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Shuttle-Based Storage and Retrieval Systems with Robotic Order-Picking Shuttle Carrier

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***Abstract*—Shuttle-based storage and retrieval systems with robotic order picking shuttle carrier are relatively new systems not yet address in automated storage system research. The objective of this research paper is to propose analytical model for robotic order picking shuttle carriers. Analytical expressions for the calculation of multi cycle have been determined by assuming uniform distributed order-picking locations and the probability theory. The proposed model enables the calculation of the expected travel (cycle) time for multiple command cycles of the robotic order picking shuttle carrier, from which the throughput performance can be evaluated.**

***Keywords*—warehousing, robotic order picking shuttle carrier, analytical modelling, performance analysis.**

I. INTRODUCTION

Today's global economy is oriented towards more product diversifications, their shorter life-cycles within a demanding competitive market. For this reason many companies are investing considerable efforts (financial funds) for the development of new technologies and new products in the market. A great deal of total costs represent the cost of labor, therefore numerous companies invest significant funds in automation [22].

Warehouses are critical for production companies that work inside the supply-chain. There are many reasons why warehouses are needed and some main reasons can be distinguished [1]: to facilitate the coordination between the production and customer demand by buffering products for a certain period of time, to accumulate and consolidate products from various producers for combined shipments, to provide same-day delivery in production and to important customers, to support products customization activities, like packaging, final assembly etc.

The simplest form of storing products is block stacking in which pallets with products known as Transport Unit Loads (TUL) are stored on the floor and on top of each other. A more advanced way to store TUL is with the use of storage racks, which are metal constructions that make it possible to stack TUL higher than with block stacking. The more advanced material handling system is the Automated Storage and Retrieval System (AS/RS). AS/RS consists of storage racks served by a Storage and Retrieval (S/R) machine on rails. It is capable to handling TUL without the interference of an operator, since the system is fully automated [22].

The major advantages of the AS/RS are: high throughput performance, efficient utilization of warehouse space, high reliability and better control of inventory, improved safety conditions and decreases in damages and shortages of products.

On the other hand, AS/RS are rather expensive and inflexible in future changes; therefore a careful design is essential for the success of such a system [3].

The throughput performance of the mini-load AS/RS is limited with number of cycles per hour (FEM 9.851), which can note cope with today's e-commerce order fulfilment services. For this reason, major material-handling providers have introduced to the market a new technology known as Shuttle-Based Storage and Retrieval Systems (SBS/RS), which enables higher throughput, flexibility and scalability.

SBS/RS is a special design of an automated warehouse, which is assembled with an elevator with a lifting table, shuttle carriers that could be tier-captive or non tier-captive, buffer positions in each tier and the storage rack (See Fig. 1).

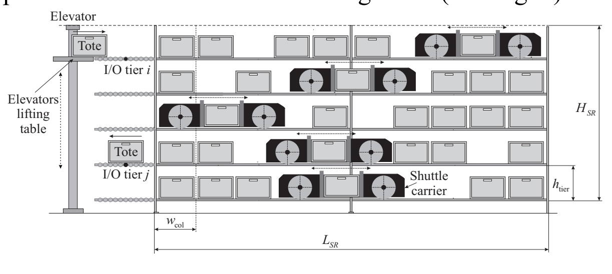

Fig. 1. Shuttle-based storage and retrieval system

In SBS/RS the elevator with lifting table provides vertical movement for totes to reach the buffer position in the i^{th} tier of the storage rack. The elevator's lifting table can reach a velocity of up to 5 m/s. Elevators are usually the bottleneck in the SBS/RS, therefore they determine the performance of the SBS/RS as a whole [18], [19], [20], [21], [22].

The shuttle carrier is an autonomous vehicle that transports totes from the buffer position to the i^{th} storage locations in the storage racks. The shuttle carrier is equipped with telescopic attachment for manipulating totes in the first lane (single-deep) or in the second lane (double-deep) of the storage rack. The maximal payload of a tote should not exceed 50 kg/shuttle carrier and its dimensions should be within the range of: min. (150 x 200 x 80) mm, max. (600 x 400 x 250) mm. A shuttle carrier can reach a velocity up to 4 m/s or more [18], [19], [20], [21], [22].

