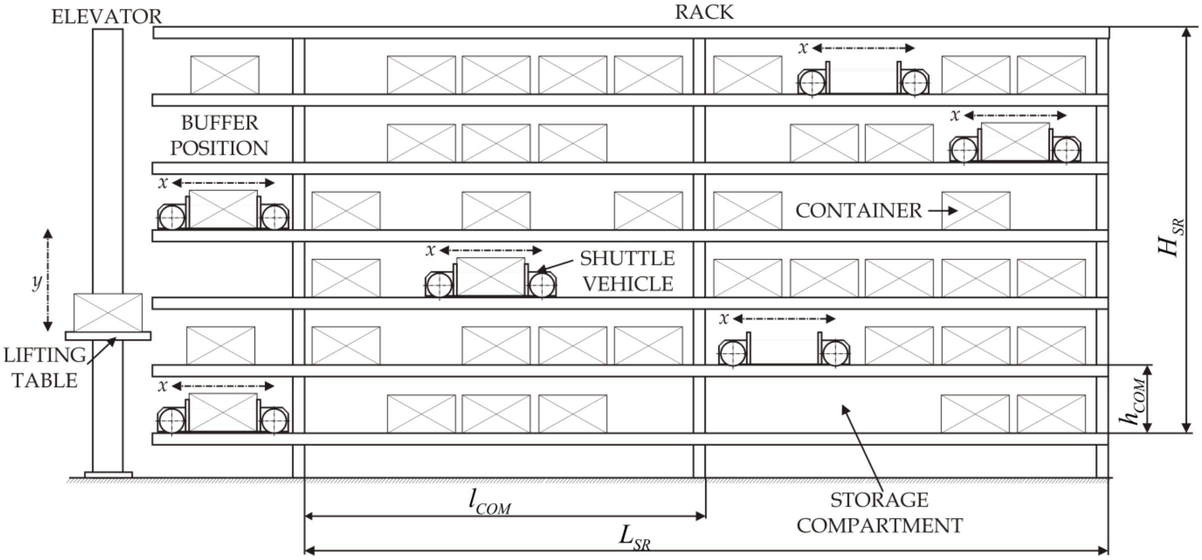


Figure 1: Shuttle based AS/RS (side view)

The elevator through the lifting table moves containers(usually called tote) up and down to the prescribed level in the storage rack. To be able to process more work two independent lifting tables canbe attached to the elevator one of which is on the right side and the other one is on the left side of the mast. In this case, the performance of the elevator can be doubled. An elevator can move lifting tables up to v_y = 1.5 m/s theoretically. Elevators are usually bottleneck in this system so that they determine the performance of the whole system.

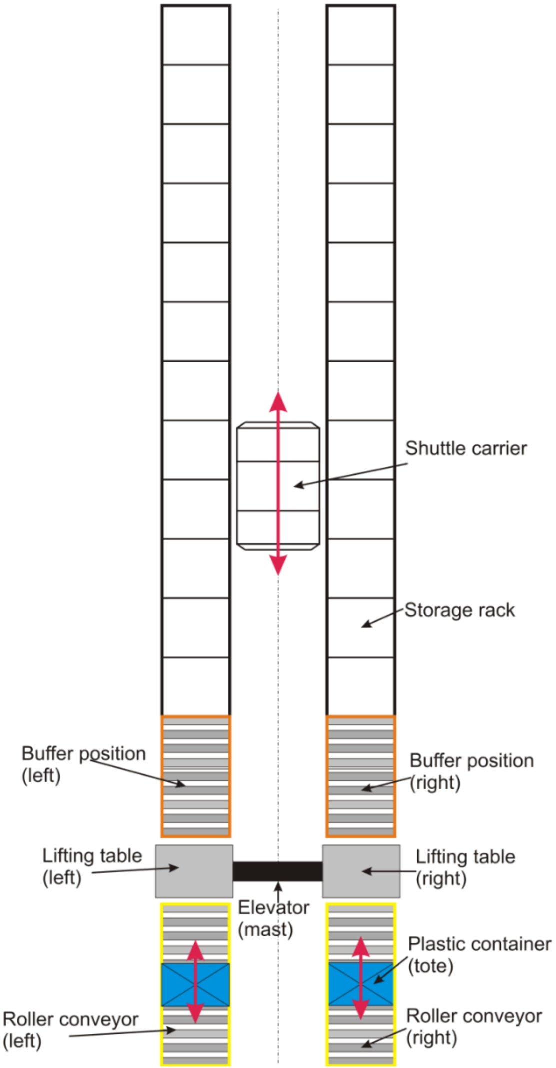


Figure 2: Shuttle based AS/RS (top view)

