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Modeling Post-disruption Equilibrium for Circular Supply Chains and New Measures for Resilience

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***Abstract*—Among concerns on sustainability and environmental protection, there is a growing interest in the adoption of circular supply chains (CSCs). This work introduces a first-cut approach to capturing the resilience of supply chains and their components related to material handling. We develop a methodology for computing the new equilibrium states after disruptions and use this to measure the resilience of circular supply chains. Numerical simulations illustrating our concept are performed on a synthetic toy-example. In addition, we introduce a secondary measure of resilience that encapsulates the transient stages of a supply chain after disruption. This measure is able to highlight the potential strength of CSCs over linear supply chains (LSCs) resulting from the circulation of reused materials. Despite the increasing hype in CSCs, caution is needed when analyzing its benefits over LSC. A CSC must be designed in way such that the positive effects of the CSC outweigh the negative effects.**

***Index Terms*—Resilience, circular supply chains, disruptions, material handling, recycling.**

I. Introduction

As illustrated by the 2030 Agenda for Sustainable Development[1] and the Paris Agreement on Climate Change, the EU is committed to consistently progressing towards sustainable industrial practices. Circular economy (CE) is a sustainable economic paradigm that aims at prolonging the resource value as long as feasible, by promoting the continuous reuse of resources and products, recapturing value from by-products and end-of-life resources, and minimising resource leakage out of the systems[2]. Circular Supply Chain (CSC) is a self-regenerative ecosystem adopting the principles of Circular Economy to extract new value from end-of-life resources, extend material life, and increase resource efficiency toward zero-waste operating conditions. CSCs extend the boundaries of closed loop supply chains (SCs) by involving multiple stakeholders, with both firms belonging to the linear supply chain stages and organizations from the external industrial networks, carrying out the circular activities. Several examples have been provided by previous works, where recycled polyethylene terephthalate (PET) bottles and textile materials may be used for construction [3], [4] or waste cooking oil from a food supply chain utilized to produce bio-fuels [5]. Despite the highly-anticipated benefits of CSCs, the current literature on the topic is rather limited and invites further studies [6]–[8].

A. Resilience of circular supply chains

Earlier studies defined Resilience as the supply chain's ability to react to unexpected events, to restore normal operating conditions [9] or to return to its original state or move to a new, more desirable one after being disturbed [10]. Alternative definitions are based on the adaptive capability of a supply chain to prepare for unexpected events, react to disruptions, and recover from them, by maintaining continuity of operations and control over structure and function [11]. In addition to the crucial role that material handling of recyclable goods (e.g., moving, lifting/dumping, fluffing, sorting) plays in the efficient and sustainable operations of CSCs, it has been acknowledged that these operations also contribute significantly to the resilience of supply chains as a whole [12]–[15]. However, the connection between circular supply chains and resilience is not clear and recently researchers have warned against making premature judgements [16]. A better understanding of this interaction can help companies re-evaluate their logistics system or choice of technology for material handling.

B. Contributions

In this paper, we propose a general framework for analyzing the resiliency of CSCs. Without a doubt, CSC networks should be resilient to better perform in the advent of frequent and unpredictable disruptions. Similar to linear supply chains, the stability of CSC networks can be jeopardized by adverse events that can occur at different levels, for instance, within each facility (e.g. warehouse), within single firms (intrafirm), between different firms (inter-firm), and at the global level. Machine malfunctioning, natural disasters (earthquakes, volcano eruptions), terrorist attacks, political turbulence, economic crises, and diseases (Covid-19, SARS) are just a few examples. If a CSC network is represented by a graph, a negative disruption would correspond to the deletion/weakening of a node or an arc on the graph. In an earlier work [17], four different types of disruptions (random node removal, random link removal, random node and link removal, and targeted node removal) were considered and the number of short cycles (cycles with less than or equal to three arcs) and long cycles (cycles with more than three arcs) after each disruption were computed for different types of network topology. The ratio between the number of cycles

