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Optimizing Distribution Center Configuration: A Practical View of a Multi-objective Problem

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Abstract

The shape of a distribution center, as well as whether dock locations are on one side or two sides of the facility, impacts measures like travel distances and the number of dock locations that may be utilized. Thus, for a required number of pallet locations, there are multiple combinations of distribution center shape and dock configurations that should be evaluated against multiple measures. We have developed a practical model for making such evaluations and illustrate the model with data reflective of a partner in the Center for Engineering Logistics and Distribution.

1 Introduction

One of the most fundamental decisions in designing a new distribution center (DC) is the shape of the facility. Along with facility shape, the decision of utilizing dock doors on one or two sides of the facility can affect the performance of the DC [3]. Some of the performance factors impacted by these two decisions include: the number of dock doors, the storage density, the average distance traveled to store/retrieve product, and the size of the external truck pad. Even though these decisions are fundamental to DC design, there is not a prescriptive — much less a descriptive — model associated with this multi-objective problem [2]. This is even more unsettling when one considers the trend towards larger DCs, sometimes referred to as “mega DCs” [1].

The above problem was brought to the authors as part of a research project in the Center for Engineering Logistics and Distribution (CELDi). CELDi is a university-based enterprise providing innovative solutions for logistics and distribution excellence. As an applied research and education consortium, CELDi is a partnership between the National Science Foundation (NSF), nine major research universities and more than 25 member organizations in commercial, military and government sectors of the economy. Research endeavors are driven and sponsored by the member organizations that specialize in distribution, transportation, manufacturing, information technology, and software solutions. The mission of CELDi is to enable member organizations to achieve logistics and distribution excellence by delivering meaningful, innovative and implementable solutions that provide a return on investment.

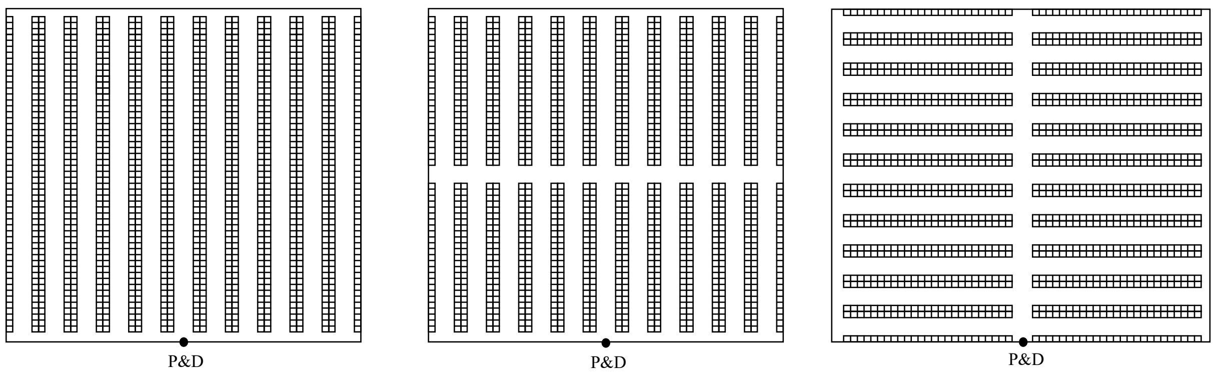
The organization that requested the project that led to this work is building three DCs per year for the next five years. The lead engineer responsible for DC design is often presented with a site plan where the shape of the facility has already been determined. He has noted over the years that facilities of some shape perform better than others, but has difficulty in characterizing this given the different requirements over the various DCs. He would like a descriptive model that can be used to predict the performance of the facility before operations begin. Additionally, his long-term goal is to use the model to influence the site-selection process with a prescriptive model that quantifies the trade-offs associated with facilities of various shapes. And although this project was initially requested by one member organization of CELDi (as is typical) at the recent CELDi meeting, this project received the most "votes" for follow-on project consideration.

In this paper we will define the problem and present a descriptive model relating warehouse shape and dock door configuration (a one-sided versus two-sided configuration) with such factors as facility square footage, external truck pad square footage, number of dock doors provided, storage density achieved, and the average distance traveled to store/retrieve product. For the purposes of this paper, we only consider a facility with single-command unit-load operations, random storage and a traditional aisle arrangement. For such facilities, a set of warehouses that differ on the basis of pallet requirements, shape and configuration will serve as input to the descriptive model and the output will be presented.

The paper is organized as follows. In the next section we review the related literature, supporting our claim that a suitable model for this problem does not exist. In Section 3 we formally define the problem, including which factors we consider in our objective function. The next two sections, Sections 4–5, present our mathematical model for DC (and associated components) dimensions and the algorithm we used to determine the facility that meets our target pallet position constraint and the travel-time model used to assess the labor in our DC, respectively. We present an example problem in Section 6. We conclude the paper in Section 7 and present our future research on this problem.

2 Related Research

There is a vast body of research to assist in designing and operating a DC [6, 7]. One aspect of this body of research concerns estimating the expected travel distances for a worker. Figure 1 shows three traditional warehouse designs, with parallel picking aisles and orthogonal cross aisles at each end of the picking aisles to facilitate flow from a picking and deposit (P&D) point. Layouts B and C in Figure 1 each have an additional cross aisle that divides the picking space into two sections. Such a cross aisle increases the efficiency of traveling between storage locations, which would be advantageous when storage and retrieval tasks are interleaved in a unit-load warehouse (also referred to as dual-command operations in contrast to single-command operations where a worker stores or retrieves a load in a cycle).



