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Andria Disney
Georgia Southern University, adisney@georgiasouthern.edu

Heidi Eisenreich
Georgia Southern University, heisenreich@georgiasouthern.edu

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This paper seeks to describe the importance of using the context of a candy shop and how focusing on discourse can deepen place value understanding in the base ten number system. Using the language of pieces (ones), rolls (tens), and boxes (hundreds) helps situate place value in a familiar context for K-2 students. Best practice in mathematics instruction is also addressed, including examining the progression of learning for place value concepts, using effective tools to support place value learning, and using explanation and justification to help students deepen their understanding of place value. The authors focus not only on content and pedagogy, but also show how the content connects to the Mathematical Practices and ELA Speaking and Listening standards. The purpose of this paper is to give current teachers something to think about as they plan lessons on place value in their K-2 classrooms. The authors also want to give teachers a reason why discourse can help both them and their students as they progress through content.

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Deepening Place Value Understanding in K-2

Place value is an important concept that crosses multiple grade levels in elementary mathematics, which is why it is critical to build a strong foundation for making sense of place value in primary grades. Discourse plays a key role in helping young children develop a deep understanding of place value concepts. Discourse should be perceived as the process students go through to make sense of mathematics, not as the tool students use to talk about mathematics (Hiebert & Wearne, 1993; Imm & Stylianou, 2011). This is a shift in the role of the student and the role of the teacher, because in the past teachers controlled much of the class conversation (Falle, 2004). Making a shift towards a more student-centered classroom focused on discourse may not be easy in classrooms that are more teacher-centered, where teachers are the only ones expected to give explanations and justifications. Students tend to look at the teacher and expect teachers to give these explanations instead of relying on themselves or their peers (Yackel & Cobb, 1996). However, encouraging students to dig deeper into their solutions and clarify their explanations through oral and written communication can lead students to be more autonomous in the classroom (Kamii, 1985). Thus, place value is the mathematical concept through which discourse will facilitate sense- making among young children.

Explanation and Justification

Communication plays a vital role in mathematics learning, both in terms of developing conceptual understanding among students and helping teachers to develop a deeper understanding of student thinking (Kosko, Rougee, & Herbst, 2014). The National Council for Teachers of Mathematics (NCTM) has long advocated for students to engage in discourse when learning mathematics (NCTM, 1989, 1991, 2000). These expectations for communication position children across the K-12 grade span as active participants in the mathematics classroom where they engage in explanation, justification, questioning, and sense making. In fact, “interacting with classmates helps children construct knowledge, learn other ways to think about ideas, and clarify their own thinking” (NCTM, 1989, p. 26). Studies indicate students should actively construct new information through classroom activities and discussions (Fosnot & Perry, 1996, Nathan, Eilam, & Kim, 2007), and students should ask questions from their peers to dictate the direction of these discussions (Bennett, 2013; Hiebert & Wearne, 1993; Imm & Stylianou, 2011). When students explain and justify different strategies to solve problems and share those strategies with their peers, students have a deeper understanding of the problem and are able to make connections between different strategies, both of which leads to a richer discussion (Nathan et al., 2007) and a deeper understanding of the content.

Currently, a continued focus on discourse as an integral part of mathematics teaching is reflected in both the Common Core State Standards for Mathematics (CCSSM) and NCTM’s *Principles to Actions* (2014). The CCSSM includes the Standards for Mathematical Practice, which focus on the process of doing mathematics and mathematical habits of mind that must be developed in students (NGA Center & CCSSO, 2010b). The need to communicate and engage in discourse is essential to student learning as identified in the Mathematical Practices, including when students “construct viable arguments and critique the reasoning of others” and when students

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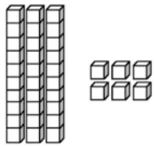
"attend to precision" as part of sharing their explanations and justifications (NGA Center & CCSSO, 2010b, pp. 6–7). Additionally, NCTM identified eight Mathematics Teaching Practices in *Principles to Actions* that reflect research-based best practices that will ensure deep mathematics learning (2014). One of these practices is focused on discourse: "Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments" (NCTM, 2014, p. 29). To facilitate student learning, teachers must purposefully plan opportunities for students to engage in discourse and the Standards for Mathematical Practice as they explain and justify their understanding of mathematics concepts.