From-node	To-node	Materials (unit)	Product (unit)	Material:Product
2	3	Bauxite (g)	Aluminum (g)	6:5
3	4	Aluminum (g)	Aluminum can (#)	10:1
4	5	Aluminum can (#) Corn syrup (ml) Carbonated water (L)	soft drink can (#)	1:1 10:1 1:4
5	6	soft drink can (#)	soft drink can (#)	1:1
6	7	soft drink can (#)	soft drink can (#)	5:1
6	8	soft drink can (#)	soft drink can (#)	5:1
6	9	soft drink can (#)	soft drink can (#)	5:3
7	4	soft drink can (#)	Aluminum can (#)	2:1
8	4	soft drink can (#)	Aluminum can (#)	2:1
9	3	soft drink can (#)	Aluminum(g)	1:5
11	12	Water (L)	Corn (#)	5:1
12	13	Corn (#)	Corn syrup (ml)	1:10
13	4	Water (L)	Carbonated Water (L)	1:1

TABLE I: Description of processes and the material-to-product ratios in the hypothetical CSC.

before and after a disruption was used as a proxy for the resilience of the CSC network. Acknowledging the ripple effect of disturbances, we present a framework that models the dynamics and equilibrium state of a supply chain and offer an exact algorithm for computing the new equilibrium state of CSC after a disruption. Using this methodology, we are able to accurately analyze the resilience of CSC in the face of different types of disturbances occurring when handling, storing and transporting materials. Moreover, we provide a measure that quantifies the benefits of CSC when compared to linear supply chains. Using this framework, we are not only able to assess the resilience of existing CSCs but also determine which nodes (e.g. warehouse, firm) and arcs (e.g. connection, flow, operation) are the most important contributors to the resilience of the network. Finally, the metric can also be readily used to assess future investments/expansions on material handling systems of the existing supply chain network.

II. Methodology

Let us represent a circular supply chain (CSC) by a weighted directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{W})$, where $\mathcal{V} = \{1, \ldots, N_v\}$ denotes the set of nodes and $\mathcal{E} = \{1, \ldots, N_e\} \subseteq \mathcal{V} \times \mathcal{V}$ denotes the set of directed arcs. A node in this graph represents an agent (e.g. stakeholder, firm, country depending on the problem) and an arc between two nodes signifies the interaction between two agents. In other words, arc $\ell = (i, j) \in \mathcal{E}$ if and only if some resource (e.g. product, materials) flows from node i to node j. The weights of the arcs represent the capacity of flow between two nodes. Let $\mathcal{D} = \{1, \ldots, N_d\}$ denote the set of potential disruptions that can happen in the CSC. Each disruption $d \in \mathcal{D}$ is associated with a probability of the disruption happening, p_d, and an updated graph of the supply chain after the disruption, $\mathcal{G}_d = (\mathcal{V}_d, \mathcal{E}_d, \mathcal{W}_d)$. Obviously, $\sum_{d=1}^{N_d} p_d + p_{\text{base}} = 1$, where p_{base} is the probability of the base-case scenario (normal conditions), and $\mathcal{V}_d \subseteq \mathcal{V}, \mathcal{E}_d \subseteq \mathcal{E}$.

A. Modeling CSC dynamics and equilibrium state

Suppose that the CSC has source nodes, denoted by the set $\mathcal{S}$ and terminal nodes (final customers), denoted by the set $\mathcal{T}$. For instance, source nodes may refer to water bodies (water source) or mines (metal source). Each node i in the system produces/manufacturers a single item by using materials in the set $\mathcal{M}(i)$. The amount of material m received by node i and the amount of material received by node i from k are denoted by R_i^m and $R_{k,i}$, respectively. The set of nodes that produce material m is denoted by $\mathcal{P}(m)$. Note that a single item can be viewed as either a material or a product depending on how it interacts with a node. Furthermore, for each node i, define two sets $\hat{\mathcal{N}}(i)$ and $\check{\mathcal{N}}(i)$, to be the set of ancestor nodes and descendent nodes, respectively. The amount of material m required to produce one unit of product at node i is denoted by α_i^m. The total amount of product manufactured at node i and the amount that is sent to node j are denoted by S_i and $S_{i,j}$, respectively. In order to model the manufacturing capacity at node i, S_i is taken to be the minimum between S_i', the potential production level based on incoming materials, and $\bar{P}_i$, the maximum production capacity based on factory limits. Due to potential capacity restrictions on the connection, the actual amount of product that is received by node j is the upper-bounded by the capacity, $W_{i,j}$. Finally, each terminal node $i \in \mathcal{T}$ is associated with a fixed demand quantity, D_i. These dynamics are encoded in the equations below.

