

Investigating Mathematical Literacy through Teacher Language

Alyson Lischka
Kennesaw State University

Abstract

Communication about mathematical concepts using appropriate terminology is a standard established by the National Council of Teachers of Mathematics. However, international test results show that United States' students are lagging in mathematical literacy. This case study analyzes the ways in which instructors use language to help students move toward conceptual understanding of mathematical vocabulary. Three mathematics education professors at a mid-size four year institution were observed teaching math classes to students enrolled in elementary or secondary certification programs. Collected data included: audio-recorded observations and field notes, lesson artifacts such as quizzes and handouts, and audio-recorded interviews with each participant. Findings showed that instructors used extended classroom discourse, asked information-seeking questions, and modeled appropriate use of mathematical language to elicit understanding of mathematical language. Implications for future study include investigation into the methods of instructors at various levels of education and the differences in discourse related to language between high performing and low performing schools.

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Students are gathering in the classroom and there is much conversation as they get settled. Bags are shifting, folders are opening, and snacks are being eaten. One student moves to the front and says “I don’t think I really understand the term *relatively prime*.” The teacher does not explain it to her but instead asks more questions. “Tell me what you do think about it.” The student has an idea about numbers that are divisible by themselves and one, but she uses terms in a mixed up fashion. “Factor,” “multiple,” “divides,” and “divisible by” are jumbled in her explanation. The instructor turns to another student, who has been listening, and says “Can you help her out?”

How do we help students make sense of mathematical terminology? Mathematical terms are tightly tied to the concepts which they represent. Using language alone is not sufficient. This student uses all of the appropriate terms, but not in a way that makes mathematical sense. What can instructors do to help students develop a mathematical vocabulary that is conceptually based?

Purpose and Significance

Current comparisons between the United States (U.S.) and other countries show that U.S. students are lagging in mathematical literacy (Organisation for Economic Co-operation and Development, 2006). In order to change this standing, mathematics educators are moving toward a more conceptual framework for teaching mathematics. Rather than solely assessing procedural accuracy, educators are focusing on the broader picture of the concepts themselves (National Council of Teachers of Mathematics, 2009). Dewey (1933) described the idea of developing conceptual understanding in learning by stating:

To grasp the meaning of a thing, an event or situation is to see it in its relations to other things: to note how it operates or functions, what consequences follow from it, what causes it, what uses it can be put to (p. 137).

In order to accomplish this goal, language development should also be approached conceptually. The National Council of Teachers of Mathematics (NCTM) established a process standard that states students should be able to “use the language of mathematics to express mathematical ideas precisely” (NCTM, 2000, p. 348). In order for this standard to be met, mathematical language should be emphasized in mathematics classes since a large portion of mathematical language is used only in the study of mathematics. Not only should terminology be provided, but the understanding of those meanings must be connected to previous knowledge. Words become labels for concepts and concepts are the basis of understanding (Schunk, 2008).

Current discourse analysis research in mathematics classrooms highlights the importance of questioning techniques used by teachers including extending discourse through questioning and developing community discourse (Davis, 1997; Franke, Webb, Chan, Ing, Freund, & Battey, 2009; Hufferd-Ackles, Fuson & Sherin, 2004; Schleppenbach, Perry, Sims, Miller, & Fang, 2007). This study strives to connect the findings of discourse analysis to the development of mathematical vocabulary. If students are expected to communicate using mathematical language, instruction must provide pathways to do so.

This research focuses on the discourse of instructors as they help students build a conceptual understanding of mathematical language. Research questions include: How do teachers use mathematical language in their verbal instruction? What instructional tools do teachers use during lessons that focus student attention on conceptual understanding of mathematical vocabulary? How is questioning used to establish conceptual understanding?

Method

Volunteers for the study were solicited from the Department of Mathematics Education at a mid-size university. Participants were selected as a purposeful sample from the pool of volunteers to represent variety in stage of career. The participants selected in this study (Dr. Baker, Dr. Jones, and Dr. Smith) are three mathematics education professors with teaching experience ranging from seven to twenty-five years at the college level. Pseudonyms are being used so as to protect the identity of participants. Two participants (the most experienced and the least experienced) also taught at the high school level in previous employments. One participant is a black male and the other two are white females. All three participants are actively involved in the teacher preparation program at their university and hold strong beliefs about reform efforts in mathematics education.