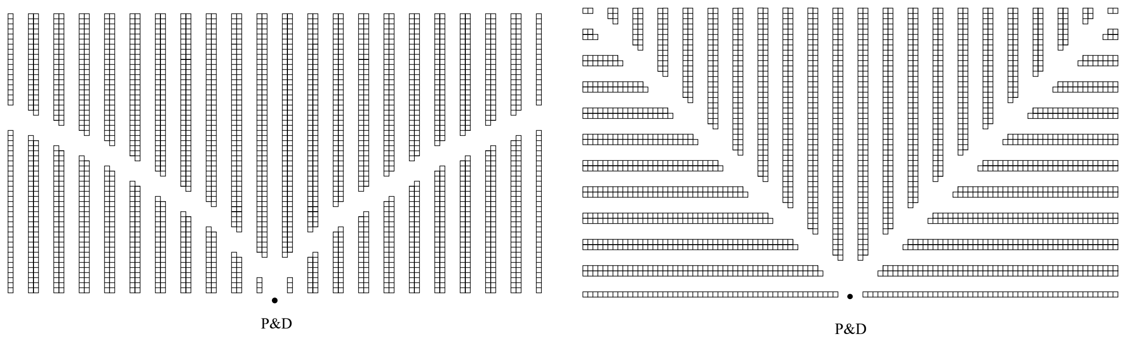
(a) Layout A (b) Layout B (c) Layout C

Figure 1: Traditional warehouses.

Researchers have modeled single-command travel distance in Layout A [5, 4, 12] and Layout C [4] and have presented some well-known results on optimal warehouse shape and P&D location with the objective of minimizing the distance traveled. A few papers that model dual-command travel consider only Layout A [11, 10], and do not use their results to determine warehouse design parameters, such as the number and length of aisles. In [11], cross aisle travel is also restricted to only the bottom cross aisle. Pohl *et al.* [12] assumed the cross aisle that provided the shortest path between pallet locations is used. They used their model to investigate the best shape of a facility operating under dual-command operations. We are unaware of any published analytical models for the optimal dual-command travel distance in Layouts B and C. For single-command travel, Layouts A and C are preferred, while Layouts B and C are generally preferred for dual-command travel [12].

Gue and Meller [8] point out that traditional designs, such as those in Figure 1, appear to be subject to the unspoken constraints that picking aisles must be parallel to one another, and cross aisles must be perpendicular to the picking aisles. When they relax these constraints, they show that non-traditional aisle layouts can reduce the expected travel distance to a single pallet location. Gue and Meller [8] proposed two new designs for a unit-load warehouse, where only single-command cycles are considered. These two new designs, Flying-V and Fishbone, which are presented in Figures 2(a) and 2(b), respectively, reduce single-command travel distance by about 10% and 20%, respectively, when compared to Layout A [8]. In all of the above models a single P&D point is assumed, through which all goods must flow. Ivanović *et al.* [9] investigated the performance of the Flying-V layout with multiple P&D points.

The other literature upon which we base our work can be summarized in textbooks like Tompkins *et al.* [13], where there are many space models that convert pallet dimensions



(a) (b)

Figure 2: (a) Flying-V warehouse optimized for single-command operations; (b) Fishbone warehouse optimized for single-command operations.

and requirements to storage area dimensions. For example, determining the space needed to store, say, two pallets on a shelf of pallet rack, including all necessary clearances, and then taking this value and determining the length of a picking aisle that is, say, 100 pallets deep, can be accomplished by implementing the models in textbooks. Other models can then be used to determine the distance between adjacent picking aisles by using standard picking aisle widths that can be determined by examination of tables in textbooks. These models and straightforward extensions to these models are employed in our work.

In summary, as evidenced by recent survey articles in the area of warehousing, there is a vast body of research on warehouse design and operation [6, 7] (with over 100 citations in each). However, the vast majority of this research is concerned with analyzing subsystems of a warehouse that have already been designed. And the research that is focused on the design of a warehouse attempts to formalize a design process for a given, simplified objective of the actual tradeoffs in warehouse design (more on this later when we discuss the warehouse performance measures that we consider in this paper). Thus, there does not appear to be a model that explicitly attempts to optimize the configuration of a warehouse considering the capital, operational, and performance specifications of interest to CELDi members.

3 Problem Statement

The objective of this research is to understand the dynamics of changing the configuration of a DC on various measures related to the performance of the DC. Here we define the *configuration of a DC* as the shape of the facility and whether there are dock doors on one side (which we define as a *1-sided configuration*) or two sides of the facility (which we define as a *2-sided configuration*). We restrict the door placement to the "long" side(s) of the facility, which we refer to as the facility's *width*. We refer to the other dimension of the facility as the facility's *depth*. Together the facility's width and depth define the facility's area, as we assume the facility is rectangular. We denote the ratio between the facility's width and depth to be the facility's *shape factor*.

In this paper we study a *unit-load warehouse*, which means that all product is received, stored and shipped in the same unit load. The operations that we explicitly model are the single-command put-away and retrieval operations. We assume the DC employs a truck-based material handling system (i.e., lift trucks) with a staging area between any dock doors and the storage area and cross aisles between any dock doors and the storage area to facilitate travel from any dock door to any storage location. Such a facility is presented in Figure 3, which also indicates notation that we will define and use in our models. As shown in the figure, we assume that the first section of the facility has no aisles except for the half-aisle generated from the second section. Specific receiving and shipping operations are assumed to be constant over all DC configurations, and thus are not explicitly modeled. Likewise, the labor associated with vertical travel to store and retrieve product is constant over all DC configurations and is not modeled.

