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Marc Goetschalckx
Georgia Institute of Technology

Edward Huang
Georgia Institute of Technology

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Marc Goetschalckx
School of Industrial and Systems Engineering, Georgia Institute of Technology
Edward Huang
School of Industrial and Systems Engineering, Georgia Institute of Technology

Abstract

The strategic design of a robust supply chain has as goal the configuration of the supply chain structure so that the performance of the supply chain remains of a consistently high quality for all possible future scenarios. We model this goal with an objective function that trades off the central tendency of the supply chain profit with the dispersion of the profit as measured by the standard deviation for any value of the weights assigned to the two components. However, the standard deviation, used as the dispersion penalty for profit maximization, has a square root expression which makes standard maximization algorithms non applicable. The focus in this article is on the development of the strategic and tactical models. The application of the methodology to an industrial case will be reported. The optimization algorithm and detailed numerical experiments will be described in future research.

1 Introduction

The design and planning of an efficient supply chain is of critical importance for the competitive success of manufacturing corporations. Many definitions for a supply chain have been proposed. One of the early definitions is *"A supply chain is a network of organizations that are involved through upstream and downstream linkages in the different processes and activities that produce value in the form of products and services in the hands of the ultimate customer,"* Christopher (1998).

The planning decisions with respect to a supply chain range from short-term decisions such as vehicle dispatching and routing to long-term decisions such as the definition of the corporate mission. Depending on the permanence of the decisions, they are typically divided into strategic, tactical, and operational planning. At the strategic level the planning includes decisions with respect to the location and the capacity of production and distribution facilities and the selection of suppliers. At the tactical level the planning decisions include the product flows and product storage throughout the supply chain. The goal of the strategic planning is to determine the configuration of the supply chain so that its long-term performance over the planning horizon is maximized. Given the long permanence of the configuration decisions, the future conditions during the planning horizon cannot be known with certainty. Configuring a supply chain that will perform efficiently in a variety of unknown future environments belongs to the decision problem class of strategic planning under uncertainty and the supply chain configuration itself is called a robust design.

Multiple definitions of robust design exist in the literature. In the area of strategic planning under uncertainty, related notions such as agility, adaptability, responsiveness, resilience, and flexibility also have been used. A recent survey of strategic supply chain planning and robust design is given in Klibi et al. (2010). One distinction they make with respect to robust design is between model, algorithm, and solution robustness. A design is defined as "model robust" if this design is "almost" feasible when the input data varies (Mulvey et al., 1995; Yu and Li, 2000; Leung and Wu, 2004; Aghezzaf, 2005). A design is called "algorithm robust" if the algorithm performance is not affected by the presence of noise in the data. A design is called solution robust if the solution remains "close" when the input data changes (Mulvey et al., 1995; Yu and Li, 2000; Leung and Wu, 2004; Aghezzaf, 2005). Solution robustness corresponds to the desired robust supply chain configuration and is the focus of this research. Solution robustness itself has two components. Solution configuration robustness requires that the same (robust) supply chain configuration is used in the different scenarios and is required by the nonanticipativity property; solution value robustness measures the variability of the objective function value (profit) over the different scenarios.

Several modeling techniques have been used in design under uncertainty. In many stochastic programming models the uncertainty is assumed to be known and modeled as a set of scenarios with known probabilities. For the strategic supply chain design a two phased model is used, where the supply chain configuration is decided in the first stage and the material flows and inventories are treated as recourse variables in the second stage. The objective is to maximize the expected value of the profit of all scenarios, i.e. the goal is to find a median type of solution. Ahmed and Sahinidis (2003) solve a stochastic capacity expansion problem with a given set of scenarios. Santoso et al. (2005) propose the use of a random sampling strategy, the sample average approximation scheme, to solve large-scale stochastic supply chain design problems. These approaches focus on the solution configuration robustness and do not explicitly consider the robustness of the solution value, which may vary widely between the different scenarios.

One approach in the field of robust optimization when the probabilities of the scenarios are not available is to either minimize the maximum regret or to maximize the minimum profit over all possible scenarios (Atamtürk and Zhang, 2007). If a particularly bad scenario exists, even though it has a very low probability of occurrence, then this scenario may determine the supply chain configuration. The goal in this case is to find a center type of solution. Another approach assumes that the probabilities of scenarios are known and considers the tradeoff between expect value, model robustness, and solution robustness. The value robustness is evaluated either with its variance (Mulvey et al., 1995) or absolute deviation (Yu and Li, 2000; Leung and Wu, 2004).