$$S_i = S_i' \leq \bar{P}_i \qquad \forall i \in \mathcal{S} \tag{1}$$

$$R_i^m = \sum_{k \in \mathcal{P}(m) \cap \hat{\mathcal{N}}(i)} R_{k,i} \qquad \forall i \in \mathcal{V}/\mathcal{S} \tag{2}$$

$$S_i' \leq \min\{\alpha_i^m R_i^m \mid m \in \mathcal{M}(i)\} \qquad \forall i \in \mathcal{V}/\mathcal{S} \tag{3}$$

$$S_i \leq \min\{S_i', \bar{P}_i\} \qquad \forall i \in \mathcal{V}/\mathcal{S} \tag{4}$$

$$S_{i,j} = \beta_{i,j} S_i \qquad \forall j \in \check{\mathcal{N}}(i), \forall i \in \mathcal{V} \tag{5}$$

$$R_{i,j} = \min\{S_{i,j}, W_{i,j}\} \qquad \forall (i,j) \in \mathcal{E} \tag{6}$$

Equation (2) says that the amount of material m received by node i is equal to the summation of material flows coming into node i from upstream neighboring nodes that produce material m. Equation (3) shows that the amount of product manufactured/produced at node i is upper-bounded by the amount of materials received (i.e., lack of a certain material can become the bottleneck in production numbers). Equation (4) shows that the actual supply is also restricted by the production capacity. The total amount of product generated in node i is then distributed to its downstream neighbors, via equation (5), where $\beta_{i,j}$ is the distribution factor. The amount that is actually received by a downstream neighbor j is the minimum between what can be sent, $S_{i,j}$, and the current capacity of the connection between nodes i and j, $W_{i,j}$, as shown in equation (6). We define an equilibrium state to be when there is no inefficiency in the system. In other words, at an equilibrium state, no materials are wasted due to surplus, and the following equation holds for each node $i \in \mathcal{V}/\mathcal{S}$.

$$\alpha_i^m R_i^m = \alpha_i^{m'} R_i^{m'} \qquad \forall m, m' \in \mathcal{M}(i) \tag{7}$$

Furthermore, the system produces as much as it can, without exceeding terminal demands. During normal operating situations (without any disruptions in the system), we assume that the network is not constrained and the demand of the final customers are fully met, thus the following conditions are met.

$$S_{i,j} = R_{i,j} \qquad \forall j \in \check{\mathcal{N}}(i) \tag{8}$$

$$\sum_{k \in \hat{\mathcal{N}}(i)} R_{k,i} = D_i \qquad \forall i \in \mathcal{T} \tag{9}$$

B. Resilience measure based on new equilibrium after disruption

A disruption in the supply chain is reflected by a change in relevant parameters of the system equations (1)-(6). For instance, if there is a disruption in the transportation of materials, then the value of $W_{i,j}$ will be reduced. On the other hand, if a disruption happens in the production part, $\bar{P}_i$ will be set lower to accommodate the altered production capacity.

Once a disruption occurs in the CSC network, either at a node or an edge, the effect ripples throughout the network and undermines the effectiveness of the SC in meeting demand. Due to the circular nature (e.g. reuse, recapture) of CSCs, it takes some time for the effect of a disruption to settle down and drive the network to a new equilibrium. This transient phenomena is discussed in Section V. In this section, we discuss the resilience of a supply chain based on the equilibrium state.

Finding the new equilibrium state is nontrivial due to the presence of cycles. We present a methodology for computing the new equilibrium state after a disruption. Then, based on this framework, we propose a measure of resilience that captures the fulfillment level of demand after disruptions. Note that we do not consider restorative actions by the CSC when computing the resilience. In other words, the ability of the network to add new arcs, increase production at certain nodes, utilize stored inventory are not considered but left to future work.