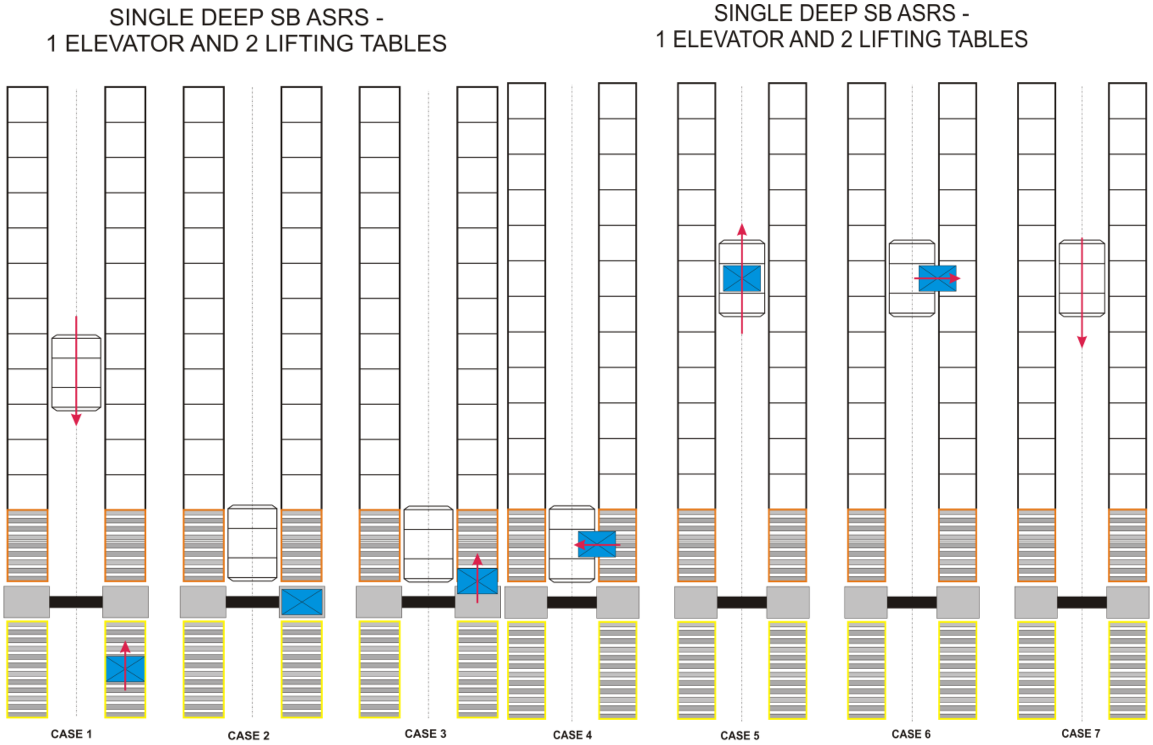
A shuttle carrier is a small autonomous vehicle with four wheels that transports containers from the buffer position to the storage locations in the storage racks. It is equipped with telescopic attachment for manipulating containers. The maximal weight of a container should not exceed 50 kg/shuttle carrier and its dimensions should range between 150 x 200 x 80 mmand 600 x 400 x 250 mm. A shuttle carrier can travel up to $v_x = 4$ m/s theoretically. Usually there is a single shuttle carrier in each level of a storage rack that is aisle-captive.This type of system is also called AVS/RS with a tier-captive configuration. This assumption can be released if we use a special shuttle elevator at the back of the storage rack, for moving shuttle carriers up and down to the prescribed level in the storage rack. In that case, the system becomes an AVS/RS with a tier-to-tier configuration.

Buffer positionsare placed at each level of the storage rack and are used for buffering the containers from the elevator (i.e. in storage process) and shuttle carriers (i.e., in retrieval process).

The storage rack is composed of storage compartments that can receive loads. By consideringone storage compartment’s length and height, it is not hard to obtain thewholestorage area’s length and height. Here, the storage racks can be considered as asingle or double deep storage rack.

The SBS/RS can perform either in single or dual command cycle operations. A single command cycle encompasses the following activity. For a storage process:1- The container (tote) arrives the I/O point by using either the right or left rolling conveyor (e.g. inFigure 3 (a), the right rolling conveyor is used). 2- Lift travels to the I/O point to pick up the tote (see Figure 3(b)). 3- Lift travels to the destination tier which is selected randomly.4- Lift discharges the tote onto the buffer position. 5- Because we consider the dwell point policy where the shuttle carrier always waits near the buffer position (see Figure 3 (c)), the shuttle carrier charges the tote from the buffer position after this tote is discharged from the lift (see Figure 3 (d)). 6- The shuttle carrier travels to the destination storage location and discharges the tote (seeFigure 3 (e) and (f)).

Recall that according to the prescribed dwell point policy, the shuttle carrier returnsnear the buffer position and waits for a task to process (seeFigure 3 (g)).



(a) (b) (c) (d) (e) (f) (g)

Figure 3: single cycle description fora SBS/RS

For dual command cycle of the lift, the following operations will be performed: 1- The tote arrives to the I/O pointvia the rolling conveyor. 2- Lifttravels to the first level, where it charges the tote on it. 3- The lifttravelsto the destination tier. 4-Liftdischarges the toteonto the buffer position. 5- Lift travels to the retrieval destination tier to pick up the tote to complete a retrieval process. 6-Lift charges the tote and travel to the first level where it discharges the tote onto the rolling conveyor.

With the same logic, the shuttle carrier can also perform a dual command cycle in the same manner: 1-Shuttle charges the totefrom the buffer position. 2- It travels to the destination storage location and discharges the tote. 3- It travels to the retrieval storage locationand charges the tote. 4- It travels to the buffer position and discharges the tote.

As mentioned previously, we aim to validate, through computer simulation, single and dual command cycle travel time models for lift and shuttle of SBS/RS.These models have been developed by Sari *et al.*[8]by taking into account acceleration and deceleration delays as well as loading and unloading delays. These latter delays are considered to be constant. In the next section,weshowthe modeling approach of single and dual command travel time models of lift and shuttle for the studied SBS/RS as developed in Sari *et al.*[8].

3 Previous Work: Travel Time Models Development

In a previous work [8], we developed closed-form travel time models for single and dual command cycle displacements of lift and shuttle in a SBS/RS by taking into account their acceleration and deceleration delays. For each cycle time scenario (single or dual command), four models were developed based ondifferent acceleration and deceleration levels: very high, high, medium and low.Inthe first level, we assumed that acceleration is so high that the lift or shuttle reaches its maximum speed instantaneously. In the second level,we considered that the maximum speed is reached in less than one half unitary displacements (from one storage segment to the following one). Inthe third level, weassumed medium level acceleration where the maximum speed is reached in more than one half unitary displacements. Finally, in the last level we supposed that the maximum speed is never reached because the acceleration is verylow.