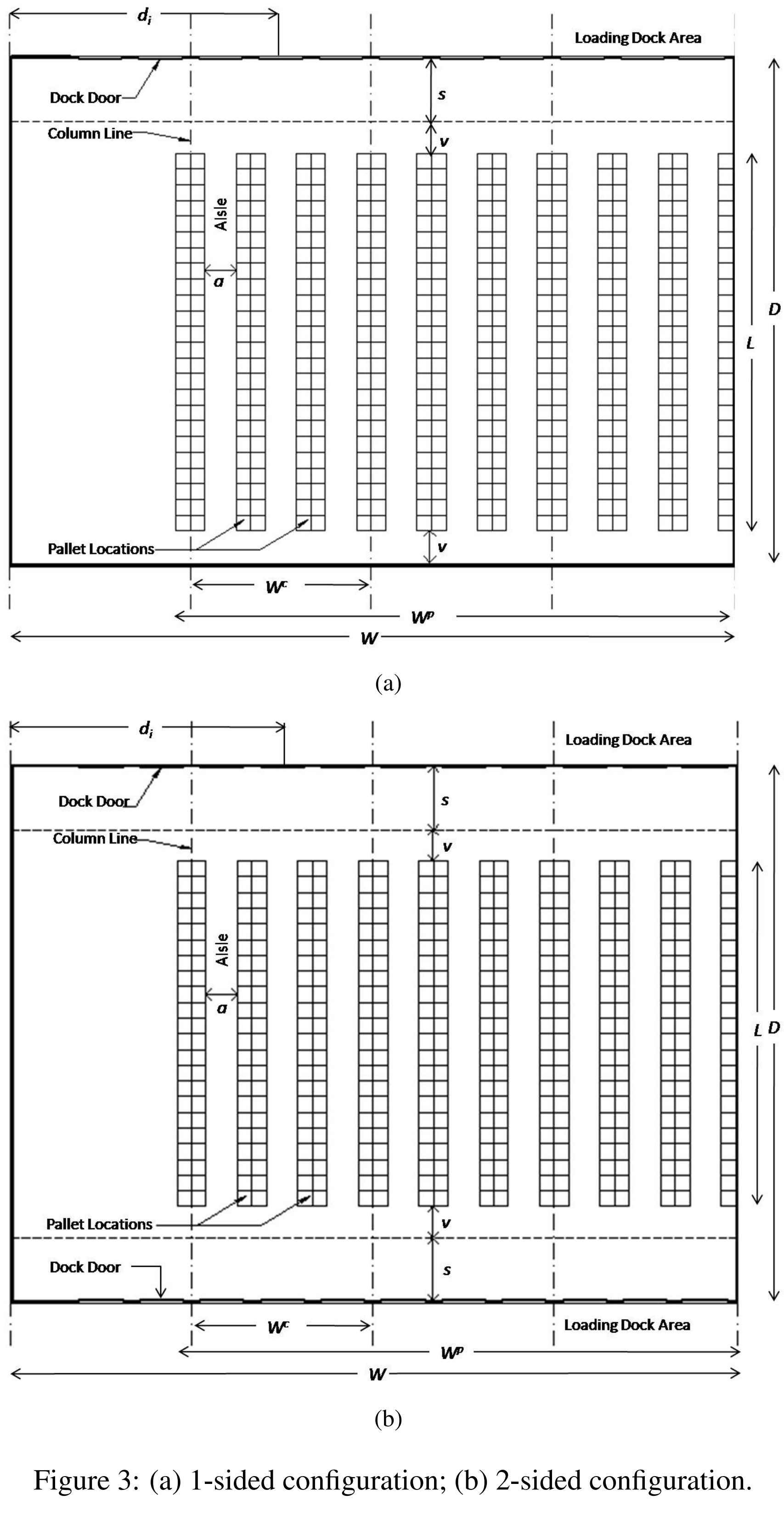
As mentioned earlier, there are many aspects of the DC that are impacted by the DC's configuration. In particular, we define the following performance measures of a DC:

1. **The number of dock doors**: This is the performance measure of a DC that is most clearly impacted by a change in the DC configuration. Namely, for a given facility shape, changing from a 1-sided to a 2-sided configuration will approximately double the number of available doors (we discuss why there is not an exact doubling later). In addition, for a given door configuration, as the facility's shape factor increases, the number of doors will increase.

2. **The operational labor**: This is the performance measure that has been most widely studied in the literature (although almost always assuming one dock door, which implies a 1-sided configuration). A 1-sided configuration is believed to hold the highest opportunity for slotting product in the warehouse. In addition, changing the facility's shape factor will influence the efficiency of travel for the put-away and retrieval operations.

3. **The facility size and utilization**: Because material handling equipment requires aisles of non-negligible width in order to operate, the size and utilization of the facility with respect to how much it can store, is impacted by the configuration of the facility. Namely, a staging area is needed on the side(s) of the facility in which dock doors are located. Also, as storage aisles are reduced in length (by increasing the facility's shape measure), for a given number of pallets to be stored, there will be more space dedicated to cross aisles.

4. **The truck pad size**: It is necessary to provide a truck pad (of a fixed depth) for access to the facility to execute receiving and storage operations. The size of the truck pad, for a given facility shape factor, doubles as one moves from a 1-sided to a 2-sided configuration. Likewise, for a given door configuration, the size of the truck pad increases linearly in the facility shape factor.



(a)

(b)

Figure 3: (a) 1-sided configuration; (b) 2-sided configuration.

The above enumerated list of performance measures illustrate that optimizing DC configuration is a multi-objective problem, consisting of both one-time expenses (facility and truck pad building) and operational costs (operational labor), as well as a performance measure that is typically viewed as a constraint over a metric that can be converted to dollars (the number of dock doors). Thus, while working with our industrial collaborator, they believed that it would be better to examine the impact of DC configuration changes on these performance measures individually instead of trying to formulate this as a weighted-objective optimization problem. Therefore, we focused our efforts on providing a descriptive model to measure the impact of changes in DC configuration on the above performance measures and a useful way in which to present the results. Furthermore, our descriptive model assumes there is a minimum number of pallet positions to be provided in any feasible DC configuration. And because there are a discrete number of pallet levels in a rack, a discrete number of pallet positions in an aisle, and a discrete number of aisles, DC configurations for a target number of pallets will vary in terms of the actual number of pallet locations provided. Therefore, to the above list we add the additional performance measure: 5. **The number of pallet locations provided**.

4 Mathematical Model and Algorithm for DC Sizing

In order to determine the best size of a DC for a required number of pallet positions, calculations for components within the facility are needed. In the next section we present equations for calculating those components, and in Section 4.2 we present an algorithm for sizing the DC to satisfy a number of required pallet positions for a given facility shape factor. Finally, in Section 4.3 we present performance measures to evaluate various warehouse shapes.