In the strategic design of supply chains there may be many thousands of parameters whose value is not known with certainty at the decision time. Even if the probability distributions of the individual parameters were known, constructing a joint probability distribution function for the scenarios in function of the parameters is not possible. In the following approach the uncertainty of the future is modeled by means of scenarios whose probabilities are assumed to be known. Scenarios may be grouped in classes or sets, such as best-guess, best case, and worst case scenarios, or represent high-impact, low-probability events.

The extended definition of solution robustness will be used in the following which considers simultaneously solution configuration and solution value robustness, i.e., the supply chain configuration has to remain unchanged over all scenarios and the variability of the solution value is penalized. Specifically, the objective is to find Pareto-optimal configurations with respect to their expected value and standard deviation of the scenario profits. The configurations can be plotted in a risk analysis graph with the expected value on the horizontal axis and the standard deviation on the vertical axis. The use of the standard deviation instead of the variance of the scenario profits allows for the computation of the coefficient of variation of the solution value. This coefficient is dimensionless which yields a more intuitive comparison between configurations and avoids dependencies on the units of the profit.

Figure 1 shows an example of a mean-standard deviation risk analysis graph. For this particular example, all possible supply chain configurations were evaluated and three Pareto-optimal configurations exist. The stochastic programming approach will find the configuration with maximum expected profit value which is generated by the configuration (110110) and shown as a triangle. If the decision maker is extremely risk-seeking, this configuration will be selected. The robust optimization approach using the min-max regret objective will select the configuration with minimum standard deviation which is the configuration (010011) and is shown as a diamond. If the decision maker is extremely risk-averse then this configuration will be selected. Neither approach will identify the configuration (011011) which is shown as a square even though this configuration is also Pareto-optimal. The methodology developed will identify all Pareto-optimal configurations and compute for which range of the coefficient of variation each of them is dominant. A particular configuration may be dominant for a large fraction of the range of the coefficient of variation but be different from the configurations found by stochastic optimization and robust optimization. The final selection of the supply chain configuration to be implemented can then be based on the risk tradeoff of the decision maker and on other considerations not included in the model, but no a priori tradeoff ratio or weight is required.

2 Model

In this section the mathematical formulations are developed that correspond to the robust design problem as defined in the previous section. The common notation is developed in the next section. The strategic model for the first stage and the tactical model for the second stage are described in the sections 2.2 and 2.3.

2.1 Notation

Because supply chains have many components the corresponding planning models have many variables and constraints. However, their structure is relatively simple, even though the replication of the components may yield very large problem instances.

2.1.1 Components

The logistics objects in the tactical supply chain model are collected in the following sets.

SF	Suppliers, indexed by *i*
P	Products, indexed by *p* (and *v*)
CF	Customers, indexed by *k*
T	Periods, indexed by *t* (and *u*)
TF	transformation facilities or transformers, indexed by *j*
R	Resources required for product flows in supplier and transformation facilities, indexed by *r*
TR, AR*, *IR	Resources required for product throughput (*TR*), assembly (*AR*) and product inventory (*IR*) in transformation facilities, respectively. These are sub sets of the resource set *R*.
$O = SF \cup TF$	Origin facilities, i.e. suppliers and transformation facilities
$D = TF \cup CF$	Destination facilities, i.e. transformation facilities and customers
OD	Transportation channels, indexed by the combination of their origin and destination facilities
S	Scenarios, index by *s*

2.1.1 Decision Variables

The symbols for most decision variables related to material flows end on the letter *q* which indicates a quantity.

pq_{ipt}	amount purchased from supplier i of product *p* during period *t*
x_{ijpt}	amount of product *p* transported from facility *i* to facility *j* during time period *t*
itq_{jpt}, otq_{jpt}	amount of product *p* respectively transported into and out of facility *j* during time period *t*
iq_{jpt}	amount of product *p* stored (carried as inventory to the next period) at facility *j* from time period *t* to time period *t+1*
bq_{kptu}	amount of product *p* delivered to customer *k* during period *t* that is used to satisfy the demand of this customer for this product during time period *u*, where *u* is smaller than *t*. This is the backorder quantity.
aq_{jpt}	amount of product *p* assembled, i.e. manufactured or produced, at facility *j* during time period *t*
cq_{jpt}	amount of component product p used in assembly (manufacturing) at facility *j* during time period *t*
dq_{kpt}	amount of product *p* delivered to customer *k* during period *t* to satisfy the demand during this period and possible backordered quantities of prior periods. The presence of backordering allows the quantities delivered to be different from customer demand for a particular period
y_j	binary status of transformation facility *j* indicating if the facility is established (open), i.e. part of the supply chain configuration, or not
p_i	probability that scenario *i* will occur
z_s	maximum profit achievable through tactical planning in scenario *s* and without the profit ceiling constraint. This will also be called the unconstrained scenario profit.
v	ceiling for the profits in all scenarios