We streamline this section by using an exemplary CSC that is created synthetically (see Figure 1). Information regarding the connectivity of arcs, which materials are used by each node, what is manufactured at each node, and the material-to-product ratio are provided in Table I. During normal operations where the demand is equal to 100 soft drink cans, the equilibrium state of the CSC is given by Figure 1(a). When a disruption happens in the CSC, the system undergoes transient states to arrive at a new equilibrium state. Mathematically, the new equilibrium can be obtained by executing the following steps.

Step 1. Given a graph $\mathcal{G}$ representing the CSC, determine a set of directed cycles, $\mathcal{C}$, that constitutes a cycle basis of $\mathcal{G}$.

Step 2. For each directed cycle $c_i \in \mathcal{C}$, choose an arc belonging to c_i, and assign a flow variable x_i to it. The choice of flow variables must be in a way such that the arc chosen for c_i cannot belong to any other cycle in $\mathcal{C}$. Express all the other flows in the network as a function of $\{x_i\}$ using the system dynamics (equations (2)-(7)).

Step 3. For each cycle in $\mathcal{C}$, establish a flow balance constraint. In addition, for the disruption under consideration, add its corresponding operational constraint.

Step 4. Maximize the demand fulfillment minus raw material usage subject to the equations in Step 3 as constraints.

Step 5. Using the result of Step 4 (values of optimized flow variables x_i), compute the equilibrium flows in the entire network.

Step 6. Compute $Z_d = \sum_{i \in \mathcal{T}} \widetilde{D}_i / \sum_{i \in \mathcal{T}} D_i$, where $\widetilde{D}_i$ denotes the demand fulfillment at node i after disruption.

For a partial production disruption in node 3, the new equilibrium state is given in Figure 1(b), and for a full disruption in node 7, the new equilibrium state is given in Figure 1(c). The value of Z_d, the system's resilience in the face of disruption d, is 0.5 for the first disruption and 0.88 for the second disruption. Once all the values of Z_d are computed, we can compute $Z(\mathcal{D})$, the *overall resilience of a supply chain in the face of disruptions in set* $\mathcal{D}$.

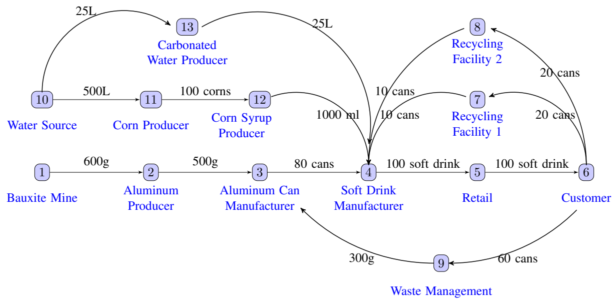
$$Z(\mathcal{D}) = \sum_{d \in \mathcal{D}} p_d Z_d + p_{\text{base}} \tag{10}$$

This metric can be interpreted as the probability weighted demand fulfillment ratio.

