Sorting with Robots: where to drop off the parcel?

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Abstract—This paper presents a method for assigning destinations to drop-off points in robotic sorting systems, taking into account robot congestion.

Keywords—robotic sorting system, destination to drop-off point assignment, open queuing network, parcel throughput time

I. INTRODUCTION

Automated, conveyor-based sorting systems are used in many distribution centers. They can sort products by order, or parcels by destination. A problem with these systems is that they are expensive, require much space and an expensive building housing it, are inflexible in throughput capacity or expansion, and the machines are big, blocking traffic. An alternative solution is offered by robotic sorters. The sorting robots typically are equipped with tilt-trays and they can drop off products or parcels in a destination chute. Although the robots are still expensive, they need a much smaller footprint, thereby saving space and costs (Zou et al., 2021). See Figure 1 for an impression of such a system. A typical sorting system (like used at Deppon) may have dozens to hundreds of sorting robots.

The robots operate on the top tier, the drop-off points are connected to delivery destination roll cages at the bottom tier. Source: Deppon Express.

Robotic sorting system performance is, however, quite sensitive to the assignment of destinations to drop-off chutes. If high-density destinations are grouped in a confined area, this might lead to congestion of the robots, thereby increasing the order throughput time. Figure 2 Shows the effect of congestion for a robot parcel sorting operation in China. It shows for a particular parcel insertion and drop-off point, the travel time between these points during one day. While the uncongested time is about 20 sec, the congested travel time is frequently above 100 sec, with an outlier close to 290 sec.

If high density destinations are assigned to areas close to the insertion stations, this reduces travel times for robots, thereby reducing the throughput time, but it may also lead to congestion, thereby increasing the throughput time. This suggests an optimum assignment may exist balancing the effects of congestion and travel distance.

This paper addresses this problem. We introduce a model to estimate the parcel throughput time, taking into account congestion, as a function of the destination assignment (and other parameters, such as the drive-path topology and the number of robots), and use the congested throughput time estimate in an optimization model to optimally assign the destinations. Since the generalized assignment problem is NP-hard and since we do not have a closed-form expression for the throughput capacity estimate for a given assignment, we revert to adaptive large neighborhood search (ALNS) to arrive at near optimal assignments.

Robotic parcel sorting systems are fairly new and have not yet been studied extensively. Zou et al. (2021) study the throughput capacity performance of different RSS system layouts and compare them for operational cost. They model the system using a queuing network approach. Zou et al. (2022) study the assignment of destinations to loading stations, for a given assignment of destinations to drop-off point, using a queuing network approach. The idea is that if parcels arrive at a loading station close to their destination point, the travel time of robots can be reduced. However, in practice it is not really possible to carry out such a pre-sort without a high capacity (expensive) pre-sort operation. Boysen et al. (2023) study a
robotic piece sorting system, where pieces of an order must be collected by robots to minimize the makespan. They heuristically optimize the piece to order assignment and the order to collection point assignment, using pre-arrival information on the arriving pieces. There is also resemblance with assigning truck destinations to dockdoors in a cross-dock. Typically, MILPs are developed to solve this problem focusing on minimizing travel distance or cost (Rijal et al., 2019, Gelareh et al. 2020). Congestion is not included in these models. We focus on the key decision in an RSS: the assignment of destinations to drop-off points, taking congestion of the robots into account and parcels arriving online.

II. SYSTEM AND QUEUING MODEL

A robotic sorting system (RSS) consists of two tiers (see Figure 1). The robots operate on the top tier (Figure 1(a)), which contains the parcel induct stations and the drop-off points. These drop-off points are connected to the destination roll containers at the bottom tier (Figure 1(c)). Typically, the parcel loading (insertion) stations are grouped around the periphery and possibly connected via a conveyor feeding the parcels. See Figure 2.

To estimate the order throughput time performance for a given assignment of destinations to drop-off points, we use an open queuing network (OQN) model. The open queuing network model is shown in Figure 3.

In this open network, parcels arrive at rate $\lambda$ (Poisson distributed). Then they are sorted to insertion station $\mu_i$, with probability $p_i = \frac{1}{n}$, with $n$ the number of insertion stations. At the insertion station, the parcel is loaded on a robot which transports it via the cross aisle $c_{a0}$ (see Figure 2), modeled as node $\mu_{c_{a0}}$, then via travel aisle $a_k$ to the $j$th drop off point in the aisle $\mu_{k,j}$. The sorting job leaves the system after drop off point $\mu_{k,j}$, with probability $p_{k,j}$. At each drop off point, there is limited space for $C_{k,j}$ robots to wait. The probability $p_{k,j}$, with $k = 1, \ldots, W$ and $j = 1, \ldots, L - 1$, where $W$ represents the number of travel aisles and $L$ the number of cross aisles (see Figure 2) equals $\lambda_{k,j}/\lambda_{a_k}$, the ratio of the demand rate at drop point $(k,j)$ and the demand rate at aisle $a_k$.