$zc_s(v)$ maximum profit achievable through tactical planning in scenario s when subject to the profit ceiling constraint equal to v. This will also be called the ceiling-constrained scenario profit

SR total sales revenue for a scenario. Used in a the tactical model for an individual scenario so scenario subscript is required.

TC total system cost for a scenario. As above used for a specific scenario in the tactical model.

2.1.1 Parameters

The symbols for most unit cost parameters end with the letter (lower case) c which indicates the cost rate. Parameters related to capacities on flow start with the letter t, while capacities related to production and inventory start with the letters a and i, respectively. The latter two are only defined at transformation facilities.

$tcap_{jrt}$ aggregate capacity of throughput resource r at supplier or at transformation facility j during period t for all products combined. Note if the capacity is by product and supplier or facility capacity, cap_{jpt} with three subscripts is to be defined and is no longer called aggregate.

$acap_{jrt}$ aggregate capacity of resource r at transformation facility j during period t for all products combined produced. Note if the capacity is by product and transformation facility capacities, $acap_{jrt}$ with three subscripts is to be defined and is no longer called aggregate.

$icap_{jrt}$ aggregate capacity of resource r at transformation facility j during period t for all products combined held in inventory, respectively. Note if the capacity is by product and transformation facility capacities, $icap_{jrt}$ with three subscripts is to be defined and is no longer called aggregate.

$tres_{jprt}$ units of resource r consumed by one unit of product p at facility j (be it either a supplier or transformer facility) during period t. The model can incorporate resource consumption rates that vary by period, e.g. to approximate learning curves.

$ares_{jprt}$ units of resource r consumed by one unit of product p produced at transformation facility j during period t. The model can incorporate resource consumption rates that vary by period, e.g. to approximate learning curves.

$ires_{jprt}$ — units of resource r consumed by one unit of product p stored at transformation facility j during period t. The model can incorporate resource consumption rates that vary by period, e.g. to approximate learning curves.

trc_{jrt} — unit resource cost for resource r at facility j during period t

arc_{jrt} — unit resource cost for production resource r at transformation facility j during period t

irc_{jrt} — unit resource cost for inventory resource r at transformation facility j during period t

dem_{kpt} — aggregate demand for product p at customer k during period t

pc_{ipt} — purchase cost for a unit of product p from supplier i during period t

tc_{ijpt} — transportation cost for a unit of product p from facility i to facility j during period t

fc_{jpt} — flow (throughput) cost for a unit of product p at facility j during period t

ac_{jpt} — assembly (production, manufacturing) cost for a unit of product p at facility j during period t

hc_{jpt} — holding (inventory) cost for a unit of product p at facility j from time period t to the next period $t+1$

bc_{kptu} — delay cost, i.e. delay penalty or backorder cost, for delivering one unit of product p during period t to satisfy demand during period u at customer k

$1bom_{jpvt}$ — number of units of component p required to assemble one unit of assembly v during period t in facility j where component p is an element of the single level bill of material of product v

$init\_inv_{jp}$ — initial inventory of product p at facility j

fac_{jt}	fixed cost for having transformation facility *j* established during period *t*. A facility is established for the full time horizon or not, but it may have different fixed costs during the different periods in the horizon
sr_{kpt}	sales revenue for selling on unit of product *p* at customer *k* during period *t*. This revenue does not incorporate any backorder penalty.

2.2 Strategic Model

The strategic model has as goal to identify the supply chain configuration and the profit ceiling that will maximize the weighted sum of the expected value and the standard deviation of the constrained scenario profits.