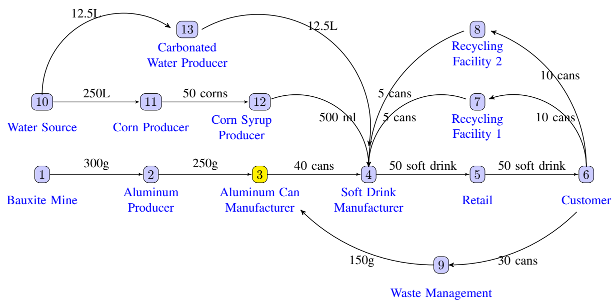
III. Numerical Results

In this section, we present numerical results that illustrate the aforementioned concepts and methodologies. We first implement algorithm 1 to the synthetic supply chain that is introduced in SectionII. Under normal conditions, this circular supply chain is described by Figure 1(a), where its structure is optimized to deliver 100 units of soft drinks to the customers. The original production capacity of the nodes and the transportation capacity of the arcs are specified in the third column of Table II. We consider disruptions that undermine the capacity of a node or an arc. Depending on the intensity of the disruption, the capacity of a node or an arc can be reduced down to either 80%, 60%, 40%, 20% or 0% of its original capacity. The probability of each type of disruption is set to be 0.5%, 0.4%, 0.3%, 0.2%, 0.1%, respectively (in real-world analysis, this probability distribution must be decided by the decision maker based on expert judgement or historical data). In other words, higher intensity disruptions happen with a lower probability. This results in 145 (29×5) distinct disruption scenarios. For each disruption scenario d, the new equilibrium state of the supply chain is computed by using algorithm 1. The satisfied demand levels of the corresponding equilibrium are also found and reported in columns 4-8 of Table II. Based on these values, we are also able to compute the overall resilience of the supply chain, $Z(\mathcal{D}) = 0.85$.

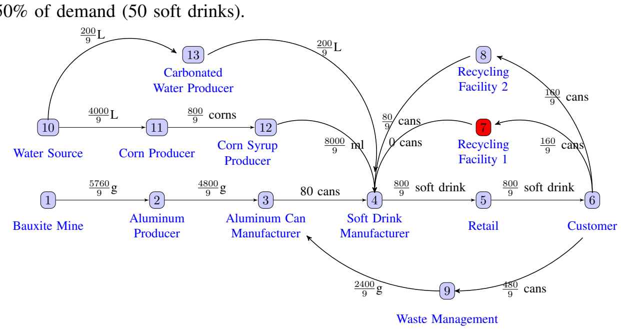
In order to capture the additional resilience obtained from operating a CSC over a LSC, we consider an LSC that is comparable to the CSC discussed above. This LSC has the same structure as the CSC in Figure 1, but without nodes 7,8,9, and all the arcs that are connected to these nodes. Essentially, all the backward arcs (operations) are deleted from the CSC. This results in a LSC with 10 nodes and 10 arcs. In terms of the production and transportation capacities of the LSC, we have two choices: (i) keep the same capacity values as the CSC, (ii) select the minimum capacity values that will deliver the same amount of products to end-customers as the CSC. These two options are depicted in Figure 2. Note



(a) Equilibrium operating state of CSC under normal conditions. System meets the full demand (100 soft drinks).



(b) Equilibrium operating state of CSC after partial disruption in node 3. System meets 50% of demand (50 soft drinks).



(c) Equilibrium operating state of CSC after full disruption in node 7. System meets 88% of demand (88 soft drinks).

Fig. 1: Flowchart of equilibrium flows in the CSC under the (a) Base-case, (b) Disruption in node 3, and (c) Disruption in node 7.

that the first option will not necessarily be able to deliver the same amount of products to end-customers, due to the lack of recycled materials.

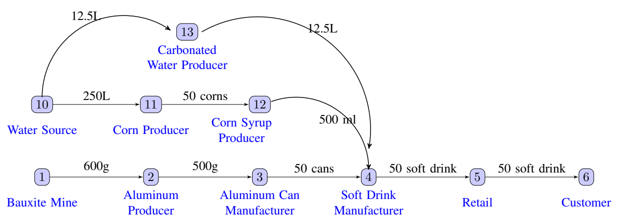
We start with option (i); similar to Table II, the original production capacity of the nodes and the transportation capacity of the arcs in the LSC are specified in the third column of Table III. Under this setting, the LSC is able to deliver 50 units of soft drinks to the customers under normal operating conditions. Again, We consider disruptions that undermine the capacity of a node or an arc. This results in 100 (20×5) distinct disruption scenarios. For each disruption scenario d, the new equilibrium state of the supply chain is computed by using algorithm 1. The satisfied demand levels of the corresponding equilibrium are found and reported in columns 4-8 of Table III. The overall resilience metric, $Z(\mathcal{D})$, is also computed and has a value of 0.46. This implies that the LSC is expected to deliver approximately half as many products in the face of disruptions compared to the CSC, while using the same amount of resources for components that coexist in both supply chains. Of course, the amount of this benefit highly depends on the structure of the supply chain, its baseline operating conditions, and the probability distribution of disruptions. Furthermore, we have to note that the CSC has additional components (nodes 7,8,9 and their connected arcs), which leads to additional costs. Therefore, it is important to appropriately compare the additional costs of creating a CSC versus the additional benefits coming from operating it.