Discourse can encompass both oral and written communication. In primary grades, there is a strong emphasis to build literacy skills across the subject areas. Because young children are at various stages in developing their reading and writing skills, the mathematics classroom is a natural place to emphasize the skills of speaking and listening. The Common Core State Standards for English Language Arts (ELA) have specific standards for Speaking and Listening, which include engaging in discussions with both peers and adults in small and whole group settings (NGA Center & CCSSO, 2010a). Many of these standards in the K-2 grade span are applicable to the mathematics classroom, including engaging in conversations, building on the responses of others, asking questions for clarification, adding drawings or visual representations to provide detail, and expressing ideas clearly ((NGA Center & CCSSO, 2010a, p. 23). As part of creating and sharing explanations and justifications during mathematics instruction, it is necessary for students to share their thinking using words, numbers, objects, and/or drawings, compare their strategies to the strategies of others, and ask questions to clarify meaning. These Speaking and Listening standards link to many of the Standards for Mathematical Practice, including "making sense of problems," "constructing viable arguments," and "attending to precision" (NGA Center & CCSSO, 2010b, pp. 6–7). Thus, the nature of mathematics learning with its emphasis on communication supports both mathematics and literacy development in young children.

Place Value

Developing a strong foundation with place value is a key learning goal in the primary grades. When students first learn about the idea of place value, they make sense of regrouping. This understanding is vital as they move towards number operations and beyond. Because of the mathematical significance of this concept, place value is a critical area in both first and second grades (NGA Center & CCSSO, 2010b). The critical areas in the CCSSM are essential learnings for a given grade level that should be taught for depth and, thus, should have a significant amount of instructional time devoted to them. Place value understanding is scaffolded across the primary grades: (a) In kindergarten, students understand teen numbers as ten ones and some more ones; (b) In first grade, students understand that 10 ones is the same as 1 ten; and (c) In second grade, students extend place value patterns to understand that 10 tens is the same as 1 hundred (NGA Center & CCSSO, 2010b, pp. 12, 14, & 19).

In addition to the instructional progression of place value reflected in the K-2 standards, students move through a defined learning progression as they come to understand place value. As students work within the base ten number system, they initially count by ones, then they count by groups and singles, and finally they count by tens and ones as illustrated in Figure 1 (Van de Walle, Lovin, Karp, & Bay-Williams, 2014). Children will progress through these stages at different paces, but they will all need multiple and varied experiences to construct these relationships among the place values.



One, two, three, four, five, six, seven…thirty-four, thirty-five, thirty six **Counting by ones**	*One, two, three groups of 10* *One, two, three, four, five, six singles* **Counting by groups and singles**	*Ten, twenty, thirty, thirty-one, thirty-two, thirty-three, thirty-four, thirty-five, thirty-six* **Counting by tens and ones**

Figure 1. Progression of counting within base ten number system

Another element that will support student learning is the choice of mathematical tools or materials that teachers select. It is imperative that initial physical models for place value must be proportional (Van de Walle, et al., 2014). It is best to begin with groupable models where students can construct tens using single items, such as placing counters into ten frames, connecting individual snap cubes into chains of ten, or bundling Popsicle sticks with rubber bands (Van de Walle, et al., 2014). Not only do children need experience with bundling 10 ones into 1 ten, they also need to unbundle 1 ten into 10 ones (Dougherty, Flores, Louis, & Sophian, 2010). It for this reason that groupable models are so important for supporting the development of place value concepts. Too often we begin with or transition to pregrouped models, like Base Ten blocks, before students have a solid foundation of constructing and deconstructing tens and hundreds. Van de Walle et al. (2014) noted, "A significant disadvantage of the pregrouped physical models is the potential for children to use them without reflecting on the ten-to-one relationships" (p. 181). Therefore, students need ample time and multiple experiences with groupable models to construct meaning of our base ten number system.