In SBS/RS, there is usually a single shuttle carrier in each tier of storage rack (the application of tier-captive system). This assumption can be released if a special shuttle elevator at the back of the storage rack is used for moving shuttle carriers up and down to the prescribed tier in the storage rack. There are two buffer positions, each serving one side of the storage rack at each tier. These positions are used for buffering totes carried by the elevator's lifting table for the storage process and by shuttles for the retrieval process. The storage rack is composed of storage columns C. By multiplying storage columns C in the horizontal and tiers M in the vertical direction, the length L_{SR} and the height H_{SR} of the storage rack are achieved [18], [19], [20], [21], [22].

Estimating the throughput performance of the SBS/RS is an essential step in SBS/RS design. One way to improve the throughput performance is to finding new shuttle carrier's design that could achieve more throughput capacity.

In this paper, the proposed analytical model for the throughput performance calculations of SBS/RS with robotic order-picking shuttle carrier is presented and discussed.

II. Literature Review on SBS/RS

Generally, SBS/RS have been the subject of many researchers over the last decade.

Previous researchers have focused mostly on Autonomous Vehicle Storage and Retrieval Systems (AVS/RS) configurations whereas "tier captive" AVS/RS and SBS/RS seem to have been disregarded [5], [6], [7], [8], [9], [10], [11], [14], [15], [16], [17], [23], [24], [28], [29], [30].

One of the first study on SBS/RS has been developed for Vanderlande Industries, where two non-passing lifting systems are mounted along the SR. In this research a scheduling problem of lifts with the development of the look-ahead heuristic for the solution procedure to improve the total handling time were introduced [4].

Roy et al. [28] presented the performance impact of AVS/RS design variables as: configuration of aisles and columns, allocation of resources to zones, and vehicle assignment rules by using an analytical model. The AVS/RS has been modeled as a multi-class semi-open queuing network with class switching and a decomposition-based approach in order to evaluate the system performance of AVS/RS.

Following a sequential processing policy is the modelling of SBS/RS via an open queuing network to estimate the performance of the SBS/RS in terms of utilizations of lifts and shuttles, as well as waiting times for lifts and queues. Several performance measures from the utilizations of lifts and shuttles, average flow time, waiting times, as well as the costs for the pre-defined SBS/RS designs have been analysed and discussed [25], [26].

Analytical travel time models for the computation of Single Command (SC) and Double Command (DC) travel (cycle) times for single- and double-deep SBS/RS, have been introduced by using the probabilistic theory. The proposed models consider the operating characteristics of the elevator's lifting table and the shuttle carrier, such as acceleration and deceleration and the maximum velocity. The proposed models enable the calculation of the expected travel (cycle) times for the SC and DC cycles, from which the performance of the SBS/RS can be evaluated. Based on the proposed models of SBS/RS, the throughput performance of the SBS/RS were presented in terms of utilizations of the elevator's lifting table and shuttles [18], [19], [20], [21], [22].

For the optimization of decision variables in SBS/RS, a multi-objective optimization solution procedure for the design of the SBS/RS has been proposed. In this research, three objective functions, minimization of average cycle time of transactions (average throughput time), amount of energy (electricity) consumption and total investment cost, have been considered. Due to the non-linear property of the objective function, the NSGA II genetic algorithm was used for facilitating the solution. Pareto optimal solutions have been searched to find out the optimum results [2].

A multi-elevator tier-captive SBS/RS, which is associated with tier-captive shuttle carriers, multiple elevators with a lifting table and storage racks has been proposed by Ning et al. [27]. The authors present a simulation model of a multi-elevator tier-captive SBS/RS, from which the throughput performance can be evaluated.

By using a fork-join queueing network, a parallel processing policy for tier-captive autonomous vehicle storage and retrieval systems (SBS/RS), under which an arrival transaction can request the lift (elevator) and the vehicle (shuttle carrier) simultaneously has been proposed by Zou et al. [31]. A fork-join queueing network has been formulated in which an arrival transaction is split into a horizontal movement task served by the vehicle and a vertical movement task served by the lift. For validation of analytical models, a simulation model has been used by the authors. The results show that the fork-join queueing network is accurate in estimating the system performance under the parallel processing policy.