The expected value, variance, and standard deviation the for the scenario value of $zc_s(v)$ are then defined as the following functions.

$$zc_s(v) = \min\{z_s, v\} \tag{1}$$

$$exp_{zc}(v) = \sum_i p_i \cdot zc_i(v) = \sum_i p_i \cdot \min\{z_i, v\} \tag{2}$$

$$var_{zc}(v) = \sum_i p_i \cdot \left(zc_i(v) - \sum_j p_j \cdot zc_j(v) \right)^2 \tag{3}$$

$$std_{zc}(v) = \sqrt{\sum_i p_i \cdot \left(zc_i(v) - \sum_j p_j \cdot zc_j(v) \right)^2} \tag{4}$$

The robust objective functions *MVO* and *MSDO* are defined in function of the scenario profits z_j. In both cases the central tendency characteristic is equal to the expected value. In the *MVO* and the *MSDO* the dispersion is equal to the variance and the standard deviation of the scenario profits, respectively. The penalty factors are λ and κ, for the *MVO* and *MSDO* respectively, are both nonnegative. Both objectives belong to the class of bi-criteria objective functions.

$$MVO = \max\left\{exp\left[Z\right] - \lambda \cdot var\left[Z\right]\right\}$$
$$= \max\left\{\sum_i p_i\left(zc_i - \sum_j p_j zc_j\right) - \lambda\left(\sum_i p_i\left(zc_i - \sum_j p_j zc_j\right)^2\right)\right\} \tag{5}$$

$$MSDO = \max\left\{exp\left[Z\right] - \kappa \cdot std\left[Z\right]\right\}$$
$$= \max\left\{\sum_i p_i\left(zc_i - \sum_j p_j zc_j\right) - \kappa\left(\sqrt{\sum_i p_i\left(zc_i - \sum_j p_j zc_j\right)^2}\right)\right\} \tag{6}$$

A single supply chain configuration can thus be plotted in the mean-variance or mean-standard deviation graph. In the following the mean will be plotted by increasing values along the horizontal axis and the variance and standard deviation will be plotted by increasing values along the vertical axis. A point in the graphs can then be Pareto-optimal or dominant. Points which are not Pareto-optimal are said to be dominated. For different values of the penalty factors are λ and κ different configurations may become optimal.

Theorem:
The set of configurations that are Pareto-optimal with respect to the *MSDO* is identical to the set of configurations that are Pareto-optimal with respect to the *MSVO*.
The proof is omitted for brevity.

The above theorem allows the algorithm to search for all Pareto-optimal points for the MVO objective and then use the same points for the MSDO objective. This removes the square root from the objective function but introduces square terms in the objective. The strategic problem thus belongs to the class of mixed-integer quadratic objective or MIQO problems. This type of problems can be solved by CPLEX version 11 or newer.

Observe that the master problem has no constraints besides defining that the decision variables y_j have to be binary. Linear constraints in the decision variables can be added to impose additional restrictions on the supply chain configuration without changing the fundamental structure of the problem. A common example is an upper bound on the total number of established facilities.

Model 1. Strategic Robust Supply Chain Model

$$\max \quad \left\{ \sum_i p_i \left(zc_i(v) - \sum_j p_j \cdot zc_j(v) \right) - \lambda \left(\sum_i p_i \left(zc_i(v) - \sum_j p_j \cdot zc_j(v) \right)^2 \right) \right\} \tag{7}$$

$$s.t. \quad y_j \in \{0,1\} \tag{8}$$

$$v \geq 0 \tag{9}$$

The profit associated for a given supply chain configuration and for a specific scenario is maximized by the tactical model. In addition, the strategic model specifies a profit ceiling for the tactical scenario profit. Depending on this profit ceiling the tactical model will yield a different profit. The expected value and variance of all the profits will also change. A single configuration will have a continuous curve of performances in the risk analysis graph. The strategic model will then select the configurations whose curve dominates the curves of the other configurations in a least one range of the profit ceiling. Graphically this is equivalent to determining the lower-right envelope of the performance curves of the configurations. The tactical model is shown in the next section.

2.3 Tactical Model

A model is developed to support the tactical planning of the supply chain, including such decisions as supplier selection for the key components, transportation, and production planning. The model maximizes the total profit which is the difference between the sales revenue and the total cost. The total cost is computed as the sum of the purchasing (procurement), transportation, manufacturing, inventory, and backorder costs. The total demand of a customer has to be delivered, even though delivery may be delayed beyond the due date through backorders. The inventory cost at this time consists only of the holding costs at transformation facilities. The model incorporates a penalty for delayed delivery to customers, which is also denoted as the backorder cost. This model ignores the lead times for sourcing components of the various suppliers but it observes supplier capacities and transformation (manufacturing) capacities.