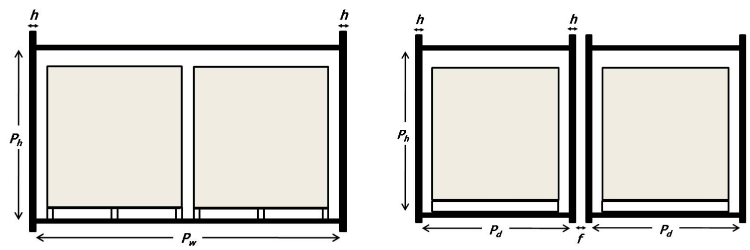
4.1 DC and Component Dimensioning

The number of dock doors and aisles in the DC are restricted by the spacing between adjacent columns within the DC. We denote the distance between two adjacent columns (i.e., a "section") with the parameter W^c and the number of warehouse sections with the variable N. Thus, the total number of dock doors can be calculated by multiplying N by the number of doors within a section of the DC and accounting for a 1-sided or 2-sided configuration. (We use Algorithm 1 in Section 4.2 to calculate N, the number of sections.) Given values for W^c, dock door width (d_w), and the distance between doors (b), the number of doors (n_d) between adjacent columns is:

$$n_d = \left\lfloor \frac{W^c}{d_w + b} \right\rfloor .$$

It should be noted here that most companies adjust the value of b so that $(W^c/(d_w+b))$ is an integer.

The dimensions of a pallet opening, P_w, P_d, P_h, are used to calculate the number of aisles (and later to calculate the length of an aisle). (We define the dimensions of the pallet opening to include the pallets as well as the clearances around the pallets.) The distance of the horizontal rack member is referred to as parameter h, and we use the parameter f to denote the clearance between back-to-back racks (flue space). Figure 4 shows a front and side view of a pallet opening.



(a) (b)

Figure 4: (a) Front view; (b) Side view.

The number of aisles in the DC can be calculated by first determining the number of aisles that can be placed between adjacent columns and then multiplying by the number of sections. Given values for W^c, aisle width (a), pallet opening depth (P_d) and f, we calculate the number of aisles between columns (n_a),

$$n_a = \left\lfloor \frac{W^c}{2P_d + f + a} \right\rfloor.$$

Again, it is typical to adjust a upward to ensure n_a is satisfied as an integer value.

4.2 Algorithm to Specify DC Dimensions to Meet a Target Number of Pallet Positions

In this section we present an algorithm to determine the required number of sections to achieve a target number of pallet positions. The number of sections defines the width of the facility, which in turn (along with shape), defines the depth of the DC. The algorithm is iterative in nature because sections of warehouse are added until a desired number of pallet positions is achieved for a given shape factor. At each iteration calculations are made to determine if the pallets in the current number of sections meets or exceeds the required number of pallet positions.

Given N sections of warehouse, the width of the facility (W) can be determined by multiplying N by the width of the warehouse sections, W^c,

$$W = NW^c.$$

The depth of the facility (D) for a given shape, r, is:

$$D = N(W^c/r).$$

The total number of aisles in the facility (A_n) can be calculated as:

$$A_n = 0.5 + (N-1)n_a.$$

We use $(N-1)$ because there are no aisles in the first section of warehouse except for the half-aisle generated from the second section.

To determine the number of pallet positions per aisle (P_A) for a 1-sided facility, we use:

$$P_A = \left\lfloor \frac{D - s - 2v}{P_w + 2h} \right\rfloor \times 2 \times p_n \times L,$$

where s is the depth of the staging area, v is the width of the cross aisles, p_n is the number of pallets per opening, L is the number of levels within the DC, and the quantity $(P_w + 2h)$ is the width of the pallet opening along with its rack members. (Note that we multiply by 2 because there are back-to-back racks in an aisle.) For a 2-sided facility, we simply substitute $2s$ for s into the previous equation, as there are two staging areas.

The total number of pallet positions (Q) in the DC is then,

$$Q = A_n P_A.$$

Substituting the previously defined equations for P_A, A_n, and D, we can rewrite Q as follows:

$$Q = (0.5 + n_a(N-1)) \times \left(\left\lfloor \frac{(W^c/r)N - s - 2v}{P_w + 2h} \right\rfloor \times 2 \times p_n \times L \right). \tag{1}$$

In addition, we can compute the aisle length (A) for a 1-sided facility,

$$A = \left\lfloor \frac{(W^c/r)N - s - 2v}{P_w + 2h} \right\rfloor \times (P_w + 2h).$$

We use $2s$ in place of s in the previous equation for a 2-sided facility.

The following algorithm is used to calculate the number of sections, N, for a facility with a given shape factor.

Algorithm 1 Calculate number of sections, N, for each given shape factor, r

```
N ⇐ 2
Compute Q via (1)
while Q < Required pallets do
  N ⇐ N + 1
  Compute Q via (1)
end while
```

We initially set the value of N to be 2 to accommodate our assumption that the first section only includes one half aisle, as shown in Figure 3. The number of sections is incremented until the desired number of pallet locations is achieved or exceeded. After each increment, a calculation is made to determine the number of pallet positions created by adding a section of warehouse. The algorithm can be executed recursively to generate a set of facilities that meet a required number of pallet positions for various shape factors.

4.3 Performance Measures

The total number of doors (D_n) can be calculated as shown below:

$$D_n = N n_d.$$

(Note that for a 2-sided configuration, the total number of doors, D_n, should be multiplied by 2.)