Each participant was observed teaching mathematics education courses for either two or three class periods over a period of three weeks. The duration of a class period was either one hour and fifteen minutes or two hours and forty-five minutes. The first participant was observed teaching a course on geometry and measurement for elementary certification candidates. The second participant was observed teaching a mathematics course for secondary mathematics certification candidates. The course focused on thinking deeply about problem solving and mathematical topics in grades 6 – 10 curriculum. The third participant was observed teaching two different courses. The first course was on the history of mathematics and designed for secondary mathematics certification candidates. The second course was designed for elementary certification candidates and focused on rational numbers and proportional reasoning. The observations were audio-recorded so as to preserve the actual word choices and questions asked by the professors.

In addition to field notes generated with the observations, lesson documents such as tests, quizzes, and handouts were collected from each participant. One participant provided a philosophical statement to support the methods used in class, and another provided historical curriculum documents that illuminated points made in discussions.

Each participant also took part in a one on one audio-taped interview with the researcher. Discussion questions focused on the participants' views of the role of vocabulary in mathematics instruction and on general thoughts on teaching for conceptual understanding. Each interview was transcribed and returned to the participant for member checking.

Analysis

Data analysis was completed using the constant comparative method. All field notes, interview transcripts, and collected documents were coded openly for themes. Triangulation was achieved by combining class observations with interview transcripts and collected documents. Member checking was conducted throughout the analysis of the data (Bodgan & Biklen, 2007).

During the initial coding process it became evident that the types and amount of discourse centered on mathematical language varied based on the type of class being observed. Discourse in the observed classes for elementary education candidates contained much discussion focused on mathematical terminology, while classes for middle grades and secondary education candidates contained very little discussion of mathematical terminology. Theoretical differences in how these two different levels of classes are taught were discussed with the participants. These conversations revealed that the instructors of secondary certification classes assume that much language is already understood on a high level, while instructors of elementary certification classes do not make this assumption. In order to capture the practices of teachers who are building conceptual understanding of terminology, only the observations of classes in the elementary education program were included in the final stages of data analysis.

The two observed courses included in the analysis, therefore, were a geometry class taught by Dr. Baker and a class on rational numbers taught by Dr. Jones. Collected documents from those courses were included in the final analysis along with interviews from all three participants. The interview with the third participant was included in analysis since the interview focused on participants' views of the role of language in mathematics teaching rather than specifically on the observed courses. Coding themes of

extended mathematical discourse, information seeking questioning, and appropriate modeling of language use evolved through the iterative process of examining current literature on classroom discourse and collected data.

Results

Extended Mathematical Discourse

Research points to the importance of extended conversations about mathematical topics in developing conceptual understanding of mathematical ideas. Sustained student conversation about ideas and terminology is essential for discourse to evolve into discussions that promote conceptual understanding (Franke et al., 2009; Hufferd-Ackles et al., 2004; Schleppenback et al., 2007). Extended discourse was one feature that was evident in the classroom conversations of both Dr. Baker and Dr. Jones.

Prior to the start of Dr. Jones' class one day, a student approached Dr. Jones and explained that she did not think she completely understood the term *relatively prime*. Dr. Jones asked her a few questions prior to class and the student took her seat while the rest of the class was gathering. Dr. Jones then started class asking the student to restate her question; the conversation that occurred follows:

- Student: I was thinking of relatively prime as a number that goes into itself and can be multiplied by one other number. But then another student explained to me that it can be multiplied by itself but then it can only be multiplied by one other number and that is one.
- Dr. Jones: What do you mean it can be multiplied by itself? Can you give me an example of two numbers that are relatively prime?
- Student: 24 and 25.
- Dr. Jones: Help me out class – what did she say about multiplying?
- Student: I said it could be multiplied by one.
- Dr. Jones: Well, can't every number be multiplied by one?
- Student: I mean one is the common number between them.
- Dr. Jones: What do you mean when you say one is the common number between them? What does that common number do?
- Student: It's a multiple.
- Dr. Jones: Is it a multiple?
- Class: It is a factor.
- Dr. Jones: Is it a factor or is it a multiple? I think this is where we are getting confused. What is a factor of a number?

This conversation continued for the first ten minutes of class during which time Dr. Jones led the class through a discussion that included the meanings of factor, multiple, composite, prime, and relatively prime. Students were directed to give examples along with verbally defining the terms and refining each others' definitions. At the end of the conversation a student explained that relatively prime means that, while the two numbers do not have to be prime themselves, they can only have a common factor of one.