2.3.1 Constraints

The model contains four types of constraints: supply capacity, transformation (production or assembly) capacity, demand satisfaction, and conservation of flow at the transformation facilities. The conservation of flow constraints may be by product or consider the bill of materials or BOM for the assembly process.

2.3.2 Model

The complete tactical production supply chain model is given next. The model can be further condensed by directly substituting variables, but it is given below in its more expanded form to clearer show its structure. Modern linear programming solvers will make the substitutions in their pre-solving phase, so this more expansive version does not increase solution time significantly.

Model 2. Tactical BOM Supply Chain Model

$$\max \quad zc \tag{10}$$

$$s.t. \quad zc = SR - TC$$

$$zc \le v \tag{11}$$

$$\begin{aligned} TC = \; & \sum_j y_j \left(\sum_t fac_{jt} \right) \\ & \sum_{i \in S} \sum_p \sum_t pc_{ipt} \cdot pq_{ipt} + \sum_{i \in O} \sum_{j \in D} \sum_p \sum_t tc_{ijpt} \cdot x_{ijpt} + \\ & \sum_{j \in TF} \sum_p \sum_t fc_{jpt} \cdot otq_{jpt} + \sum_{j \in TF} \sum_p \sum_{r \in TR} \sum_t trc_{jrt} \cdot tres_{jprt} \cdot otq_{jpt} + \\ & \sum_{j \in TF} \sum_p \sum_t ac_{jpt} \cdot aq_{jpt} + \sum_{j \in TF} \sum_p \sum_{r \in AR} \sum_t arc_{jrt} \cdot ares_{jprt} \cdot aq_{jpt} + \\ & \sum_{j \in TF} \sum_p \sum_t hc_{jpt} \cdot iq_{jpt} + \sum_{j \in TF} \sum_p \sum_{r \in IR} \sum_t irc_{jrt} \cdot ires_{jprt} \cdot iq_{jpt} + \\ & \sum_{k \in C} \sum_p \sum_{t \in T, t \ge 2} \sum_{u \in T, u < t} bc_{kptu} \cdot bq_{kptu} \end{aligned} \tag{12}$$

$$SR = \sum_k \sum_p \sum_t sr_{kpt} dq_{kpt} \tag{13}$$

$$\sum_p tres_{iprt} \cdot pq_{ipt} \le tcap_{irt} \qquad \forall i, \forall t, \forall r \tag{14}$$

$$pq_{ipt} \le tcap_{ipt} \qquad \forall i, \forall p, \forall t \tag{15}$$

$$pq_{ipt} = \sum_j x_{ijpt} \qquad \forall i, \forall p, \forall t \tag{16}$$

$$\sum_i x_{ijpt} = itq_{jpt} \qquad \forall j, \forall p, \forall t \tag{17}$$

$$itq_{jpt} + aq_{jpt} + init\_inv_{jp} - iq_{jpt} - cq_{jpt} - otq_{jpt} = 0 \qquad \forall j, \forall p, t = 1 \tag{18}$$

$$itq_{jpt} + aq_{jpt} + iq_{jpt-1} - iq_{jpt} - cq_{jpt} - otq_{jpt} = 0 \qquad \forall j, \forall p, t = 2..T-1 \qquad (19)$$

$$itq_{jpt} + aq_{jpt} + iq_{jpt-1} - cq_{jpt} - otq_{jpt} = 0 \qquad \forall j, \forall p, t = T \qquad (20)$$

$$\sum_k x_{jkpt} = otq_{jpt} \qquad \forall j, \forall p, \forall t \qquad (21)$$

$$cq_{jpt} = \sum_v 1bom_{jpvt} \cdot aq_{jvt} \qquad \forall p, \forall v, \forall j, \forall t \qquad (22)$$

$$\sum_p ares_{jprt} \cdot aq_{jpt} \le acap_{jrt} \cdot y_j \qquad \forall j, \forall t, \forall r \qquad (23)$$

$$aq_{jpt} \le acap_{jpt} \cdot y_j \qquad \forall j, \forall p, \forall t \qquad (24)$$

$$\sum_p tres_{jprt} \cdot otq_{jpt} \le tcap_{jrt} \cdot y_j \qquad \forall j, \forall t, \forall r \qquad (25)$$

$$otq_{jpt} \le tcap_{jpt} \cdot y_j \qquad \forall j, \forall p, \forall t \qquad (26)$$

$$\sum_p ires_{jprt} \cdot iq_{jpt} \le icap_{jrt} \cdot y_j \qquad \forall j, \forall t, \forall r \qquad (27)$$