The utilization of the facility can be calculated by identifying the "footprint" of the racks in the facility and dividing by the total area of the warehouse. The utilization is summarized below, where P_w is the pallet opening width and Q is the total number of pallet positions in the DC,

$$U = \frac{\frac{Q/L}{p_n}\left(P_w + 2h\right)\left(P_d + 0.5f\right)}{D \times W}.$$

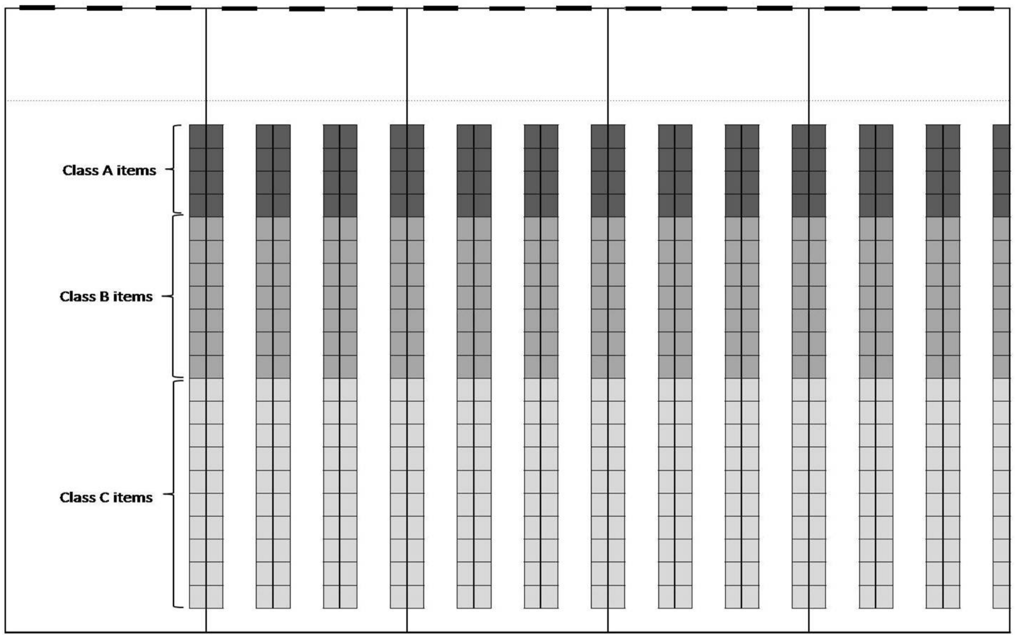
The size of the truck pad (TP) can be calculated as shown below, where t is the depth of the truck pad,

$$TP = t(NW^c).$$

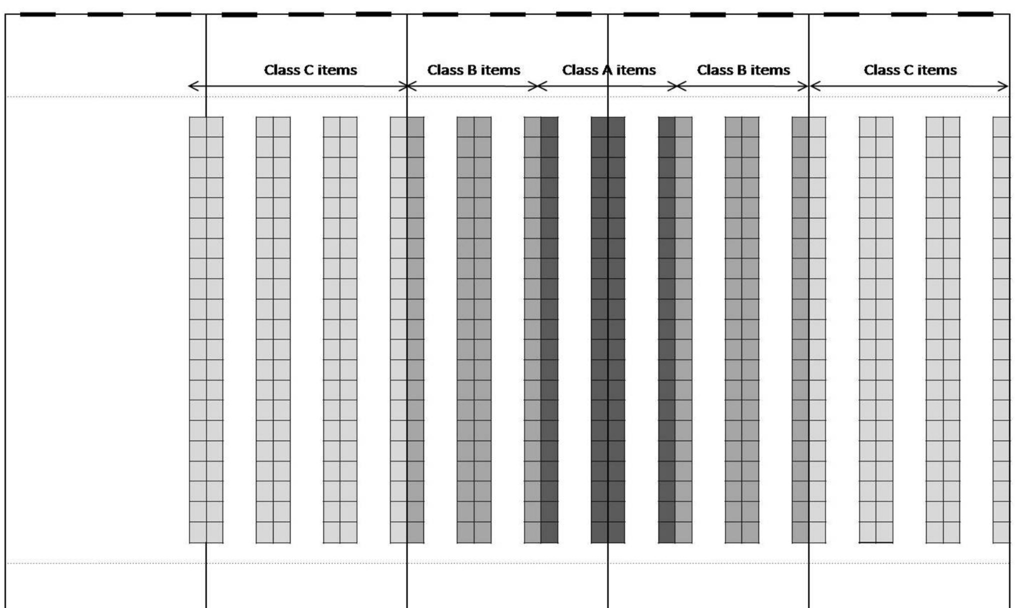
5 Travel-Time Model

The travel-time model utilizes class-based storage where the fast-moving items (class A) are located in the most-desirable positions, followed by class-B items and class-C items. Figure 5 illustrates the placement of class-A items, class-B items, and class-C items for a 1-sided and 2-sided configuration. In the travel calculations we assume a discrete number of dock doors and continuous aisles. We also assume that all of the dock doors are used with equal probability. In order to incorporate dedicated storage into a travel-time model, we first calculate the approximate number of pallets for each class. We denote the percentage of storage locations for each storage class (A, B, or C) with P and the appropriate class subscript. We also add the appropriate class subscript to the total number of storage positions, Q, to indicate the number of storage positions in a class. The approximate number of pallet locations for each class can be determined as follows:

$$Q_A = P_A \times Q, \qquad Q_B = P_B \times Q, \qquad Q_C = P_C \times Q.$$



(a)



(b)

Figure 5: (a) 1-sided class-based storage; (b) 2-sided class-based storage.

We then approximate the number of racks necessary to accommodate each class. We use R to denote the number of racks required and the subscripts A, B, and C to identify the class. The number of racks for each class is calculated as follows:

$$R_A = Q_A/(L \cdot p_n), \qquad R_B = Q_B/(L \cdot p_n), \qquad R_C = Q_C/(L \cdot p_n).$$

The putaway travel can be divided into two components: 1) travel parallel to the dock doors and 2) travel along an aisle. We refer to the former as the x-distance and the latter as the y-distance.

5.1 1-Sided Configuration

To take advantage of class-based storage in a 1-sided configuration, the class-A items are placed at the front of the aisles nearest the dock doors. Class-B items follow the class-A items, and C-items are located farthest from the dock doors as shown in Figure 5(a). For a 1-sided configuration, the x-distance can be calculated as half the distance from either side of the center of a dock door with appropriate weights for each side based on the position of the door. Figure 6 shows dock doors where d_i is the distance to the center of dock door i, W is the width of the facility, and W^p is the width of the area of the facility that contains pallet rack.