Not only do students need opportunities to use these hands-on groupable models for sense-making, they also need opportunities to reason about and discuss the action of grouping and ungrouping 10 ones or 10 tens. To promote discourse about place value concepts, teachers could ask questions like: How are 10 ones and 1 ten alike? How are they different? What happened when you put 5 ones and 6 ones together? Why did you make a ten? What does it mean to *make a ten*? Why did you decide to unbundle that ten? All of these questions seek to probe student thinking and help make the mathematics of place value, the ten-to- one relationship, visible. Thus, use of appropriate mathematical tools support primary students' engagement in discourse as a means for making sense of place value concepts.

Using the Candy Shop in K-2 Classrooms

In order to develop mathematical reasoning in young students, it is vital to present content in a familiar and understandable context, which applies to the concept of place value. Interestingly, when young children begin exploring our base ten number system, grouping by tens may not be natural for children given that they often group in smaller amounts like twos or fives (Dougherty, et al., 2010). Moreover, because children begin counting by ones, as a teacher "you cannot arbitrarily impose grouping by ten on children" (Van de Walle, et al., 2014, p. 182). Rather, you must provide some context that requires students to make sense of numbers being grouped by tens. This is where we can use the context of a candy shop to help students have a reason to group by tens based on the mathematical situation (Dixon, Nolan, Adams, Brooks, & Howse, 2016; Gregg & Yackel, 2002; Whitenack, Knipping, Novinger, & Underwood, 2001).

Candy is a context that many students can relate to. The following activity was taken from *Making Sense of Mathematics for Teaching Grades K-2* (Dixon et al., 2016). In this activity, students are given the following information: Each snap cube represents one piece of candy. We can combine 10 pieces of candy to make 1 roll. We can combine 10 rolls to make 1 box. The context of a candy shop corresponds to our base ten number system in terms of ones (pieces), tens (rolls), and hundreds (boxes). The language we use with our students (boxes, rolls, pieces) helps put place value into context so students can make sense of what each place value means in terms of packaging the candy, both in terms of physically grouping the ones and tens and explaining what these actions mean in the candy shop. Once they have an understanding of these candy shop terms and what they mean when making sense of how to package an amount, we can move towards the language of ones, tens, and hundreds. Based on the K-2 mathematics standards, packaging 10 pieces into 1 roll would be appropriate for first grade whereas packaging 10 rolls into 1 box would be appropriate for second grade (NGA Center & CCSSO, 2010b, pp. 14, & 19).

Giving students the background of the ten-to-one relationship between boxes and rolls or rolls and pieces allows teachers to then ask the following, "How many pieces are in a box? and "How do you know?" Students might make sense of this task by counting by ones or tens, but ultimately they should arrive at the same conclusion, 100 pieces are in a box, and be able to justify how they arrived at that conclusion. Giving students a set amount, for example 143 snap cubes displayed as 1 box, 4 rolls, and 3 pieces, and asking students to count how many pieces are at their table would be another task teachers can ask students to complete. To facilitate deeper understanding of place value concepts, teachers can then ask students explain how they counted the amount of pieces and justify their strategy. Identifying possible student misconceptions, such as counting a roll as 1 piece instead of 10 pieces, could also support understanding during class discussions where teachers ask students to think about the meaning of the rolls or boxes in relation to place value and discuss justifications as to why or why not this makes sense in their small groups and then again as a whole class. This context of a candy shop connects to the Mathematical Practice "make use of structure" as students are making sense of the ten-to-one relationship in this context (NGA Center & CCSSO, 2010b, p. 8). As students have continued experiences with this context and create explanations and justifications, they are "express[ing]regularity in repeated reasoning" (NGA Center & CCSSO, 2010b, p. 8). Inherent in both of these Mathematical Practices is the need for students to engage in discourse by expressing their thinking and reasoning verbally and/or in writing.