The open queuing network can be reduced to a four stage network consisting of node $\mu_e$, nodes $\mu_{a_i}$, node $\mu_{c_{a0}}$ and nodes $\mu_{a_i}$, $i = 1, \ldots, W$. Node $\mu_{a_i}$ is the aggregated node that represents the total travel time in aisle $i$, i.e., the travel and drop-off time of the parcel, like indicated for aisle 1 with the dashed line in Figure 3. In order to analyze the network, first the mean and SCV of the service time of node $\mu_{a_i}$ must be obtained, i.e. the travel time between a loading and a drop-off point. Note that travel paths are single directional (see Figure 2), so detours must be taken into account. In addition, these travel times must include possible delays due to congestion on this path, as a robot may be blocked by downstream robots queuing or dropping off parcels. We estimate this blocking behavior using finite queues and approximate the robot travel times using the decomposition method of Dallery and Frein (1993).

The resulting open queuing network can be analyzed for key performance indicators like the parcel throughput time using the method of Marchet (2012), which is again based on the queuing network analyzer of Whitt (1983). We validate the mean throughput time estimates using simulation (in Arena), in networks with up to 8 insertion stations, $L \times W \leq 192$, and $C_{k,j} = 1$, $k = 1, \ldots, W$ and $j = 1, \ldots, L$. We assume sufficient robots are present. Absolute relative errors appear to be below 3%.
III. ALIGNMENT MODEL

We then use this mean throughput time estimate $TT(x)$ for a given assignment of destinations to drop-off points $x$ as an objective to be minimized in a destination to drop-off assignment model. As each destination has its demand rate, a given assignment of destinations to drop-off points results in a different throughput time estimate. The model is formulated below.

$$\text{Min } TT(x) \quad (\text{M.1})$$

$$\sum_{j=1}^{N} x_{i,j} = 1, \quad i = 1, \ldots, M$$

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With $x_{i,j} = 1$, if destination $j = 1, \ldots, N$ is assigned to drop-off point $i = 1, \ldots, M$, and 0 otherwise.

The objective function of model (M.1) is non-linear. Since the number of drop-off points and destinations can be large, we resolve to adaptive large neighborhood search (ALNS, see Ropke and Pisinger, 2006) to heuristically solve the assignment. We assume $N = M = L \times W$ (if not, we can create artificial drop-off points, or merge destinations). The gap with optimal assignments (obtained with Gurobi) for small instances ($L \times W = 24$) appears to be less than 0.1%.

IV. RESULTS

We compare the results for large instances using the layout, robot and demand data (demand over 30 randomly selected days during 5 months from November 2018 to March 2019) of the sorting center of Deppon Express. Deppon’s assignment and the ALNS-based result are shown in Figure 4 (insertion stations are located on the south side). The relative improvement of robot congestion time of ALNS over Deppon’s assignment is 73% and the parcel throughput time reduction is 25%.

![Figure 4](image-url)  
Figure 4. Assignment of demand to drop off points by Deppon (a) and by ALNS (b), using Deppon demand data.

Figure 4(b) shows the ALNS assigns light and heavy demand destinations in a balanced fashion over the drop-off points. Based on insights obtained from this, and other experiments we also develop a simple straightforward balancing heuristic, BA. BA sorts the destinations by increasing demand rate and sequentially pairs a high with a low demand destination to a pair of adjacent drop-off points. However, over a large set of instances, BA still has an average 27% gap in throughput time compared to ALNS.

We also carry out a cost analysis, using an embedded closed queuing network analysis to estimate throughput capacity $TC$ for a given layout $W$ and $L$. The throughput should meet a given required throughput capacity $\lambda$. The model is shown below.

$$\text{min } CT(n, R, p) = C_{ls} \cdot n + C_R \cdot R \quad (\text{M.2})$$

Such that $TC \geq \lambda$, and $W, L, \lambda$ are given,

Here $CT(n, R, p)$ stands for the total (multi-annual) costs, $C_{ls}$ for the multi-annual costs of an insertion station and $C_R$ for the cost of a robot. We have $R$ robots, $n$ insertion stations and the assignment $p$ as decision variables. The closed queuing network yielding the throughput capacity at a given number of robots is solved using the AMVA algorithm (Buitenhek et al., 2000). Results for a typical instance can be found in Figure 5. Assignment $HC$ is Deppon’s assignment with high density destinations allocated close to the insertion stations at the southside (see Figure 4).

![Figure 5](image-url)  
Figure 5. 10-years total multi-annual cost, with $W = 6$, $L = 4$, $C_{ls} = €12,500$, $C_R = €25,000$.

Figure 5 shows that, for any throughput capacity level, ALNS yields the lowest cost, as it requires fewer robots and insertion stations. For low throughput capacity requirement $(TC)$ levels $BA$ outperforms $HC$, while for higher $TC$ levels, $HC$ outperforms $BA$. 

Classification: Internal
V. CONCLUSIONS

This paper develops a method to estimate the throughput time and throughput capacity in robotic sorting systems with robot congestion. It then uses the method to (heuristically) optimize the assignment of destinations to drop-off points, using ALNS. The ALNS assignment can substantially reduce the throughput time of parcels compared to assignments currently used in practice. For small instances it is near optimal. It also can substantially reduce the total cost of the system, for a given required throughput capacity. The model is valid for single directional travel aisles, but it can be extended to other topologies.

REFERENCES