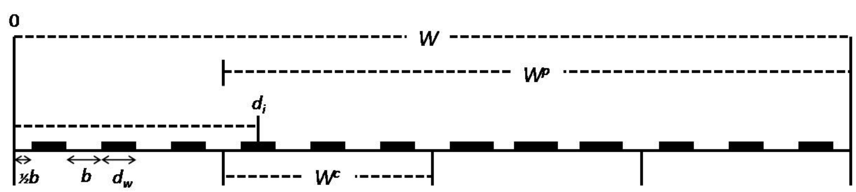


Figure 6: x-Travel for 1-sided.

The distance to dock door i can be calculated as shown below, where the distance from a column to the first door is $\frac{1}{2}b$,

$$\begin{aligned} d_i &= \frac{1}{2}(b+d_w)+(i-1)(d_w+b) \\ &= (b+d_w)(i-0.5) \qquad\qquad \forall i \in D^n. \end{aligned}$$

For a 1-sided configuration all aisles are equally-likely to be visited. The x-distance traveled from the dock door located within W^p with distance d_i from the leftmost side of the facility is given as follows:

$$x_i = \begin{cases} \left(\dfrac{d_i - W^c}{W^p}\right)\left(\dfrac{d_i - W^c}{2}\right) + \left(\dfrac{W^p - d_i}{W^p}\right)\left(\dfrac{W^p - d_i}{2}\right) & \text{if } d_i \geq W^c; \\ W^c - d_i + 0.5W^p & \text{if } d_i < W^c. \end{cases}$$

The expected value of the x-distance is calculated as follows:

$$E[x] = \frac{\sum_{i=1}^{D_n} x_i}{D_n}. \tag{2}$$

The y-distance traveled depends on the class of the item being retrieved. Thus, the aisle must be divided such that the distance for each class is known. We first approximate the number of racks per class in each aisle, where the total number of aisles in the facility is A_n,

$$L_A = \frac{R_A}{A_n} \qquad L_B = \frac{R_B}{A_n} \qquad L_C = \frac{R_C}{A_n}.$$

Then, we calculate the total number of racks in an aisle,

$$L_T = L_A + L_B + L_C.$$

Now we can define the y-distances traveled in terms of proportions for each class and multiplying by the aisle length, A (note that in order to travel to a class-B item, one must first travel past the class-A items),

$$y_A = 0.5\left(\frac{L_A}{L_T}\right)A,$$

$$y_B = \left(\frac{L_A}{L_T}\right)A + 0.5\left(\frac{L_B}{L_T}\right)A = \left(\frac{L_A + 0.5L_B}{L_T}\right)A,$$

$$y_C = \left(\frac{L_A + L_B}{L_T}\right)A + 0.5\left(\frac{L_C}{L_T}\right)A = \left(\frac{L_A + L_B + 0.5L_C}{L_T}\right)A.$$

Letting PA denote the percentage of activity for each class, the expected y-distance traveled can be calculated,

$$E[y] = s + v + PA_A y_A + PA_B y_B + PA_C y_C. \tag{3}$$

Finally, the expected travel (T) for a 1-sided configuration can be calculated by summing two times the expected x-distance, (2), and the expected y-distance (3), and multiplying by 2,

$$E[T] = 2(E[x] + E[y]).$$

5.2 2-Sided Configuration

For a 2-sided configuration, the class-A items are located in the aisles in the centermost part of the facility with class-B and class-C items in the aisles extending outward toward the walls of the facility as shown in Figure 5(b). Thus, the x-distance traveled must take into consideration the location of each class. Figure 7 shows the distance to each class from the leftmost side of the facility. (In the figure, subscripts for class B and C are only used to denote that there are two locations.) To determine the distance to the beginning of each

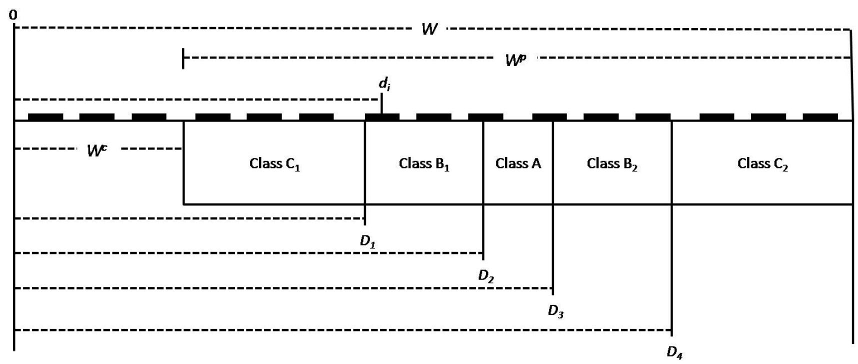


Figure 7: x-Travel for 2-sided.

class, first we approximate the number of aisles for each class, where Q is the total number of pallet positions, Q with a subscript is the number of pallets for a given class, and A_n is the total number of aisles in the facility,

$$A_A = \frac{Q_A}{Q} A_n, \quad A_B = \frac{Q_B}{Q} A_n, \quad A_C = \frac{Q_C}{Q} A_n.$$

From Figure 7 the distance to a class can be calculated as follows:

$$D_1 = W^c + 0.5 \left(\frac{A_C}{A_n} \right) W^p,$$

$$D_2 = D_1 + 0.5 \left(\frac{A_B}{A_n} \right) W^p,$$

$$D_3 = D_2 + \left(\frac{A_A}{A_n} \right) W^p,$$

$$D_4 = D_3 + 0.5 \left(\frac{A_B}{A_n} \right) W^p.$$

In the following equations we consider the travel to a specific class from the various possible dock door locations. We later weight these distances by the appropriate class activity level to determine an expected distance traveled.