Discussion

Using the context of the candy shop incorporates many elements of mathematics education best practice for K-2 learners. First and foremost, it provides a familiar and relatable context that requires students to group items into tens and/or hundreds. Because young children will not naturally group by tens, the candy shop context provides a purpose for such grouping (Dixon et al., 2016; Dougherty, et al., 2010; Gregg & Yackel, 2002; Van de Walle, et al., 2014; Whitenack et al., 2001). Additionally, in the candy shop, context brings meaning to the language of place value. When young learners first hear the words *ones, tens,* or *hundreds*, they may not have an understanding of what those words really mean as they relate to the mathematics of place value. To help give these words meaning and to support students in using the beginnings of place value language in their discourse, the students explore and explain what ones (pieces), tens (rolls), and hundreds (boxes) are in the candy shop. Teachers can use realia to further support meaning for students by bringing in Mentos, Starbursts, Life Savers, or other stacked candies to allow students to see what a piece, roll, or box looks like in real life. Then by giving students snap cubes to represent the pieces of candy, they have an opportunity to make sense of how to group by physically packaging (snapping) 10 pieces (ones) into 1 roll (ten) or unpackaging 1 roll (ten) into 10 pieces (ones) and how to describe the mathematics of packaging or unpackaging through discussions. This same idea can be extended to packaging 10 rolls (tens) into 1 box (hundred) or unpackaging 1 box (hundred) into 10 rolls (tens). Thus, the context of the candy shop provides meaning for the actions and language of bundling and unbundling in our place value system.

The candy shop activity also provides many opportunities to support and develop mathematical communication among young learners. First, the candy shop activity helps students connect contextual language (piece, roll, box) to more formalized vocabulary (ones, tens, hundreds). The use of accurate mathematical vocabulary is part of "attend[ing] to precision," one of the Standards for Mathematical Practice (NGA Center & CCSSO, 2010b, p. 7). Teachers, however, must facilitate these language connections by pairing the language of the candy shop with the language of place value. Additionally, the candy shop activity provides students with opportunities to engage in explanation and justification. Throughout the activity, the teacher should be challenging students to justify how they know 1 roll is the same as 10 pieces or 10 rolls is the same as 1 box. Likewise, the teacher should be facilitating partner or small group talk among the students where they have to explain how they packaged various amounts of candy and their classmates must agree or disagree with their solution approach and justify why they agree or disagree. Engaging in such discourse via explanation and justification reflects another Standard for Mathematical Practice, where students are "construct[ing] viable arguments and critiqu[ing] the reasoning of others" (NGA Center & CCSSO, 2010b, p. 6). Additionally, students are using the ELA Speaking and Listening skills while engaging in conversations about packaging the candy with their partners and in small and whole group settings ((NGA Center & CCSSO, 2010a, p. 23)

Finally, the candy shop activity supports the progression of place value learning

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in K-2 students. The context of the activity gives students a purpose in grouping ones (pieces) into tens (rolls), which moves them beyond counting by ones only (Van de Walle, et al., 2014). Therefore, the candy shop activity is a means by which teachers can challenge students who are only counting by ones to consider a more efficient way of counting by using groups of ten. By allowing students the opportunity to understand our base ten number system in a way that makes sense to them, the focus is on grouping and renaming values. For example, we might start with 14 individual pieces, then move towards describing that same amount as a roll and some pieces, and finally arrive at a more formalized conception of 1 ten and 4 ones. Although this progression through place value can be difficult for students to grasp, pairing the base ten language with the language of the candy shop gives students a context in which they are familiar, to move through this progression. Additionally, the candy shop activity utilizes snap cubes, which are groupable models for place value. Such groupable models support students in making sense of the ten-to-one relationship through physically bundling and unbundling (Dougherty, et al., 2010; Van de Walle, et al., 2014). The use of groupable models is essential for developing a deep understanding of place value concepts in K-2 learners. Therefore, the candy shop activity is developmentally appropriate, and it is a rich task that allows students to make sense of the complexity of our base ten number system through modeling and the use of explanations and justifications.

As suggested in this paper, allowing students to use manipulatives and engage in discourse, all grounded within the context of a candy shop, promotes a deeper understanding of place value among K-2 learners. It is essential that students have multiple experiences with making sense of the ten-to-one relationship. The candy shop activity is one such experience that provides students with opportunities to reason about our base ten number system in a familiar context and to explain and justify their thinking as they work with renaming 10 ones into 1 ten and 10 tens into 1 hundred. Thus, a rich task with an emphasis on mathematical communication, such as the candy shop, is an effective way to ensure K-2 students engage in deep learning about the complexities of place value.