Ekren [12] proposed a graph-based solution for performance evaluation of an autonomous vehicle based storage and retrieval system (SBS/RS) under various design concepts. The performance of the system was evaluated in terms of average utilization of lifts and shuttle carriers by using the simulation modelling approach.

Epp et al. [13] proposed a method for performance evaluation of autonomous vehicle storage and retrieval systems (SBS/RS) with tier-captive single-aisle vehicles. For the performance evaluation, the authors have used a discrete-time open queueing network approach.

In this study, analytical travel time model for SBS/RS with robotic order-picking shuttle carrier is proposed, from which the throughput performance can be evaluated. Managers and warehouse designers from the industry (Schäfer, KNAPP, Dematic, TGW, Vanderlande) could use key findings and observations from this study to properly understand the proposed SBS/RS with robotic order-picking shuttle carrier. This means that the warehouse designers could use the proposed model with a confidence to calculate the throughput performance of the selected SBS/RS with robotic order-picking shuttle carrier in the early stage of the project.

This paper is organised as follows: In Section 3, a model formulation of the SBS/RS with robotic order-picking shuttle carrier, is given. In Section 4, the performance of the selected SBS/RS with robotic order-picking shuttle carrier is evaluated and discussed. Finally, the conclusion is given in section 5.

III. Model Formulation

SBS/RS with robotic order-picking shuttle carrier differs from the classical SBS/RS.

In this system the shuttle carrier is order picking (collecting) the items in the i^{th} tier of the storage rack by utilizing the robotic arm (see Fig. 2).

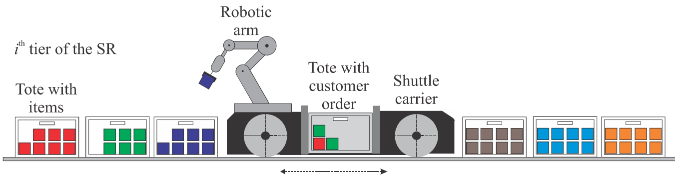

Fig. 2. Robotic order-picking shuttle carrier in a SBS/RS

The complete working cycle would look as follows:

- The elevator's lifting table starts from the ground-floor, i.e., the first tier.
- The elevator's lifting table picks up the (empty) tote and moves to the i^{th} tier.
- When the elevator's lifting table reaches the i^{th} tier, it releases the tote in the buffer position.
- The shuttle carrier in the i^{th} tier picks-up the tote from the buffer position and starts picking the items.
- When the order is finished, the shuttle carrier travels to the buffer position of the i^{th} tier.
- The shuttle carrier releases the tote in the buffer position of the i^{th} tier.
- The elevator's lifting table moves to the the i^{th} tier and picks up the tote from the buffer position.
- The elevator's lifting table moves to the ground-floor (first tier), where the tote is released.

Note that the elevator is excluded from this study. Operations regarding the storage of full totes and retrieval of empty totes with the shuttle carrier is note studied in this research, as well.

A. Assumptions

The assumptions that were used in analytical modelling are summarized as follows:

- The storage rack is divided into two sides (left and right), therefore totes with items are available on both side of the storage rack.
- The dwell-point location of the tier-captive robotic order-picking shuttle carrier in the i^{th} tier of the SR (when idle) is located at the buffer position.
- At each tier of the storage rack, there are two buffer position (left and right) and a single tier-captive robotic order-picking shuttle carrier.
- The robotic order-picking shuttle carrier is operated on a multi command cycle collecting four (4) and six (6) items on four (4) and six (6) randomly selected locations.
- The sequences of (i) Acceleration, constant velocity and deceleration has been used (See Fig. 3).
- The drive characteristics of the tier-captive robotic order-picking shuttle carrier, as well as the length L_{SR} of the storage rack, are known in advance.
- The length L_{SR} of the storage rack is large enough for the robotic order-picking shuttle carrier to reach its maximum velocity v_{max} in the horizontal direction.
- A randomized assignment policy is considered which means that any order-picking location is equally likely to be selected for picking the items with the robotic order-picking shuttle carrier.