From the system description (see Section 2) we can notice that the lift and the shuttle have similar operations. These operations can be summarized as follows: displacement of a carrier on a linear path servicing a known number of equidistant positions (levels or storage compartments). Consequently our model will be based on this statement and can be used for lift, shuttle or any other carrier satisfying the previous hypothesis. In addition to that the model will be based on the following assumptions:

- An aisle is composed of storage racks on both sides. The input/output (I/O $_{\text{aisle } i}$) location of the warehouse is located at the first level (see Figure 2).
- At each level of the storage rack there are two buffer areas where lifting tables drop off the loads. In each level *i*, there is a single shuttle carrier – (aisle-captive shuttle carrier (see Figure 2)).

- The elevator can manipulate the two lifting tables independently. Namely, the lift capacity is doubled in an aisle. A lifting table can carry one tote at a time.
- Drive characteristics of the elevator (lifting tables), shuttles as well as the height of the storage racks are known priori.
- Randomized storage policy is used which means that any storage location is equally likely to be selected.
- The acceleration and deceleration values are supposed to be equaland known priori.
- Levels and storage compartments are supposed to have equal heights and lengths, respectively.

We consider the following notations throughout the study:

a	Acceleration/deceleration.
$E(SC)$	Single command cycle expectedtravel time.
$E(DC)$	Dual command cycle expected travel time.
$E(TB)$	Expected travel time from any position (level or storage compartment) to another one.
i, j	i^{th} or j^{th} position (i^{th} or j^{th} level for lift, or i^{th} or j^{th} storage compartment for shuttle).
m	Number of unitary displacement requiredto reach the maximum velocity (after the acceleration phase) and to decrease thespeed until stop (deceleration phase).
n	Total number of positions (number of levels for lift or number of storage compartments for shuttle).
s	unitary displacement (for lift: from one level to the next one, for shuttle from one storage compartment to the next one).
t_l	Charge/discharge time.
$T(x)$	Time in dependence of distance.
v	Maximum velocity (or constant cruise velocity that is reached after the acceleration phase).

As stated before, there is no restriction on acceleration/deceleration profile except that they are known priori and equal to each other. Four cases are considered as in below:

Case 1: acceleration/deceleration value is very high that the maximum speed can be reached in very short displacement which is assumed to bemuch smaller than one unitary displacement. In that case, acceleration delay can be neglected and we can consider that the lift/shuttletravels at theirmaximum speed - v.

Case 2: acceleration/deceleration value is high so thatthe maximum speed can be reached in less than s/2 (which means that the maximum speed is always reached even when the lift/shuttlemakes a unitary displacement).

Case 3: acceleration/deceleration value is medium so that the maximum speed can be reached in more than s/2 (which means that the maximum speed may not be reached for some displacements).

Case 4: acceleration/deceleration value is very low sothatthe maximum speed cannot be reached inany displacement.

For each case above, single and dual command cycles are considered, separately.

Single command cycle: We consider a single command cycle as a round trip travel from the I/O point to the desired level (for the lift) or storage compartment (for the shuttle).

Dual command Cycle: A dual command cycle consists of storage and retrieval in the same travel. It consists of one single command cycle E(SC), plus a travel time between the storage and the retrieval points E(TB): E(DC)=E(SC)+E(TB).

To determine the expected travel time in the system where all positions have equal probability to be selected, a trivial way is to calculate the travel time for all positions and then divide it by the number of positions. This method has the advantage to be exact, but the drawback to be seldom effective because its summations are usually not computable. However, in our case we were able to use effectively this method. Most of the summations were computable, and the others were approximated with a very small error.

3.1 Travel profile of the Carriers

As stated before, we consider a carrier that travels along a linear path with a known number of equidistant positions that can be visited. The carrier has pre-known acceleration/deceleration and so pre-known maximum speed (cruise speed). Lerher [7] derived travel profile for this kind of system. He considered two type of travel profile: type I state that the displacement is short and the carrier cannot reach it maximum speed, while type II considers that the displacement is long enough so that the carrier can reach it maximum speed and travel at that speed (cruise speed) during a certain time. Figure 4 presents a schematic representation of these two travel profiles.This travel profile is adapted to our problem as follows:

$$T(x) = \begin{cases} 2.\sqrt{\frac{x}{a}} if x \leq \frac{v^2}{2a} \\ \frac{x}{v} + \frac{v}{a} if x \geq \frac{v^2}{2a} \end{cases}$$

x being any displacement. As in our system all positions are equidistant with a distance *s* between two adjacent positions, *x* can be written as *x=i.s*

$$T(i.s) = \begin{cases} 2.\sqrt{\frac{i.s}{a}} if i.s \leq \frac{v^2}{2a} & (a) \\ \frac{i.s}{v} + \frac{v}{a} if i.s \geq \frac{v^2}{2a} & (b) \end{cases} \qquad (1)$$