References

Bennett, C. A. (2013). Teachers' perspectives of whole-class discourse: Focusing on effective instruction to improve student learning. *Action in Teacher Education, 35*(5-6), 475-488.

Dixon, J. K., Nolan, E. C., Adams, T., Brooks, L.A., & Howse, T.D. (2016). *Making sense of mathematics for teaching grades K-2: (Communicate the context behind high-cognitive-demand tasks for purposeful, productive learning)*. Bloomington, IN: Solution Tree Press.

Dougherty, B. J., Flores, A., Louis, E., & Sophian, C. (2010). *Developing essential understanding of number & numeration*: Pre-K-grade 2. Reston, VA: National Council of Teachers of Mathematics.

Falle, J. (2004). Let's talk maths: A model for teaching to reveal student understandings. *Australian Senior Mathematics Journal, 18*(2), 17-27.

Fosnot, C. T., & Perry, R. S. (1996). Constructivism: A psychological theory of learning. In Fosnot, C. T. (Ed). *Constructivism: Theory, perspectives, and practice,* (pp. 8-33). New York, London: Teachers College Press.

Gregg, D. U., & Yackel, E. (2002). Helping students make sense of algebraic expressions: The candy shop. *Mathematics Teaching in the Middle School, 7*(9), 492.

Hiebert, J., & Wearne, D. (1993). Instructional tasks, classroom discourse, and students' learning in second-grade arithmetic. *American Educational Research Journal, 30*(2), 393-425.

Imm, K., & Stylianou, D. A. (2012). Talking mathematically: An analysis of discourse communities. *The Journal of mathematical behavior, 31*(1), 130-148.

Kamii, C. (1985). Young children reinvent arithmetic: *Implications of Piaget's theory.* New York, NY: Teachers College Press.

Kosko, K. W., Rougee, A., & Herbst, P. (2014). What actions do teachers envision when asked to facilitate mathematical argumentation in the classroom?. *Mathematics Education Research Journal, 26*(3), 459-476.

Nathan, M. J., Eilam, B., & Kim, S. (2007). To disagree, we must also agree: How intersubjectivity structures and perpetuates discourse in a mathematics classroom. *The Journal of the Learning Sciences, 16*(4), 523- 563.

National Council of Teachers of Mathematics [NCTM]. (1989). *Curriculum and evaluation standards for school mathematics* [3D Issue version]. Retrieved from http://www.nctm.org/flipbooks/standards/currandevalstandards/index.html

National Council of Teachers of Mathematics [NCTM]. (1991). *Professional standards for teaching mathematics* [3D Issue version]. Retrieved from http://www.nctm.org/flipbooks/standards/professionalteaching/index.html

National Council of Teachers of Mathematics [NCTM]. (2000). *Principles and standards for school mathematics* [3D Issue version]. Retrieved from http://www.nctm.org/flipbooks/standards/pssm/

National Council of Teachers of Mathematics [NCTM]. (2014). *Principles to actions: Ensuring mathematical success for all.* Reston, VA: Author.

National Governors Association Center for Best Practices [NGA Center] & Council of Chief State School Officers [CCSSO]. (2010a). *Common core state standards for English language arts.* Washington, D.C.: Author. Retrieved from http://www.corestandards.org/wp- content/uploads/ELA_Standards1.pdf

National Governors Association Center for Best Practices [NGA Center] & Council of Chief State School Officers [CCSSO]. (2010b). *Common core state standards for mathematics.* Washington, D.C.: Author. Retrieved from http://www.corestandards.org/wp- content/uploads/Math_Standards.pdf

Van de Walle, J. A., Lovin, L. H., Karp, K. S., & Bay-Williams, J. M. (2014). *Teaching student-centered mathematics: Developmentally appropriate instruction for grades Pre-K-2* (2nd ed.). Boston, MA: Pearson.

Whitenack, J. W., Knipping, N., Novinger, S., & Underwood, G. (2001). Second graders circumvent addition and subtraction difficulties. *Teaching Children Mathematics, 8*(4), 228.

Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education, 27*(4), 458-477.

https://digitalcommons.georgiasouthern.edu/stem_proceedings/vol2/iss1/9
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