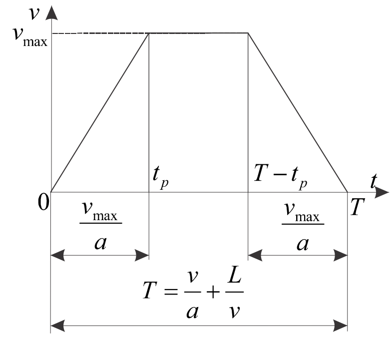

Fig. 3. Velocity-time relationship of the shuttle carrier

B. Abbreviations and Notations

The abbreviations that were used in the paper are summarized as follows:

SBS/RS	Shuttle-based storage and retrieval systems.
SR	Storage rack.
SA	One way.
TB	Travel between.
MC	Multi cycle.

To formulate the problem, the following notations is used:

a_x	Acceleration / deceleration of the robotic order-picking shuttle carrier.
v_x	Velocity of the robotic order-picking shuttle carrier.
d_x	Distance.
$t(d_x)$	Travel time of the robotic order-picking shuttle carrier to the most distance storage location (cell).
$F_x(z)$	Probability distribution function.
$f_x(z)$	Probability density function.
z	Variable.

E(SA) The expected one way travel time.
E(TB) The expected travel-between time.
$E(MC_4)$ The expected multi command cycle time for visiting four (4) locations.
$E(MC_6)$ The expected multi command cycle time for visiting six (6) locations.
τ Throughput performance
$t_{\text{P/S tote}}$ Pick-up and set-down time of a tote
$t_{\text{P/S item}}$ Pick-up and set-down time of an item
$t_{\text{robo move}}$ Time for moving the robotic arm from the right to the left position of the shuttle carrier

C. *Travel time model*

Expected one way travel time

Travel time of the robotic order-picking shuttle carrier to the most distant order-picking location in the SBS/RS is calculated by (1):

$$t(d_x) = \frac{v_x}{a_x} + \frac{d_x}{v_x} \quad (1)$$

Under the randomized storage policy, the probability distribution function $F_x(z)$ and probability density function $f_x(z)$ of z_i (i =1, 2, …, n) are as follows. Probability distribution function $F_x(z)$ from Bozer and White [3] is calculated by (2):

$$F_x(z) = \begin{cases} \frac{z}{d_x} & 0 \le z \le d_x \\ 1 & z \ge d_x \end{cases} \quad (2)$$

Probability density function $f_x(z)$ is calculated by (3):

$$f_x(z) = \frac{\mathrm{d}F_x(z)}{\mathrm{d}z} = \begin{cases} \frac{1}{d_x} & 0 \le z \le d_x \\ 0 & z \ge d_x \end{cases} \quad (3)$$

The expected one way travel time E(SA) for travelling of the robotic order-picking shuttle carrier is equal to the next expression:

$$E(\mathrm{SA}) = \frac{v_x}{a_x} + \frac{1}{v_x}\int_0^{d_x} z f_x(z)\mathrm{d}z = \frac{v_x}{a_x} + \frac{d_x}{2v_x} \quad (4)$$

Expected travel-between time

Under the randomized storage policy, the probability distribution function $F_x(z)$ and probability density function $f_x(z)$ of z_i (i =1, 2, …, n) are as follows. Probability distribution function $F_x(z)$ from Bozer and White [3] is calculated by (5):

$$F_x(z) = \begin{cases} \frac{2z}{d_x} - \frac{z^2}{d_x^2} & 0 \le z \le d_x \\ 1 & z \ge d_x \end{cases} \quad (5)$$

Probability density function $f_x(z)$ is calculated by (6):

$$f_x(z) = \frac{\mathrm{d}F_x(z)}{\mathrm{d}z} = \begin{cases} \frac{2}{d_x} - \frac{2z}{d_x^2} & 0 \le z \le d_x \\ 0 & z \ge d_x \end{cases} \quad (6)$$