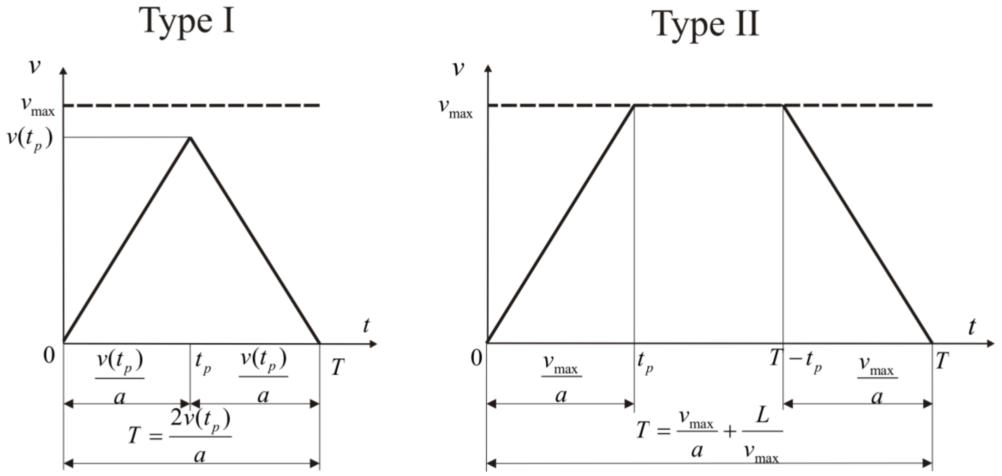


Figure4. Velocity – time relationship [7]

If we consider (1) with respect to the four above-mentioned cases, we can state that: For case 1, and since acceleration/deceleration delay is neglected, so (1) does not stand and is replaced by *T(i.s)=i.s/v.* For case 2, and since acceleration/deceleration occur in less than one unitary displacement, which means that only (1.b) is defined since maximum speed is reached for all displacements.For case 3, for short displacements (*i* small)(1.a) is defined whereas for long displacements (*i* large) (1.b) is defined. Finally, for case 4, only (1.a) is defined since maximum speed is never reached. Obviously, we can say that only Cases 2-3 are practical, whereas Cases 1-4 have little chance to exist in practice.

3.2 Case 1: Very High Acceleration/Deceleration

In this case *E(SC), E(TB)* and *E(DC)* can be easily derived as follows: recall that single command cycle represents a round trip from I/O point to any position which means 2.*i.s/v.* If we sum this for *n* positions and divide by *n* we obtain the expected travel time for single command cycle as (2).

$$E(SC) = \frac{2}{n}.\left(\sum_{i=1}^{n}\frac{s.i}{v}\right) = \left(\frac{s(n+1)}{v}\right) \tag{2}$$

Travel time between any two positions can be calculated by defining two positions *j* and *i*, then calculating travel times for each of these couples, and dividing them by the square of the total number of the positions. Since the system is symmetric (travel time from *i* to *j* is equal to travel time from *j* to *i*), one can consider only one-way travel from *i* to *j* positionand then divide by half ofsquare of the total number of position. This travel time is called "time-in-between" [9]and can be calculated by (3):

$$E(TB) = \frac{2}{n^2 - n} \sum_{j=1}^{n} \left(\sum_{i=j+1}^{n} \frac{s.(i-j)}{v} \right) = \frac{s(n+1)}{3v} \tag{3}$$

Finally, the dual command cycle expected travel time is obtained by the sum of single command cycle expected travel time plus “time-in-between”: *E(DC)=E(SC)+E(TB)* as in (4):

$$E(DC) = \frac{4s(n+1)}{3v} \tag{4}$$

3.3 Case 2: High Acceleration/Deceleration

Recall that in this case, acceleration/deceleration occurs in less than one unitary displacement, which means that only (1.b) happens since maximum speed is reached for all displacements. Consequently, single command expected cycle time can be calculated by (5):

$$E(SC) = \frac{2}{n} . \sum_{i=1}^{n} \left(\frac{s.i}{v} + \frac{v}{a} \right) = \frac{s(n+1)}{v} + \frac{2v}{a} \tag{5}$$

Similarly,as in Case 1, *E (TB)* is calculated by (6):

$$E(TB) = \frac{2}{n^2 - n} \sum_{j=1}^{n} \left(\sum_{i=j+1}^{n} \left(\frac{s.(i-j)}{v} + \frac{v}{a} \right) \right) = \frac{s(n+1)}{3v} + \frac{v}{a} \tag{6}$$

Similarly, as in Case 1, *E (DC)* is calculated by (7):

$$E(DC) = \frac{4s(n+1)}{3v} + \frac{3v}{a} \tag{7}$$

3.4 Case 3: Medium Acceleration/Deceleration

In this case, remember that we are considering that acceleration/deceleration occurs in more than one unitary displacement.It means that for short displacements (*i* small) (1.a) is validwhereas for long displacements (*i* large) (1.b) is valid. Consequently, single command expected cycle time should be calculated by considering both (1.a) and (1.b). For this, we assume a number *m*denoting the number of unitary displacements required to reach maximum speed in acceleration phase and then decelerate until stopping. In other words, we consider it as the number of unitary displacements which limit the domain of definition of (1.a) and (1.b). As a result, *m* can be written asaninteger part of the limit between (1.a) and (1.b) as in (8):

$$m = \left\lfloor \frac{v^2}{2sa} \right\rfloor \tag{8}$$

Obviously, m is smaller than the total number of positions n, since we consider that in Case 3 the carrier can reach up to its maximum speed.

So, the single command expected cycle time will be the average of two summations, the first one considers that the carrier cannot reach the maximum speed for a certain number (m) of positions, and therefore uses equation (1.a) and the second summation considers that the carrier will always reach to its maximum speed for positions larger than m.So, we obtain (9):

$$E(SC) = \frac{2}{n}\left(\sum_{i=1}^{m} 2\sqrt{\frac{s.i}{a}} + \sum_{i=m+1}^{n} \frac{s.i}{v} + \frac{v}{a}\right) \quad with \quad m = \left\lfloor \frac{v^2}{2sa} \right\rfloor \tag{9}$$

(9) is composed of two summations that should be treated differently. The second one is calculable, and can be computed by (10):

$$\frac{2}{n}\left(\sum_{i=m+1}^{n} \frac{s.i}{v} + \frac{v}{a}\right) = \frac{s(n+1)}{v} + \frac{2v}{a} - \frac{v}{2na} - \frac{5v^3}{4sna^2} \tag{10}$$