Dr. Baker's instructional model is one that relies heavily on group interactions and student presentations. Groups prepare to present assigned problems and then must be able to defend solutions when questioned by Dr. Baker. Within this model, Dr. Baker creates extended conversations about the presented topics and terminology. During the preparation portion of the class, Dr. Baker rotates between groups and assists the groups. One group asked Dr. Baker for help on a problem dealing with perimeter. Dr. Baker replied by asking the student to state the meaning of perimeter. The student stated that perimeter was found by adding all of the measurements of the object. Dr. Baker then asked the student to explain how to find the perimeter of a table and then a bookcase. This finally led to a refinement of the student's original definition of perimeter. Throughout the discussion, the students in the group were all contributing thoughts and arriving at a shared definition of the term.

During the class presentation portion of Dr. Baker's class, he frequently questioned student explanations, extending the conversation beyond a simple statement of mathematical terms. For example, when a student presented a problem involving parallel lines crossed by a transversal, Dr. Baker stopped the explanation and said:

- Dr. Baker: Tell me what that means.
 Student: That R and S are parallel?
 Dr. Baker: Yes. Explain what that means to me.
 Student: R and S parallel means that they will never meet.
 Dr. Baker: What is the other line?
 Student: The other line is a transversal and that means that it crosses the two parallel lines, R and S. When it crosses, it forms all of the angles that we are talking about and they have certain relationships.

Both Dr. Baker and Dr. Jones continually refused to accept brief statements pertaining to mathematical vocabulary. When students used mathematical terms, they were required to explain the meanings of those terms and classmates were encouraged to challenge their ideas until a common understanding was achieved.

Information-Seeking Questioning

Another feature of the discourse observed was information-seeking questioning. Relatively few response-seeking questions were asked in which a one word answer would suffice (Davis, 1997). Instead, the professors used information-seeking questions about vocabulary which required students to explain their understanding of the meanings of terms, often providing definitions in their own words. This continuous questioning on the meaning of terms supports students' building of conceptual understanding and frequently directed the next step in the class lesson (Davis, 1997; Franke et al., 2009; Hufferd-Ackles et al., 2004).

During Dr. Jones' discussion of the meaning of relatively prime, she asked multiple information-seeking questions to encourage students to arrive at a conceptual understanding of the terms. Questions included: What do factors do for a number?, What does it mean for a set to be infinite?, What do you mean by multiplied by itself?, What is the difference between a factor and a multiple?, and When do we need to use greatest common factors? Each student response led into another question to get students closer to conceptual understanding.

In a later observation, Dr. Jones was working to build understanding of the use of benchmarks in estimating with fractions. Rather than defining the term at the beginning, Dr. Jones led a discussion on estimating that was driven by a variety of information-seeking questions. Why do we estimate? How do you decide what a number is close to? Is $\frac{3}{8}$ closer to zero or to one? How do you know? After a lengthy discussion, Dr. Jones finally arrived at the question "What is a benchmark?" Students then constructed their own definition based on the previous discussion and decided that a benchmark is a landmark on the number line for fractions.

Dr. Baker's most common way of questioning for meaning with information-seeking questions was to ask students to explain a term as if the listener did not know anything about geometry. A student explaining the meaning of the term *median of a triangle* was led through the following conversation:

- Student: A median is the segment connecting the midpoint of a side to the opposite vertex.
 Dr. Baker: Say I am a UPS delivery person and I walk in off the street and need to know this. Can you explain what this is without the math jargon?
 Student: The vertex –
 Dr. Baker: That is math jargon. I don't understand that.
 Student: The angle . . . The corner of the shape. You connect the corner of the shape to the middle of the side across from it.

Dr. Baker's insistence on explaining in students' own words, forced students to make sense of the meaning rather than spouting back a memorized definition. All three participants commented in interviews about the importance of students constructing definitions for terms that were usable and made sense. Dr. Smith commented that, while mathematical precision is important, "you need to be able to use a definition for what you need to use it for at that moment." Dr. Jones said "I don't care if they memorize a definition . . . as long as you can describe what it is in clear terms." The observed characteristic of information-seeking questioning leads students to making their own sense of meanings of mathematical terminology, building toward conceptual understanding.

Modeling Appropriate Use of Language

A final theme that emerged from observation is accurate modeling of appropriate use and examples of mathematical language. Greenwood (2002) and Burns (2006) support appropriate modeling of mathematical language as essential to building student understanding. Observations showed instructors using language appropriately, modeling correct pronunciation, and providing appropriate examples or eliciting examples from students.

While Dr. Baker does not focus on the use of mathematical jargon at the beginning of a topic, appropriate use of terms was modeled throughout lessons. During a discussion of relationships between two triangles, Dr. Baker used appropriate terminology by asking "How do you know that the two triangles are congruent?" rather than referring to the triangles as being the same. Discussions about transversals crossing parallel lines included appropriate modeling of the terms interior, exterior, alternate, corresponding, and vertical. Though Dr. Baker did not insist on student use of appropriate terminology from the beginning, he did use terminology appropriately himself.