$$iq_{jpt} \le icap_{jpt} \cdot y_j \qquad \forall j, \forall p, \forall t \qquad (28)$$

$$\sum_j x_{jkpt} = dq_{kpt} \qquad \forall k, \forall p, \forall t \qquad (29)$$

$$dq_{kpt} + \sum_{t<u} bq_{kput} = \sum_{u<t} bq_{kptu} + dem_{kpt} \qquad \forall k, \forall p, \forall t \qquad (30)$$

$$pq, x, bq, iq, aq, cq \ge 0 \qquad (31)$$

The objective function computes the total cost as the sum of the individual unit costs multiplied by the corresponding quantities. The model has capacity constraints and conservation of flow constraints. Typically capacity limitations at suppliers are either for individual products or for all products combined. The model allows both simultaneously but usually either constraint (14), which models the joint capacity, or (15), which models the capacity for an individual product, are defined but not both. The equivalent is true for transformation capacities modeled by constraints (23), which models the joint capacity, or (24), which models the capacity for an individual product as well as for the throughput and inventory capacity constraints at the transformation facilities.

The remaining constraints are all conservation of flow constraints. Backorder flows can only occur at customers, inventory flows can only occur at transformation facilities. There are four types of conservation of flow constraints at the transformation facilities, indicated by space, space-time, creation-space, and creation-space-time, respectively. The flow diagrams for the four types are shown in the next figures.

In its most general form, the conservation of flow constraint for a product in a period for a transformation facility has six flows. The three input flows are transportation receipts, inventory held from the previous period, and production during the period. The three output flows are transportation shipments, inventory held to the next period, and consumption of the product during the period when it is used as a component in the production process. The most general form has been used in the tactical model. The type of conservation of flow constraint used can be adjusted based on the requirements of the particular supply chain in question. If the time dimension is present, three variants of the conservation flow constraint need to be created since the equation is different for the first, intermediate, and last periods of the planning horizon. During the first period there is only the initial inventory which is a parameter and during the last period there is no inventory held to the next period.

Constraints (16) and (29) ensure that all the products purchased get transported from the suppliers and all finished goods produced get transported to the customers, respectively. Constraints (18) through (20) ensure the conservation of flow for a transformation facility for the first, intermediate, and last periods, respectively. The model uses a parameter for the initial inventory of a product at a facility. Constraint (22) ensures that the correct amount of component products is consumed in the assembly facility to be assembled into finished goods. Finally, constraint (30) ensures that the goods delivered to a customer and backorders from future periods are allocated to satisfy either the demand of that period or satisfy backorders in previous periods.

3 Numerical Examples

3.1 Small Example

The model was applied to a small pedagogical example. The supply has two echelons. The first echelon contains 3 manufacturing plants and the second echelon contains 3 distribution centers. There are 3 customers. The robust design is based on 3 scenarios.

The MIQO optimization problem was solved by CPLEX within 20 seconds. The mean-standard deviation risk analysis graph is shown in Figure 1. Three Pareto-optimal configurations are found. Two of the Pareto-optimal configurations are the stochastic optimization (SO) configuration with a standard deviation penalty equal to zero and the robust optimization (RO) configuration with a large positive penalty for the standard deviation. But there exist a third dominant configuration with achieves 98% of the maximum expected profit of the SO configuration but at 76% of the risk as measured by the standard deviation.

3.2 Industrial Example

The model was also applied to an industrial case. Elkem is a global manufacturer of specialty additives in the metallurgic industry. The company and its supply chain are described in Ulstein (2006), but the case data used in this example are different. The supply chain has 10 products, 35 customers, 15 suppliers, 9 transformation facilities, and one period. The robust design is based on 30 scenarios.

The MIQO optimization problem was solved by CPLEX within 1200 seconds. The mean-standard deviation risk analysis graph is shown in Figure 5. Only two configurations are Pareto-optimal, namely the stochastic optimization (SO) configuration with a standard deviation penalty equal to zero and the robust optimization (RO) configuration with a large positive penalty for the standard deviation. However, the company can specify different profit ceilings and the SO configuration will have a different performance. At the crossover point between the dominance of the SO and RO configuration the company can achieve 93% of the expected profit with 19.7% of the risk as measured by the standard deviation. Another point on the performance curve of the SO configuration achieves an expected profit of $57 million, this is 97% of the maximum profit of the SO configuration ($58.5 million) but at 5% of the risk as measured by the standard deviation.