However, the first summation is not calculableand should be approximated. Using asymptotic expansion, an expression with a good fitting of the first summation isdetermined by (11).

$$\frac{2}{n}\left(\sum_{i=1}^{m} 2\sqrt{\frac{s.i}{a}}\right) = \frac{4}{n}\sqrt{\frac{s}{a}}\left(\sum_{i=1}^{m} \sqrt{i}\right) \approx \frac{4}{n}\sqrt{\frac{s}{a}}\left(\frac{2m\sqrt{m}}{3} + \frac{\sqrt{m}}{2} + \frac{1}{24\sqrt{m}} - \frac{5}{24}\right) \tag{11}$$

Replacing m by its value and after the related calculations (12) is obtained:

$$\frac{2}{n}\left(\sum_{i=1}^{m} 2\sqrt{\frac{s.i}{a}}\right) \approx \frac{2\sqrt{2}v^3}{3sa^2n} + \frac{\sqrt{2}v}{an} + \frac{\sqrt{2}s}{6nv} - \frac{5\sqrt{s}}{6n\sqrt{a}} \tag{12}$$

Finally, the single command expected cycle time is calculated by (13):

$$\begin{aligned} E(SC) &= \frac{2}{n}\left(\sum_{i=1}^{m} 2\sqrt{\frac{s.i}{a}} + \sum_{i=m+1}^{n} \frac{s.i}{v} + \frac{v}{a}\right) \\ &= \frac{s(n+1)}{v} + \frac{2v}{a} - \frac{0.3072v^3}{sa^2n} + \frac{\sqrt{2}s}{6nv} + \frac{5.4853v - 5\sqrt{sa}}{6an} \end{aligned} \tag{13}$$

For determining*E(TB)*, depending on the distance between departure and destination positions, either (1.a) or (1.b) can be used. Hence, time-in-between is calculated by (14):

$$E(TB) = \frac{2}{n^2 - n}\sum_{j=1}^{n}\left(\sum_{i=j+1}^{m+j} 2.\sqrt{\frac{s(i-j)}{a}} + \sum_{i=m+j+1}^{n}\left(\frac{s(i-j)}{v} + \frac{v}{a}\right)\right) with m = \left\lfloor \frac{v^2}{2sa} \right\rfloor \tag{14}$$

Similarly to (9), (14) is composed of one computable summation and one approximated summation. For the latter, asymptotic expansion is used. After the required calculations,finally we obtain (15):

$$E(TB)=\frac{s(n+1)}{3v}-\frac{0.3072v^3}{(n-1)sa^2}+\frac{\sqrt{2}s}{6v(n-1)}+\frac{v(n-0.08579)-\frac{5}{6}\sqrt{sa}}{a(n-1)} \tag{15}$$

Recall that dual command cycle expected travel time is obtained by (16):

$$E(DC)=E(SC)+E(TB) \tag{16}$$

So, (16) becomes as in (17):

$$\begin{aligned}E(DC)=&\frac{4s(n+1)}{3v}+\frac{2v}{a}-\frac{0.3072v^3(2n-1)}{n(n-1)sa^2}+\frac{\sqrt{2}s(2n-1)}{6vn(n-1)}+\frac{nv}{a(n-1)}\\&+\frac{v(0.8284n-0.9142)-5\sqrt{sa}(2n-1)}{an(n-1)}\end{aligned} \tag{17}$$

3.5 Case 4: Low Acceleration/Deceleration

In Case 4, we consider that due to low acceleration/deceleration, the carrier will never reach up to its maximum speed. In this case, only 1.a) is validall over the system speed profile. Consequently, single command cycle expected travel time can be evaluated as in follow. As mentioned previously, $\sum_{i=1}^{n}\sqrt{i}$ cannot be calculated exactly and should be approximated (see case 3). After the approximation, calculations and simplifications, *E(SC)* is found as in (18):

$$E(SC)=\frac{2}{n}\sum_{i=1}^{n}2\sqrt{\frac{s.i}{a}}=\frac{\sqrt{s}}{6n\sqrt{na}}\left(16n^2+12n+1-5\sqrt{n}\right) \tag{18}$$

Similarly, time-in-between can be calculated by (19):

$$E(TB)=\frac{2}{n^2-n}\sum_{j=1}^{n}\left(\sum_{i=j+1}^{n}2.\sqrt{\frac{s(i-j)}{a}}\right) \tag{19}$$

The double summation can be simplified to two single summations as in (20):

$$\begin{aligned}E(TB)&=\frac{4\sqrt{\frac{s}{a}}}{n^2-n}\sum_{j=1}^{n}\sum_{i=j+1}^{n}\sqrt{i-j}=\frac{4\sqrt{\frac{s}{a}}}{n^2-n}\sum_{i=1}^{n}(n-i)\sqrt{i}\\&=\frac{4\sqrt{\frac{s}{a}}}{n^2-n}\left(\left(n\sum_{i=1}^{n}\sqrt{i}\right)-\left(\sum_{i=1}^{n}i\sqrt{i}\right)\right)\end{aligned} \tag{20}$$

Using asymptotic expansion for both summations, approximated expressions are derived. After the calculations and simplifications, an approximated expression with a good fitting of *E(TB)* is determined by (21):

$$E(TB) = \frac{\sqrt{s}}{30n\sqrt{na}(n-1)}\left(32n^3 - 25n\sqrt{n} + 5n - 15\right) \tag{21}$$

Similarly to previous cases, dual command cycle expected travel time is the sum of the single command cycle expected travel time plus “time-in-between”: *E(DC)=E(SC)+E(TB)*and it becomes as in (22):

$$E(DC) = \frac{\sqrt{s}}{30n\sqrt{na}(n-1)}\left(112n^3 - 140n^2 + 60n - 10 - 25\sqrt{n}\right) \tag{22}$$