The expected travel-between time E(TB) for travelling of the robotic order-picking shuttle carrier between two randomly selected order-picking locations is equal to the following expression:

$$E(\mathrm{TB}) = \frac{v_x}{a_x} + \frac{1}{v_x}\int_0^{d_x} z f_x(z)\mathrm{d}z = \frac{v_x}{a_x} + \frac{d_x}{3v_x} \quad (7)$$

Expected multi cycle travel time

If the items on the order-picking list are sequenced sequentially and according to the random policy, the expected travel time for multiple command cycles is equivalent to the expected travel time for a single-command cycle and necessary number of travel-between times. The algorithm for performing multi command cycle in case of visiting four (4) order-picking locations works on the following sequence:

1. Selection of two random order-picking locations on the right side of the i^{th} tier of the storage rack.

   $$(x_{1\mathrm{R}}, x_{2\mathrm{R},\dots,} x_{n\mathrm{R}}) \in x_\mathrm{R}$$

2. Selection of two random order-picking locations on the left side of the i^{th} tier of the storage rack.

   $$(x_{1\mathrm{L}}, x_{2\mathrm{L},\dots,} x_{n\mathrm{L}}) \in x_\mathrm{L}$$

3. Sequence of the multi command cycle in case of visiting four picking locations (see Fig. 4).

   Order picking on the right side of the SR (increasing *Strategy x*):

   - Pick the first item, if the condition $x_{1\mathrm{R}} < x_{2\mathrm{R}}$ holds true.
   - Pick the second items.

   Order picking on the left side of the SR (decreasing *Strategy x*):

   - Pick the first item, if the condition $x_{1\mathrm{L}} > x_{2\mathrm{L}}$ holds true.
   - Pick the second items.

4. The expected multi command cycle time for visiting four (4) locations is calculated by (8):

$$E(\mathrm{MC}_4) = 2E(SA) + 3E(TB) + 2t_{\text{P/S tote}} + 4t_{\text{P/S item}} + t_{\text{robo move}} \quad (8)$$

5. Throughput performance τ is calculated by (9):

$$\tau(\mathrm{MC}_4) = \frac{3600}{E(\mathrm{MC}_4)} \cdot 4 \left[\frac{\text{items}}{\text{hour}}\right] \quad (9)$$

Note: on each order-picking location, a robotic order-picking shuttle carrier picks one item, only.

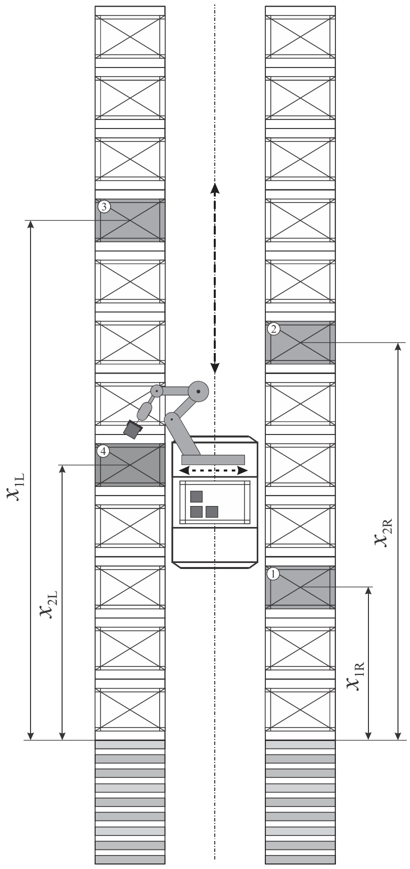

Fig. 4. Multi command cycle in case of visiting four (4) picking locations

The algorithm for performing multi command cycle in case of visiting six (6) order-picking locations works on the following sequence:

1. Selection of three random order-picking locations on the right side of the i^{th} tier of the storage rack.

$$(x_{1R}, x_{2R}, x_{3R,\dots,} x_{nR}) \in x_R$$

2. Selection of three random order-picking locations on the left side of the i^{th} tier of the storage rack.

$$(x_{1L}, x_{2L}, x_{3L,\dots,} x_{nL}) \in x_L$$

3. Sequence of the multi command cycle in case of visiting six order-picking locations (see Fig. 5).

   Order picking on the right side of the SR (increasing *Strategy x*):

   - Pick the first item, if the condition $x_{1R} < x_{2R} < x_{3R}$ holds true.
   - Pick the second items, if the condition $x_{2R} < x_{3R}$ holds true.
   - Pick the third item.