4 Conclusions

The robust design methodology described above allows manufacturing companies to design a supply chain that corresponds to their risk preferences. The full gamma of Pareto-optimal configurations can be identified and shown in the mean-standard deviation graph and desirable candidate configurations can be selected for further detailed study. By specifying a profit ceiling, the corporation can make their supply chain have a more or less risky performance. This specification of the profit ceiling can even be done for existing supply chains.

This methodology is currently being validated with an extensive numerical experiment. Inclusion of a larger number of scenarios increases the problem instance size significantly. The number of continuous variables in the tactical model grows linearly with the number of scenarios but the number of discrete variables remains constant. This indicates that for very large number of scenarios a primal decomposition strategy may have to be employed.

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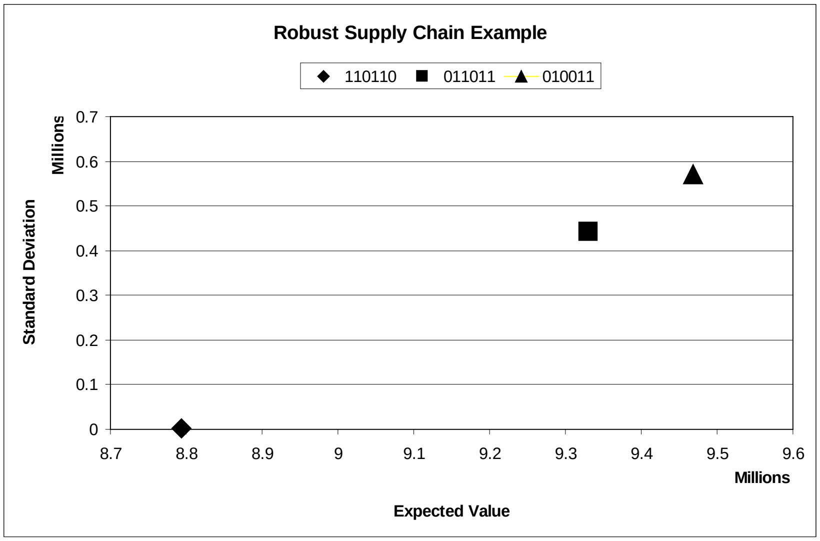

Figure 1: Mean-Value versus Standard Deviation Risk Analysis Graph.

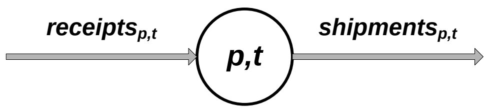

Figure 1. Conservation of Flow of Type 1 (Space).

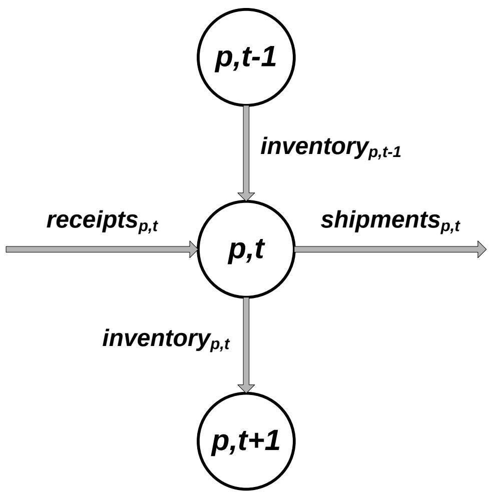

Figure 2. Conservation of Flow of Type 2 (Space-Time).

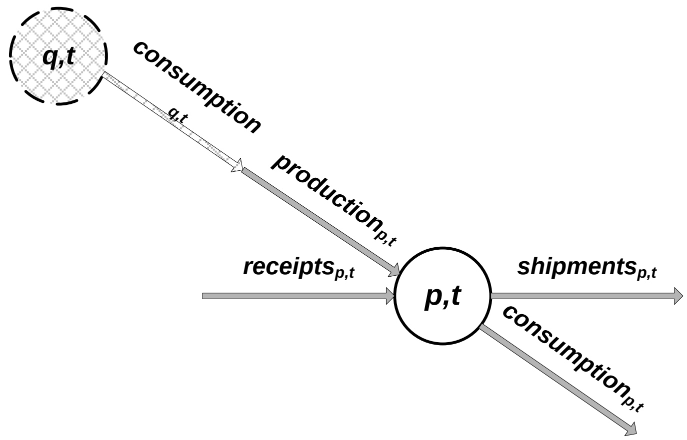

Figure 3. Conservation of flow of type 3 (Creation-Space).