3.5 Generic Models

In this section, we provide the generic models for travel times ofSBS/RS. The modelsare based on the assumption that a carrier with priori known speed profiles (constant acceleration/deceleration and limited maximum speed) that is used to serve a set of equidistant positions. All positions have equal probability to be served for storage or retrieval operations. The carrier can perform a single or a dual command cycle operation. In single cycle, it either stores or retrieves items. In dual command cycle it performs both storage and retrieval processes in the same journey. Depending on the speed profile of the carrier, four different models have been developed. The first one concerns the situation where the acceleration/deceleration is not taken into account. This case is of little use because it has been proved that it gives a bad approximation of the real system operation. The other models concern the importance of acceleration/deceleration with respect to travel distance. In the following a summary of these models is providedfor both single and dual command expected travel times as in (23) and (24):

$$E(SC) = \begin{cases} \dfrac{s(n+1)}{v} + \dfrac{2v}{a} & if \quad \dfrac{v^2}{2sa} \leq 1 \\ \dfrac{s(n+1)}{v} + \dfrac{2v}{a} - \dfrac{0.3072v^3}{sa^2n} + \dfrac{\sqrt{2}s}{6nv} + \dfrac{5.4853v - 5\sqrt{sa}}{6an} & if \quad 1 < \dfrac{v^2}{2sa} \leq n \\ \dfrac{\sqrt{s}}{6n\sqrt{na}}\left(16n^2 + 12n + 1 - 5\sqrt{n}\right) & if \quad n < \dfrac{v^2}{2sa} \end{cases} \tag{23}$$

$$E(DC) = \begin{cases} \dfrac{4s(n+1)}{3v} + \dfrac{3v}{a} & if \quad \dfrac{v^2}{2sa} \leq 1 \\ \dfrac{4s(n+1)}{3v} + \dfrac{2v}{a} - \dfrac{0.3072v^3(2n-1)}{n(n-1)sa^2} + \dfrac{\sqrt{2}s(2n-1)}{6vn(n-1)} + \dfrac{nv}{a(n-1)} + \dfrac{v(0.8284n - 0.9142) - 5\sqrt{sa}(2n-1)}{an(n-1)} & if \quad 1 < \dfrac{v^2}{2sa} \leq n \\ \dfrac{\sqrt{s}}{30n\sqrt{na}(n-1)}\left(112n^3 - 140n^2 + 60n - 10 - 25\sqrt{n}\right) & if \quad n < \dfrac{v^2}{2sa} \end{cases} \tag{24}$$

These models haveadvantages to take into account the speed profile of the carrier. These models can be used for throughput calculations, system optimization or as a basis for comparison of control algorithms or heuristics.

4 Experimental Validation

To perform experimental validation, the best method is to realize real onsite experiment on a real system. Unfortunately this is not always possible because of many reasons like: lack of time, inexistence of the system, expensiveness of the experiment etc. In our case we developed simulation models of the system to validate the analytical model results. These simulation models have been developed for cases 2-4, where acceleration&deceleration are effectively taken into account, are considered for experimental validation. Actually, case 1 is trivial and has no interest in real systems. Single and dual cycle travel time models are considered. For each model, two types of simulation are considered: exact and random travel simulations

4.1 Single Cycle

4.1.1 Exact Travel Simulation

In this simulation model, the carrier will visit each storage segment once. To do so, n items are created (where n is equal to the number of storage segments). Then, each item is transported to one storage segment and the travel time is measured. Finally the expected travel time is calculated as the average of travel times of each item.Algorithm 1 is used for the simulation model:

Algorithm 1: Exact travel simulation for single cycle travel time evaluation;

```
Assign a numerical value to n and s;
Declare n+1 stations, indexed from 0 to n, station 0 being pickup/delivery station
and dwell point of the carrier;
Declare a transporter TRANS and assign it a numerical value for maximum speed
v and acceleration deceleration rate a;
for i varying from 1 to n do:
    CréerCreate an entity ITEM(i) at station 0
    Transport ITEM(i) via TRANS from station 0 to station i and measure travel
    time T(i)  (T(i)=round trip from station 0 to i and back)
End do
Calculate average travel time as: $E(SC)_e = \frac{\sum_{i=1}^{n} T(i)}{n}$
```

4.1.2 RandomTravel Simulation

In this simulation model, we proceed differently. Actually, in order to determine an average travel time of the carrier, we construct an SBS/RS with n storage segment. Then we create 200000 entities. Each entity is transported to a storage segment selected randomly with a uniform distribution function varying from 1 to n. The expected travel time is calculated as the average of the 200000 travels. The simulation is repeated with 5 replications.The following algorithm is used to for the simulation model:

Algorithm 2: Random travel simulation for single cycle travel time evaluation;

```
Assign a numerical value to n and s;
Declare n+1 stations, indexed from 0 to n, station 0 being pickup/delivery station
and dwell point of the carrier;
Declare a transporter TRANS and assign it a numerical value for maximum speed v
and acceleration deceleration rate a;
Repeat 5 times☹number of replications)
    Repeat 200000 times (number of items)
        Assign to i a random value from uniform distribution function (1, n)
        Create an entity ITEM(j) at station 0
        Transport ITEM(i) via TRANS from station 0 to station i and measure
        travel time T(j)  (T(j)=round trip from station 0 to i and back)
    End repeat;
    Calculate average travel time for each replication as: Tk $=\frac{\sum_{j=200000}^{n} T(j)}{n}10$
End repeat;
Calculate average single cycle time as: $E(SC)_r = \frac{\sum_{j=5}^{n} Tk}{n}$ ;
```