   Order picking on the left side of the SR (decreasing *Strategy x*):

   - Pick the first item, if the condition $x_{1L} > x_{2L} > x_{3L}$ holds true.
   - Pick the second items, if the condition $x_{2L} > x_{3L}$ holds true.
   - Pick the third item.

4. The expected multi command cycle time for visiting six (6) locations is calculated by (10):

$$E(\mathrm{MC}_6) = 2E(SA) + 5E(TB) + 2t_{\mathrm{P/S\ tote}} + 6t_{\mathrm{P/S\ item}} + t_{\mathrm{robo\ move}} \quad (10)$$

5. Throughput performance τ is calculated by (11):

$$\tau(\mathrm{MC}_6) = \frac{3600}{E(\mathrm{MC}_6)} \cdot 6 \left[\frac{\mathrm{items}}{\mathrm{hour}}\right] \quad (11)$$

Note: on each order-picking location, a robotic order-picking shuttle carrier picks one item, only.

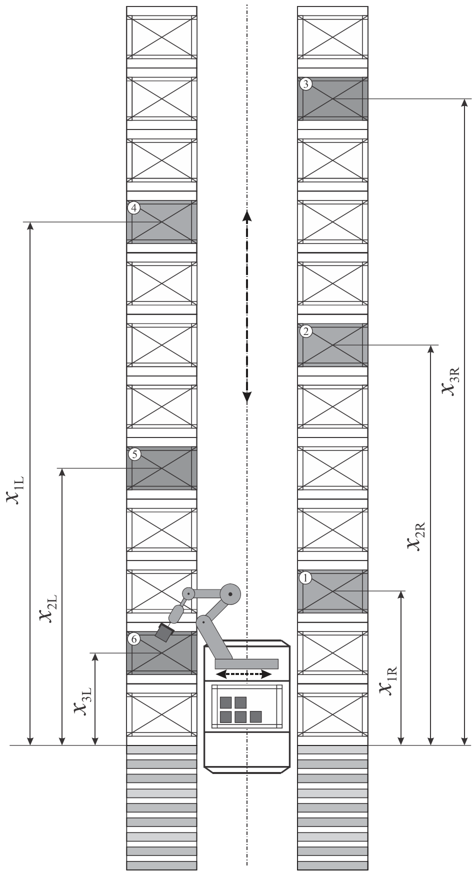

Fig. 5. Multi command cycle in case of visiting six (6) picking locations

IV. EXPERIMENTAL ANALYSIS

A. Input data for the analysis

In this study totes with the following dimensions: length l_{tote} = 0.6 m, width w_{tote} = 0.4 m and height h_{tote} = 0.24 m have been used. With regard to the tote, the order-picking location has the following dimensions: length (depth) of the column l_{COM} = 0.6 m, width of the column w_{COM} = 0.5 m and height of one column (tier) h_{COM} = 0.5 m. Dimensions of the storage rack depends on the number of columns C in the horizontal direction and number of tiers M in the vertical direction. Note that the elevator was excluded from this study, which means that the number of tiers M equals 1.

For the calculation of the throughput performance of the robotic order-picking shuttle carrier, the following lengths (L_1 = 30 m, L_2 = 40 m, L_3 = 50 m, L_4 = 60 m, L_5 = 70 m, L_6 = 80 m, L_7 = 90 m, L_8 = 100 m, L_9 = 110 m, L_{10} = 120 m) of the storage rack were used.

Since the throughput performance depends on the velocity characteristics of the robotic order-picking shuttle carrier, the following velocity profiles vp_i were used in this study.

TABLE I. VELOCITY PROFILE OF THE SHUTTLE CARRIER

	v_x (m/s)	a_x^+ (m/s^2)
vp_1	2	1
vp_2	3	2
vp_3	4	3

Note: Velocity scenarios vp_i were selected according to the references of material handling equipment producers and practical experiences of the authors.