Dr. Jones was more purposeful in her modeling of appropriate language, pointing to a school system's curriculum document from 1988 that required kindergarten students to use the term *sphere*. Dr. Jones explained that finding that statement caused reform in the language that students in mathematics education classes were allowed to use. In geometry, the word *ball* could no longer be used to replace the word *sphere*.

In observations, Dr. Jones continually required students to use appropriate terminology. While a student was explaining a problem involving equivalent fractions, the following conversation took place.

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| Student: | If you take the one-eighth and put them on the one-half, you can see that Four-eighths match up with one-half. |
| Dr. Jones: | What does that show? |
| Student: | It shows that four-eighths is equal to one-half. |
| Dr. Jones: | That they are equivalent. |
| Student: | That four-eighths and one-half are equivalent fractions. |

Dr. Jones not only required students to use appropriate words, but also insisted on appropriate pronunciation of fractions and decimals. The fraction " $\frac{3}{8}$ " was not read as "three over eight" but pronounced "three-eighths." Students were not allowed to read the number "1.25" as "one point two five" but instead were required to say "one and twenty-five hundredths." This insistence on appropriate use of terminology may help students build toward conceptual understanding. The phrase "three-eighths" indicates that one has three of the pieces that are one-eighth of a whole. The phrase "one and twenty-five hundredths" indicates that one has one whole and 25 pieces that are one-hundredths. Students in this course were grappling with fractions and the pronunciation can support the development of meaning.

Dr. Baker and Dr. Jones both supported modeling of appropriate language with examples that furthered conceptual understanding. Dr. Baker directed the group struggling with the perimeter problem to consider the perimeter of various shapes until they arrived at the consensus that perimeter only involved the measures around the outside of a shape. In a discussion involving congruent right triangles, Dr. Baker directs the group to draw two right triangles and then attempts to lead them to understanding by

discussing their example. Dr. Jones frequently asked for examples of terms once they had been defined by students. After a student described multiples as *skip counting*, Dr. Jones followed by requesting a list of the multiples of 12. Dr. Jones provided an example to explain the term *common denominator* by stating “You need a piece that is going to fit into both of those fractions. Something that is going to fit into thirds and fourths at the same time. That piece would be twelfths.” Examples, provided by instructors and solicited from students, served to solidify the appropriate use of terminology and aid students in further building conceptual understanding.

Discussion

This research indicates that instructors apply established discourse practices to the teaching and learning of mathematical vocabulary. The observed instructors encouraged extended conversation about mathematical terminology, questioned students in ways that sought meaning of terms and directed classroom discourse, and modeled appropriate use of mathematical language. This study can serve as a preliminary investigation to fuel further research into the teaching and learning of mathematical language.

Limitations

This study does highlight important ideas in mathematics instruction. However, it may be limited by the setting. The students in the observed classes were primarily white females therefore only representing a portion of the population of pre-service teachers. Another limitation arose during analysis due to the variation in types of classes observed. Selection of participants and classes should have been focused on one particular type of mathematics course.

Implications

There are a variety of implications that arise from this study. The participants commented that they were not aware of statements they had made throughout the observations. This supports the idea that language in a classroom is organic and emerges as a part of classroom discourse. Making language more visible to classroom teachers can help them to redirect their use of language into more effective ways of building conceptual understanding. Acknowledging that use of language is often rooted in culture, bringing awareness to this aspect of classroom instruction can improve learning for all.

Educational leaders and policy makers should encourage teachers to investigate this aspect of classroom instruction and assist them in doing so. Cultural dialogue about use of language should be encouraged and incorporated in professional development that is focused on improving teaching practice. As teachers continue to work with required standardized testing, it is necessary to align language practices in the classroom with the language of these assessments.

This preliminary investigation into mathematics instructors' use of mathematical language reveals many questions for future study. Differences were noted in observations of the two levels of teacher preparation (elementary and middle grades / secondary). Is there variation in the use of language at different levels of education, such as elementary, middle school, and secondary? If there are differences, are transitions made between the levels? In terms of culture, do distinctions exist in the ways various cultures make sense of mathematical vocabulary? If so, how can we take this into account and differentiate to make mathematics accessible to all learners? Finally, do differences relate to culture only, or is socioeconomic level involved? Further investigation into the relationship between language and conceptual understanding can be a tool for improving teaching practice.

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