4.1.3 Some Results& Discussion

The computer simulation has been performed on a large number of configurations of the system, taking into account all its parameters. Only a sample of these simulation results is presented in this paper because of lack of space. Table 1 presents analytical and simulation results for different speed and acceleration/deceleration scenarios. These scenarios will cover all the domain of functioning of the system. The total number of levels is fixed to 10 whereas the unitary displacement is fixed to 4. Column 1 provides speed profile case corresponding to models presented above (*Cases 2-4*). Column 2 gives the lift acceleration and deceleration whereas column 3 presents speed of the lift. Column 4 provides single cycle travel time using analytical models. Columns 5 and 7 show the same travel time but determined by the simulation models and finally columns 6 and 8 present the percent error between the analytical and simulation travel time models. We can notice from Table 1 that the percent error between analytical and simulation models is very small. For case 2 this error is often zero. This can be easily explained by the fact that no approximation was used to determine the analytical travel time model for case 2 which means that this model is exact. For case 3 and 4 some approximation were used but as seen in Table 1, they do not affect the quality of case 3 and 4 analytical model results.

Table 1: Analytical vs. simulation results for single cycle travel times of SBS/RS.

case	*a*	*v*	*E(SC) Analytical*	*E(SC)$_e$ exact travel simulation*	*% error for exact travel simulation*	*E(SC)$_r$ random travel simulation*	*% error for random travel simulation*
2	1	1	46.0000	46.0000	0.0000%	46.0086	-0.0187%
2	2	1	45.0000	45.0000	0.0000%	44.9626	0.0831%
2	4	1	44.5000	44.5000	0.0000%	44.5111	-0.0249%
2	8	1	44.2500	44.2500	0.0000%	44.2611	-0.0251%
2	16	1	44.1250	44.1250	0.0000%	44.1361	-0.0252%
2	14	10	5.8286	5.8195	0.1556%	5.8204	0.1402%
2	20	10	5.4000	5.3989	0.0204%	5.3961	0.0722%
3	1	10	17.9743	17.9746	-0.0019%	17.9768	-0.0141%
3	2	10	12.8287	12.7100	0.9252%	12.7049	0.9649%
3	4	10	9.0746	9.0527	0.2419%	9.0485	0.2882%
3	10	10	6.3713	6.3708	0.0086%	6.3679	0.0541%
3	2	15	12.7097	12.7100	-0.0022%	12.7049	0.0380%
3	4	15	9.0791	8.9873	1.0113%	8.9837	1.0509%
3	10	15	5.7648	5.7539	0.1899%	5.7512	0.2367%
3	20	15	4.4061	4.4052	0.0208%	4.4027	0.0776%
4	1	15	17.9743	17.9746	-0.0019%	17.9763	-0.0113%
4	1	20	17.9743	17.9746	-0.0019%	17.9763	-0.0113%
4	2	20	12.7097	12.7100	-0.0022%	12.7112	-0.0116%
4	4	20	8.9871	8.9873	-0.0019%	8.9837	0.0382%

4.2 Dual Cycle

Similarly to single cycle travel time evaluation, we performed two types of discrete event simulation of the SBS/RS using ARENA 12 software: exact travel simulation and random travel simulation.

4.2.1 Exact Travel Simulation

As stated for single cycle simulation, the aim of this simulation is to find the average travel time of all possible travels in a dual cycle operation. For a SBS/RS of *n* storage segments, we create $n(n-1)$ items. Each item will induce a displacement of the S/R machine. Each displacement is performed from P/D station to storage segment *i,* then from storage segment *i*to storage segment *j*, and finally from storage segment *j* to P/D station. *i* and *j* indices are varied from 1 to *n* with$i \neq j$. Travel times of the S/R machine are measured for each displacement and finally an average time is calculated. The following algorithm illustrates how exact travel simulation is performed.

Algorithm 3: Exact travel simulation for dual cycle travel time evaluation;

```
Assign a numerical value to $n$ and $s$;
Declare $n$+1 stations, indexed from 0 to n, station 0 being pickup/delivery station
Declare a transporter TRANS and assign it a numerical value for maximum speed $v$ and acceleration deceleration rate $a$;
for $i$varying from 1 to $n$ do:
    for $j$ varying from 1 to $n$ do:
        if$i \neq j$do:
            Créer Create an entity ITEM($i,j$) at station 0;
            Transport ITEM($i,j$) via TRANS from station 0 to station i and measure travel time $T_{ij1}$;
            Transport ITEM($i,j$) via TRANS from station i to station j and measure travel time $T_{ij2}$;
            Transport ITEM($i,j$)  via TRANS from station j to station 0 and measure travel time $T_{ij3}$;
            Calculatetravel time$T_{ij} = T_{ij1} + T_{ij2} + T_{ij3}$;
        End if:
    End do;
End do;
Calculate average travel time as: $\overline{E(DC)_e} = \frac{\underset{(i\neq j)}{\sum_{i=1}^{n}\sum_{j=1}^{n} T_{ij}}}{n(n-1)}$;
```

4.2.2 Random Travel Simulation

For random travel simulation, dual cycle travel time is measured as follows: a SBS/RS with n storage segment is modeled, than a simulation is performed with 5 replications. At warm-up, the system is filled at a load rate of 50%. Then at steady state two millions of dual cycles are realized and their travel times are measured, then an average dual cycle travel time is calculated. The following algorithm illustrates how the random travel simulation of dual cycle time is performed.