Next, we consider option (ii); see Table IV. Under this setting, the LSC is able to deliver the full 100 units of soft drinks to the customers under normal operating conditions. However, note that is at the cost of increased capacities at nodes corresponding to Bauxite mine, Aluminum producer, Aluminum can producer, and four arcs that are connected to them. This is accompanied by increased resource usages that contribute to the supply chain's operating costs. Same as with option (i), there are 100 (20×5) distinct disruption scenarios, and for each disruption scenario d, the new equilibrium state of the supply chain is computed by using algorithm 1. The satisfied demand levels of the corresponding equilibrium are found and reported in columns 4-8 of Table IV. The overall resilience metric, $Z(\mathcal{D})$, has a value of 0.87. Why is the resilience of this LSC greater than that of the CSC? There are several factors contributing to this result. First, as mentioned before, this LSC has higher capacities at certain components of the supply chain and therefore have higher flexibility. Second, since the CSC has more components than the LSC, the probability of any disruption happening in the CSC is higher than in the LSC. This second factor raises an important design problem for the CSC; Given limited capital, which components should be strengthened (i.e. lower the probability of disruption) so that the overall resilience of the supply chain is maximized? This problem is discussed in more detail in Section IV.

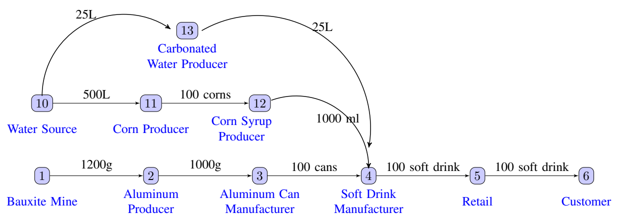
The above examples together offer important insights for designing CSCs. Does a circular supply chain offer more resilience than the traditional open-loop supply chains? Not necessarily. Despite the increasing hype in CSCs, caution is needed when analyzing its benefits over LSC. A CSC must be designed in way such that the positive effects of the CSC (e.g., prolonging resource value, reducing utilization of materials directly coming from the source, reducing waste) outweigh the negative effects (e.g., increased components in the supply chain, increased vulnerability to disruptions).

IV. Optimal Investments of Circular Supply Chains

Given an existing circular supply chain, one may ask the question of what will be the optimal set of investments going forward? These investments reinforce the supply chain and include a variety of options such as renovating existing facilities, changing old machinery (into better performing, higher technology machinery), updating contracts with suppliers, improving transportation routes, etc. Within the framework that is proposed in this paper, the effects of these investments are realized through a change in the disruption probabilities, p_d. For instance, an investment on better-performing machinery will reduce the probability of production decreases



(a) Equilibrium operating state of LSC option (i) under normal conditions. System delivers 50 soft drinks to end-consumers.



(b) Equilibrium operating state of LSC option (ii) under normal conditions. System delivers 100 soft drinks to end-consumers.

Fig. 2: Flowchart of equilibrium flows in the LSC under option (i) and option (ii), located on top and bottom, respectively.