1. **Travel to an A-item**

$$x_{iA} = \begin{cases} \left(\frac{d_i - D_2}{D_3 - D_2}\right)\left(\frac{d_i - D_2}{2}\right) + \left(\frac{D_3 - d_i}{D_3 - D_2}\right)\left(\frac{D_3 - d_i}{2}\right) & \text{if } D_2 \le d_i \le D_3; \\ (D_2 - d_i) + 0.5(D_3 - D_2) & \text{if } d_i < D_2; \\ (d_i - D_3) + 0.5(D_3 - D_2) & \text{if } d_i > D_3. \end{cases}$$

2. **Travel to a B-item in zone B$_1$**

$$x_{iB_1} = \begin{cases} \left(\frac{d_i - D_1}{D_2 - D_1}\right)\left(\frac{d_i - D_1}{2}\right) + \left(\frac{D_2 - d_i}{D_2 - D_1}\right)\left(\frac{D_2 - d_i}{2}\right) & \text{if } D_1 \le d_i \le D_2; \\ (D_1 - d_i) + 0.5(D_2 - D_1) & \text{if } d_i < D_1; \\ (d_i - D_2) + 0.5(D_2 - D_1) & \text{if } d_i > D_2. \end{cases}$$

3. **Travel to a B-item in zone B$_2$**

$$x_{iB_2} = \begin{cases} \left(\frac{d_i - D_3}{D_4 - D_3}\right)\left(\frac{d_i - D_3}{2}\right) + \left(\frac{D_4 - d_i}{D_4 - D_3}\right)\left(\frac{D_4 - d_i}{2}\right) & \text{if } D_3 \le d_i \le D_4; \\ (D_3 - d_i) + 0.5(D_4 - D_3) & \text{if } d_i < D_3; \\ (d_i - D_4) + 0.5(D_4 - D_3) & \text{if } d_i > D_4. \end{cases}$$

4. **Travel to a C-item in zone C$_1$**

$$x_{iC_1} = \begin{cases} \left(\frac{d_i - W^c}{D_1 - W^c}\right)\left(\frac{d_i - W^c}{2}\right) + \left(\frac{D_1 - d_i}{D_1 - W^c}\right)\left(\frac{D_1 - d_i}{2}\right) & \text{if } W^c \le d_i \le D_1; \\ W^c - d_i + 0.5(D_1 - W^c) & \text{if } d_i < W^c; \\ (d_i - D_1) + 0.5(D_1 - W^c) & \text{if } d_i > D_1. \end{cases}$$

5. **Travel to a C-item in zone C_2**

$$x_{iC_2} = \begin{cases} \left(\frac{d_i - D_4}{W - D_4}\right)\left(\frac{d_i - D_4}{2}\right) + \left(\frac{W - d_i}{W - D_4}\right)\left(\frac{W - d_i}{2}\right) & \text{if } d_i \geq D_4; \\ (D_4 - d_i) + 0.5(W - D_4) & \text{if } d_i < D_4. \end{cases}$$

The expected x-distance for a class-A item is:

$$E[x_A] = \frac{\sum_{i=1}^{D_n} x_{iA}}{D_n}.$$

The expected x-distance to the first section of class-B items is:

$$E[x_{B_1}] = \frac{\sum_{i=1}^{D_n} x_{iB_1}}{D_n}.$$

The expected x-distance to the second section of class-B items is:

$$E[x_{B_2}] = \frac{\sum_{i=1}^{D_n} x_{iB_2}}{D_n}.$$

The expected x-distance to the first section of class-C items is:

$$E[x_{C_1}] = \frac{\sum_{i=1}^{D_n} x_{iC_1}}{D_n}.$$

The expected x-distance to the second section of class-C items is:

$$E[x_{C_2}] = \frac{\sum_{i=1}^{D_n} x_{iC_2}}{D_n}.$$

Again, using PA as the percentage of activity for each class, the expected x-distance traveled is calculated as follows:

$$E[x] = PA_A E[x_A] + PA_B\left(0.5E[x_{B_1}] + 0.5E[x_{B_2}]\right) + PA_C\left(0.5E[x_{C_1}] + 0.5E[x_{C_2}]\right). \tag{4}$$

Travel along an aisle for a 2-sided configuration is simply the depth of the staging area and the cross-aisle plus half the length of the aisle,

$$E[y] = s + v + 0.5A. \tag{5}$$

The expected travel (T) for a 2-sided configuration can be calculated by summing the expected x-distance, (4), and y-distance, (5), and multiplying by 2,

$$E[T] = 2(E[x] + E[y]).$$

6 Example Problem

The following example illustrates how the mathematical model can be used to evaluate the performance measures from Section 3. Consider a facility with a requirement of 10,000 pallet positions and the following component dimensions:

$$\begin{array}{rcl}
W^c = 54 \text{ ft} & b = 5 \text{ ft} & d_w = 8.5 \text{ ft} \\
v = 10 \text{ ft} & s = 40 \text{ ft} & a = 9.5 \text{ ft} \\
P_w = 13 \text{ ft} & P_d = 4 \text{ ft} & p_n = 3 \\
h = 0.\bar{3} \text{ ft} & f = 0.41\bar{6} \text{ ft} & L = 5 \\
P_A = 0.20 & P_B = 0.30 & P_C = 0.50 \\
PA_A = 0.80 & PA_B = 0.15 & PA_C = 0.05 \\
t = 100 \text{ ft} & &
\end{array}$$

The number of doors per section is:

$$n_d = \left\lfloor \frac{54}{8.5+5} \right\rfloor = 4.$$

The number of aisles per section is:

$$n_a = \left\lfloor \frac{54}{2(4)+0.41\bar{6}+9.5} \right\rfloor = 3.$$

Using Algorithm 1 the number of required sections to achieve a minimum of 10,000 pallet positions for a 1-sided facility with a shape factor of 1.0 is 7 ($N = 7$). Thus, the total number of aisles in the warehouse is:

$$A_n = 0.5 + (7-1)(3) = 18.5.$$

The actual aisle length (based on the number of racks) is:

$$A = \left\lfloor \frac{(54/1.0)7 - 40 - 2(10)}{13 + 2(-.\bar{3})} \right\rfloor \times \left(13 + 2(0.\bar{3})\right) = 314.\bar{3} \text{ ft}.$$

The total number of pallet positions is:

$$Q = (0.5 + 3(7-1)) \times \left(\left\lfloor \frac{(54/1.0)7 - 40 - 2(10)}{13 + 2(0.\bar{3})} \right\rfloor \times 2 \times 3 \times 5 \right) = 12{,}765.$$