Algorithm 4: random travel simulation for dual cycle time

```
Assign a numerical value to $n$ and $s$;
Declare $n$+1 stations, indexed from 0 to n, station 0 being pickup/delivery station
Declare a transporter TRANS and assign it a numerical value for maximum speed $v$ and acceleration deceleration rate $a$;
Repeat $n$/2 times:
Assign to $i$ a random value from uniform distribution function (1, $n$)
```

```
        Create an entity ITEM(i) at station 0
        Transport ITEM(i) via TRANS from station 0 to station i
End Repeat
Repeat 5 times (number of replications)
    Repeat 200000 times (number of items)
        Assign to $i$ a random value from uniform distribution function (1, $n$)
        among empty storage segments
        Assign to $j$ a random value from uniform distribution function (1, $n$)
        among filled storage segments
        Create an entity ITEM($k$) at station 0
            Transport ITEM($k$) via TRANS from station 0 to station $i$ and
            measure travel time $\boldsymbol{T_{k1}}$;
            Transport ITEM($k$)  via TRANS from station $i$ to station $j$ and
            measure travel time $\boldsymbol{T_{k2}}$;
            Transport ITEM($k$) via TRANS from station j to station 0 and
            measure travel time $\boldsymbol{T_{k3}}$;
            Calculate travel time $\boldsymbol{T_k = T_{k1} + T_{k2} + T_{k3}}$ ;
    End repeat;
    Calculate average travel time for one replication: $T_{rep} = \frac{\sum_{k=1}^{2000000} T_k}{2000000}$;
End repeat;
Calculate average travel time as: $\overline{\mathrm{E(DC)_r}} = \frac{\sum_{rep=1}^{5} T_{rep}}{5}$
```

4.2.3 Some Results& Discussion

As for single command cycle, computer simulation has been performed for dual command cycle on a large number of configurations. Only a sample of these simulation results is presented in this paper because of lack of space. Table 2 presents analytical and simulation results for different speed and acceleration/deceleration scenarios that cover all the domain of functioning of the system. The total number of levels is fixed to 10 whereas the unitary displacement is fixed to 4. Column 1 provides speed profile case corresponding to models presented above (*Cases 2-4*). Column 2 gives the lift acceleration and deceleration whereas column 3 presents speed of the lift. Column 4 provides single cycle travel time using analytical models. Columns 5 and 7 show the same travel time but determined by the simulation models and finally columns 6 and 8 present the percent error between the analytical and simulation travel time models. It can be noticed from Table 2 that the percent error between analytical and simulation models is very small for case 2 but increases to reach almost 10% for case 3 and 4. For case 2 therelative small error can be easily explained by the fact that no approximation was used to determine the analytical travel time model, which means that this model is exact. For case 3 and 4 some approximations were used that led to a significant error.

Table 2: Analytical vs. simulation results for dual cycle travel times of SBS/RS.

case	*a*	*v*	*E(DC) Analytical*	*$E(DC)_e$ exact travel simulation*	*% error for exact travel simulation*	*$E(DC)_r$ random travel simulation*	*% error for random travel simulation*
2	1	1	61,6667	61,6667	0,0001%	61,6686	0,0031%
2	2	1	60,1667	60,1667	0,0001%	60,1561	-0,0176%
2	4	1	59,4167	59,4167	0,0001%	59,4233	0,0112%
2	8	1	59,0417	59,0417	0,0001%	59,0483	0,0112%
2	16	1	58,8542	58,8542	0,0001%	58,8608	0,0113%
2	4	5	15,4833	15,4633	-0,1294%	15,4643	-0,1229%
2	8	5	13,6083	13,6083	-0,0002%	13,6097	0,0100%
2	16	5	12,6708	12,6708	-0,0003%	12,6694	-0,0113%
2	14	10	8,0095	7,9914	-0,2263%	7,9919	-0,2200%
2	20	10	7,3667	7,3644	-0,0308%	7,3649	-0,0240%
3	4	10	11,8002	12,7023	7,6450%	12,7014	7,6374%
3	8	10	8,8781	9,5033	7,0421%	9,5038	7,0478%
3	12	10	7,8254	8,3351	6,5136%	8,3356	6,5200%
3	4	15	11,4226	12,6322	10,5899%	12,6327	10,5942%
3	8	15	8,3099	8,9555	7,7696%	8,9555	7,7696%
3	16	15	6,1763	6,6186	7,1615%	6,6181	7,1534%
3	10	20	7,3646	7,9893	8,4821%	7,9893	8,4821%
3	20	20	5,3400	5,7383	7,4597%	5,7379	7,4522%
4	1	10	23,0755	25,2644	9,4859%	25,2657	9,4916%
4	1	15	23,0755	25,2644	9,4859%	25,2644	9,4859%
4	2	15	16,3168	17,8647	9,4864%	17,8654	9,4907%
4	1	20	23,0755	25,2644	9,4859%	25,2654	9,4903%
4	2	20	16,3168	17,8647	9,4864%	17,8654	9,4907%
4	4	20	11,5377	12,6322	9,4859%	12,6329	9,4920%

4 Conclusion

In this study, we develop simulation models that are used to validate single and dual command cycle expected travel timesfor SBS/RS. These models are developed bytaking into account the speed profiles of lift and shuttles. Acceleration and deceleration phases of the carriers are also considered and travel time models are developed based on different acceleration/deceleration values. Depending on the pre-defined scenarios, the maximum speed of the carrier can be reached instantaneously, in a short distance less than half unitary displacement, in more than one half unitary displacements or it can never be reached. For each speed profile, single and dual command cycle expected travel times are derived. To validate these models, two simulation protocols are developed that do not use the same work methodology. The first one called exact simulation is based on the visit of all storage segments whereas the second one called random simulation works like a real system operates. The error between simulation and analytical models is found to be very small except for dual command cycle in Cases 3 and 4.

As a future extension of this work, developed travel times models can be used to determine the optimal dimensions of the system in order to minimize the expected travel times.

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