Constant times were used as follows: $t_{P/S\ tote}$ = 3 sec., $t_{P/S\ item}$ = 8 sec. and $t_{robo\ move}$ = 5 sec.

B. Case study

The expected multi command cycle times along with the throughput performance are given based on the performed analysis. Analysis has been conducted for the selected length of the storage rack with three different velocity profiles (see Table I.) of the robotic order-picking shuttle carrier. Throughput performance analysis in Table II. relates to the velocity profile vp_1, meanwhile the throughput performance analysis in Tables III. and IV. relates to the velocity profile vp_2 and vp_3.

TABLE II. TROUGHPUT PERFORMANCE ANALYSIS I.

	E(SA) sec.	E(TB) sec.	E(MC$_4$) sec.	τ(MC$_4$) items/h	E(MC$_6$) sec.	τ(MC$_6$) items/h
L_1	9.50	7.00	40.00	173	54.00	191
L_2	12.00	8.67	50.00	155	67.33	171
L_3	14.50	10.33	60.00	140	80.67	155
L_4	17.00	12.00	70.00	127	94.00	141
L_5	19.50	13.67	80.00	117	107.33	130
L_6	22.00	15.33	90.00	108	120.67	120
L_7	24.50	17.00	100.00	101	134.00	112
L_8	27.00	18.67	110.00	94	147.33	105
L_9	29.50	20.33	120.00	88	160.67	98
L_{10}	32.00	22.00	130.00	83	174.00	93

TABLE III. TROUGHPUT PERFORMANCE ANALYSIS II.

	E(SA) sec.	E(TB) sec.	E(MC$_4$) sec.	τ(MC$_4$) items/h	E(MC$_6$) sec.	τ(MC$_6$) items/h
L_1	6.50	4.83	27.50	204	37.17	225
L_2	8.17	5.94	34.17	187	46.06	206
L_3	9.83	7.06	40.83	172	54.94	190
L_4	11.50	8.17	47.50	159	63.83	176
L_5	13.17	9.28	54.17	148	72.72	164
L_6	14.83	10.39	60.83	139	81.61	154
L_7	16.50	11.50	67.50	130	90.50	144
L_8	18.17	12.61	74.17	123	99.39	136
L_9	19.83	13.72	80.83	116	108.28	129
L_{10}	21.50	14.83	87.50	110	117.17	123

TABLE IV. TROUGHPUT PERFORMANCE ANALYSIS III.

	E(SA) sec.	E(TB) sec.	E(MC$_4$) sec.	τ(MC$_4$) items/h	E(MC$_6$) sec.	τ(MC$_6$) items/h
L_1	5.08	3.83	21.67	223	29.33	245
L_2	6.33	4.67	26.67	207	36.00	227
L_3	7.58	5.50	31.67	193	42.67	212
L_4	8.83	6.33	36.67	181	49.33	199
L_5	10.08	7.17	41.67	170	56.00	188
L_6	11.33	8.00	46.67	161	62.67	178
L_7	12.58	8.83	51.67	152	69.33	168
L_8	13.83	9.67	56.67	144	76.00	160
L_9	15.08	10.50	61.67	138	82.67	152
L_{10}	16.33	11.33	66.67	131	89.33	146

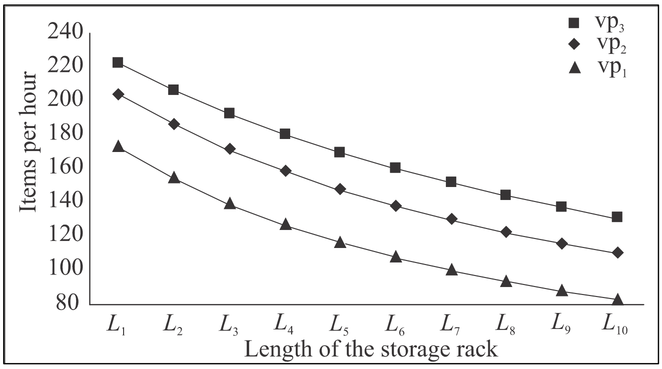

Fig. 6. Throughput performance analysis of the robotic order-picking shuttle carrier for visiting four (4) locations