The width of the facility is:

$$W = 7(54) = 378 \text{ ft.}$$

The depth of the facility with a shape factor of 1.0 is:

$$D = 378/1.0 = 378 \text{ ft.}$$

This results in a facility that is 142,884 ft^2 with the total number of doors available in the 1-sided facility as:

$$D_n = 7(4) = 28.$$

The utilization of the facility is:

$$U = \frac{\frac{12{,}765/5}{3}(13 + 2(0.\bar{3}))(4 + 0.5(0.41\bar{6}))}{378 \times 378} = 0.34.$$

The truck pad size is:

$$TP = 100(378) = 37{,}800 \text{ ft}^2.$$

The x-distance traveled from all doors can be calculated as a summation of the x-distance traveled from each door, $\sum_{i=1}^{D_n} x_i$ (3,346.875 ft for this example). Then, the expected x-distance, (2), is:

$$E[x] = 3,346.875/28 = 119.53 \text{ ft.}$$

The y-distance, (3), can be calculated as follows:

$$Q_A = 0.2(12{,}765) = 2{,}553, \quad Q_B = 0.3(12{,}765) = 3{,}829.5, \quad Q_C = 0.5(12{,}765) = 6{,}382.5;$$

$$R_A = 2{,}553/(5 \cdot 3) = 170.2, \quad R_B = 3{,}829.5/(5 \cdot 3) = 255.3, \quad R_C = 6{,}382.5/(5 \cdot 3) = 425.5;$$

$$L_A = 170.2/18.5 = 9.2, \quad L_B = 255.3/18.5 = 13.8, \quad L_C = 425.5/18.5 = 23.0;$$

$$L_T = 9.2 + 13.8 + 23.0 = 46;$$

$$y_A = 0.5\left(\frac{9.2}{46}\right)314.\bar{3} = 31.4\bar{3};$$

$$y_B = \left(\frac{9.2 + 0.5(13.8)}{46}\right)314.\bar{3} = 110.01\bar{6};$$

$$y_C = \left(\frac{9.2 + 13.8 + 0.5(23.0)}{46}\right)314.\bar{3} = 235.75;$$

$$E[y] = 40 + 10 + 0.8(31.4\bar{3}) + 0.15(110.01\bar{6}) + 0.05(235.75) = 103.44 \text{ ft.}$$

The total distance traveled for a 1-sided facility with a 1.0 shape factor and 10,000 required pallets is:

$$E[T] = 2(119.53 + 103.44) = 445.9 \text{ ft.}$$

The performance measures for the other 1-sided facilities with various shape factors are presented in Table 1, where the units for putaway travel, $E[T]$, depth, and width are in feet, and the units for truck pad and area are ft^2. Table 2 presents the results for the 2-sided facilities.

Table 1: 1-Sided Facility.

Shape	Width	Depth	Doors	Aisles	Aisle Length	Pallets	*E[T]*	Area	*U*	*TP*
1.0	378.0	378.0	28	18.5	314	12,765	446	142,884	0.34	37,800
1.5	432.0	288.0	32	21.5	219	10,320	449	124,416	0.32	43,200
2.0	540.0	270.0	40	27.5	205	12,375	515	145,800	0.33	54,000
2.5	594.0	237.6	44	30.5	164	10,980	537	141,134	0.30	59,400
3.0	648.0	216.0	48	33.5	150	11,055	568	139,968	0.30	64,800
3.5	702.0	200.6	52	36.5	137	10,950	599	140,821	0.30	70,200
4.0	756.0	189.0	56	39.5	123	10,665	630	142,884	0.29	75,600

Table 2: 2-Sided Facility.

Shape	Width	Depth	Doors	Aisles	Aisle Length	Pallets	*E[T]*	Area	*U*	*TP*
1.0	378.0	378.0	56	18.5	273	11,100	466	142,884	0.30	75,600
1.5	486.0	324.0	72	24.5	219	11,760	606	157,464	0.29	97,200
2.0	594.0	297.0	88	30.5	191	12,810	746	176,418	0.28	118,800
2.5	648.0	259.2	96	33.5	150	11,055	816	167,962	0.25	129,600
3.0	756.0	252.0	112	39.5	150	13,035	956	190,512	0.26	151,200
3.5	810.0	231.4	120	42.5	123	11,475	1,026	187,458	0.23	162,000
4.0	864.0	216.0	128	45.5	109	10,920	1,096	186,624	0.22	172,800

In general, as the shape of the facility increases, the number of doors, truck pad size, and putaway distance increase. Also, space utilization is higher for a 1-sided facility because the 2-sided facility includes an additional staging area.

The results of this analysis can be used in multiple ways. For example, if the designer believes that they will require 56 doors, they can evaluate a 1-sided facility with a shape factor of 4.0 against a 2-sided facility with any shape factor greater than 1.0. For example the 2-sided facility with a shape factor of 1.0 will have lower labor costs than the 1-sided facility with a shape factor of 4.0 (based on $E[T]$) while having similar building and truck pad costs. Other comparisons may be made to determine the relative cost of providing more doors within a particular door configuration.

7 Future Research

The model we present allows us to provide a decision maker with data on the tradeoff between the number of pallets and/or doors needed and the capital and unit-load operational costs of a distribution center. The company that initiated this project (which covers more than unit-load operations) envisions using the results to both influence the shape of new distribution centers and also to convert some of their existing facilities from a 2-sided to a 1-sided configuration.

There are many avenues that may be further explored with this research as a foundation. First, a detailed study could be conducted, using the model to — under a set of parameters — establish the best shape factor for a multiple-dock operation under class-based storage. As noted in Section 2, all prior studies assume a single dock, and most assume random storage. Second, the model could be extended beyond unit-load operations by incorporating replenishment and order-picking activities. Third, an engineering economy perspective could be taken to examine the multi-objective nature of this problem, attempting to find ways in which to express the tradeoffs that exist between the performance measures of interest over the time horizon of the facility. And finally, any modeling of the "best" shape and configuration is dependent on specifying the number of dock doors needed. We have found that this issue is not well understood in industry and may benefit from further investigation.

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