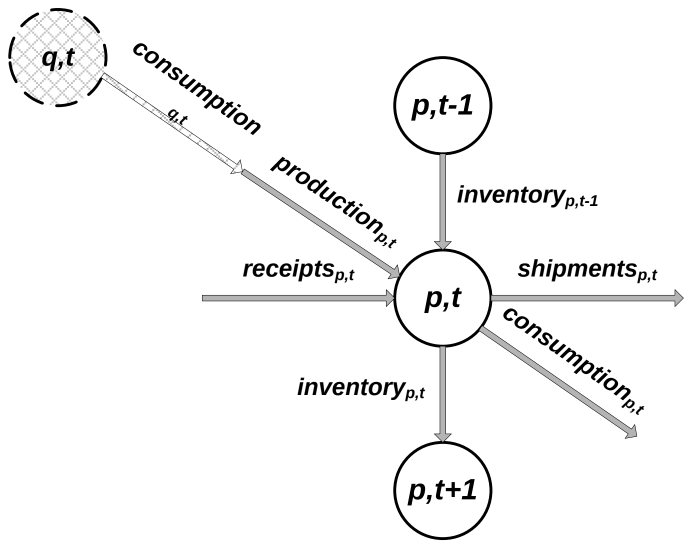

Figure 4. Conservation of flow of type 3 (Creation-Space-Time).

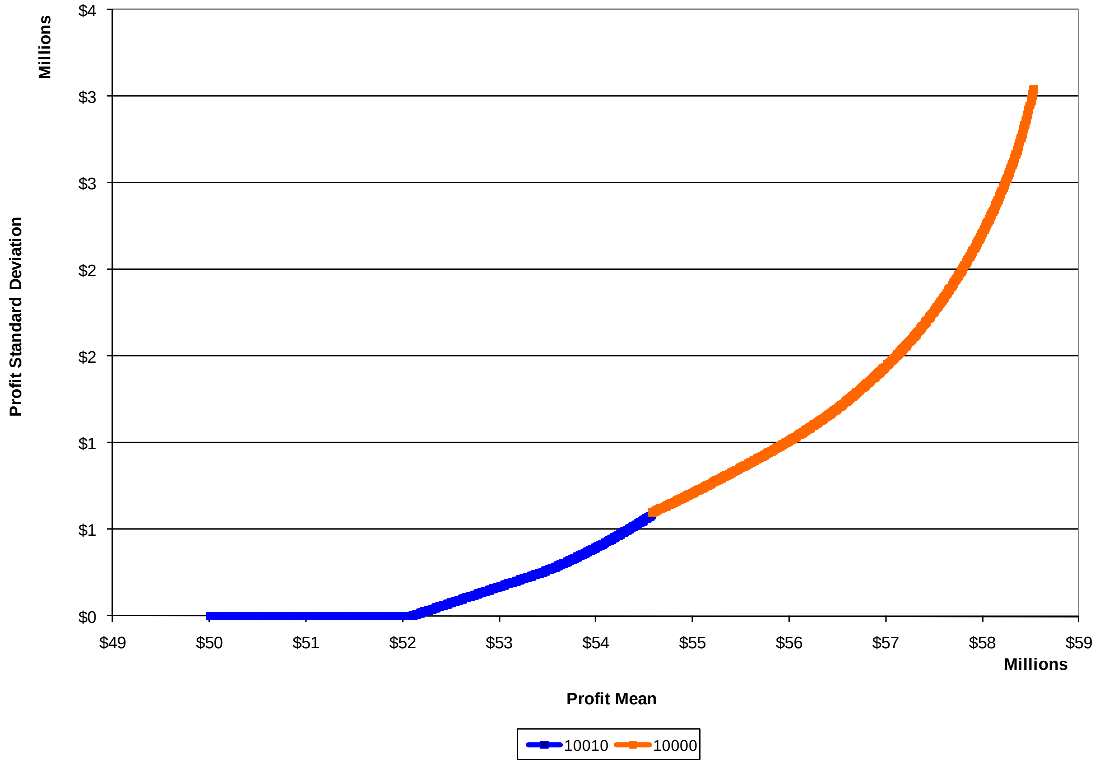

Figure 5. Mean-Standard Deviation Risk Analysis Graph for Industrial Case