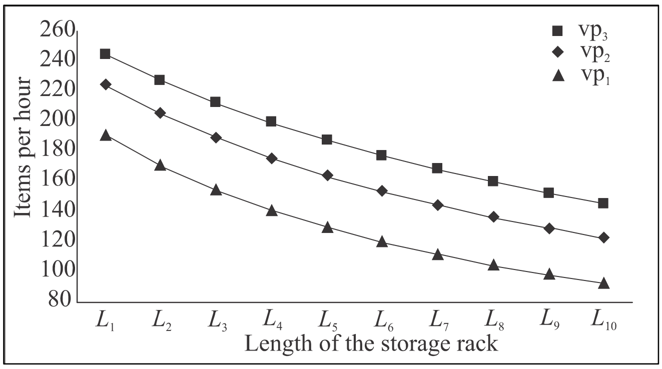

Fig. 7. Throughput performance analysis of the robotic order-picking shuttle carrier for visiting six (6) locations

Note: Throughput performance of the elevator's lifting table (lift) was not the case of this study.

The expected multi cycle travel times $E(MC_4)$ and $E(MC_6)$ along with the throughput performance $\tau(MC_4)$ and $\tau(MC_6)$ depend on the length L_i of the storage rack and the velocity profile vp_i of the robotic order-picking shuttle carrier (see Table I.).

The fastest transactions belong to the robotic order-picking shuttle carrier with fast drives (vp_3 and vp_2), meanwhile the slowest transactions belong to the robotic order-picking shuttle carrier with moderate drive (vp_1).

According to the distribution of $E(MC_4)$ and $E(MC_6)$, velocity profile vp_i has a significant impact on the expected cycle time (see Tables II., III. and IV.). An increasing tendency of $E(MC_4)$ and $E(MC_6)$ is observed for the velocity profile vp_1 and vp_2, compared to vp_3. This relationship shows the influence of the horizontal velocity v_x and acceleration a_x in accordance to the length L_i of the storage rack. Generally, the best results are achieved with the robotic order-picking shuttle carrier having fast drives in the horizontal travelling direction. Because the throughput capacity $\tau(MC_4)$ and $\tau(MC_6)$ is inversely dependent on the expected cycle time $E(MC_4)$ and $E(MC_6)$, the highest throughput capacity belongs to the robotic order-picking shuttle carrier with fast drives (vp_3). On the contrary, the lowest throughput capacity belongs to the robotic order-picking shuttle carrier with moderate drive (vp_1).

Although the expected travel (cycle) time for visiting six (6) locations $E(MC_6)$ is longer compared to the travel (cycle) time for visiting four (4) locations $E(MC_4)$, the throughput performance in case of $\tau(MC_6)$ will be higher, since we are able to pick six items in one run.

Therefore, the performance of the system will be highly influenced by the number of items to be collected and the velocity profile of the of the robotic order-picking shuttle carrier.

V. Conclusion

The aim of this research study is to present the analytical model that can estimate the throughput performance of the robotic order-picking shuttle carrier. The proposed model considers a shuttle carrier that is performing order-picking by utilizing the robotic arm. Travel-time model for multiple command cycles in the i^{th} tier of the storage rack has been determined by applying the probability theory. The sequences of (i) Acceleration, constant velocity and deceleration has been used in the proposed model. A randomized policy is considered which means that any order-picking location (tote with items) is equally likely to be selected for order-picking sequence to be processed. The proposed model allows the calculation of the expected cycle time for multiple command cycles, from which the performance of the robotic order-picking shuttle carrier can be evaluated. Various parameters were examined such as: Velocity (v_x), acceleration / deceleration (a_x), length (L_i) of the storage rack.

The proposed analytical model demonstrated good performances and satisfactory deviations and could be a very helpful tool for designing automated order-picking systems with robotic order-picking shuttle carriers. It could be of considerable help to professionals in practice, when making decisions in the early stages of design project and when deciding which type of the storage rack configuration or robotic order-picking shuttle carriers will be most promising.

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