	Index	Original capacity	Satisfied Demand ($\widetilde{D}$)				
			80%	60%	40%	20%	0%
Nodes	1	600g	80	60	40	20	0
	2	500g	80	60	40	20	0
	3	80 cans	80	60	40	20	0
	4	100 drinks	80	60	40	20	0
	5	100 drinks	80	60	40	20	0
	6	100 drinks	90	80	70	60	50
	7	10 cans	96.7	93.3	90	86.7	83.3
	8	10 cans	96.7	93.3	90	86.7	83.3
	9	300g	92.5	85	77.5	70	62.5
	10	525L	80	60	40	20	0
	11	100 corns	80	60	40	20	0
	12	1000mL	80	60	40	20	0
	13	25L	80	60	40	20	0
Arcs	(1,2)	600g	80	60	40	20	0
	(2,3)	500g	80	60	40	20	0
	(3,4)	80 cans	80	60	40	20	0
	(4,5)	100 drinks	80	60	40	20	0
	(5,6)	100 drinks	80	60	40	20	0
	(6,7)	20 cans	96.7	93.3	90	86.7	83.3
	(7,4)	10 cans	96.7	93.3	90	86.7	83.3
	(6,8)	20 cans	96.7	93.3	90	86.7	83.3
	(8,4)	10 cans	96.7	93.3	90	86.7	83.3
	(6,9)	60 cans	92.5	85	77.5	70	62.5
	(9,3)	300g	92.5	85	77.5	70	62.5
	(10,13)	25L	80	60	40	20	0
	(13,4)	25L	80	60	40	20	0
	(10,11)	500L	80	60	40	20	0
	(11,12)	100 corns	80	60	40	20	0
	(12,4)	1000mL	80	60	40	20	0

TABLE II: Original capacities of the CSC and satisfied demand values under each type of disruption scenario. The percentage values in the header indicate the reduced capacity level of the CSC component under each disruption scenario.

	Index	Original capacity	Satisfied Demand ($\widetilde{D}$)				
			80%	60%	40%	20%	0%
Nodes	1	600g	40	30	20	10	0
	2	500g	40	30	20	10	0
	3	80 cans	50	48	32	16	0
	4	100 drinks	50	50	40	20	0
	5	100 drinks	50	50	40	20	0
	6	100 drinks	50	50	50	50	50
	7	525 L	50	50	40	20	0
	8	100 corns	50	50	40	20	0
	9	1000mL	50	50	40	20	0
	10	25L	50	50	40	20	0
Arcs	(1,2)	600g	40	30	20	10	0
	(2,3)	500g	40	30	20	10	0
	(3,4)	80 cans	50	48	32	16	0
	(4,5)	100 drinks	50	50	40	20	0
	(5,6)	100 drinks	50	50	40	20	0
	(6,7)	25L	50	50	40	20	0
	(7,4)	25L	50	50	40	20	0
	(6,8)	500L	50	50	40	20	0
	(8,4)	100corns	50	50	40	20	0
	(6,9)	1000mL	50	50	40	20	0

TABLE III: Original capacities of the LSC (option (i)) and satisfied demand values under each type of disruption scenario. The percentage values in the header indicate the reduced capacity level of the LSC component under each disruption scenario.

	Index	Original capacity	Satisfied Demand ($\widetilde{D}$)				
			80%	60%	40%	20%	0%
Nodes	1	1200g	80	60	40	20	0
	2	1000g	80	60	40	20	0
	3	100 cans	80	60	40	20	0
	4	100 drinks	80	60	40	20	0
	5	100 drinks	80	60	40	20	0
	6	100 drinks	100	100	100	100	100
	7	525 L	80	60	40	20	0
	8	100 corns	80	60	40	20	0
	9	1000mL	80	60	40	20	0
	10	25L	80	60	40	20	0
Arcs	(1,2)	1200g	80	60	40	20	0
	(2,3)	1000g	80	60	40	20	0
	(3,4)	100 cans	80	60	40	20	0
	(4,5)	100 drinks	80	60	40	20	0
	(5,6)	100 drinks	80	60	40	20	0
	(6,7)	25L	80	60	40	20	0
	(7,4)	25L	80	60	40	20	0
	(6,8)	500L	80	60	40	20	0
	(8,4)	100corns	80	60	40	20	0
	(6,9)	1000mL	80	60	40	20	0

TABLE IV: Original capacities of the LSC (option (ii)) and satisfied demand values under each type of disruption scenario. The percentage values in the header indicate the reduced capacity level of the LSC component under each disruption scenario.

in a manufacturer, and improving transportation routes will reduce the probability of transportation delays. Taking that into consideration, a decision maker could find the optimal set of investments that will maximize the CSC resilience by solving the following optimization problem, where $\mathcal{I}$ is the set of candidate investments, x_i is a binary variable deciding whether or not to make the i-th investment (reinforcement), C_i is the cost of making that investment, and B is the available budget.

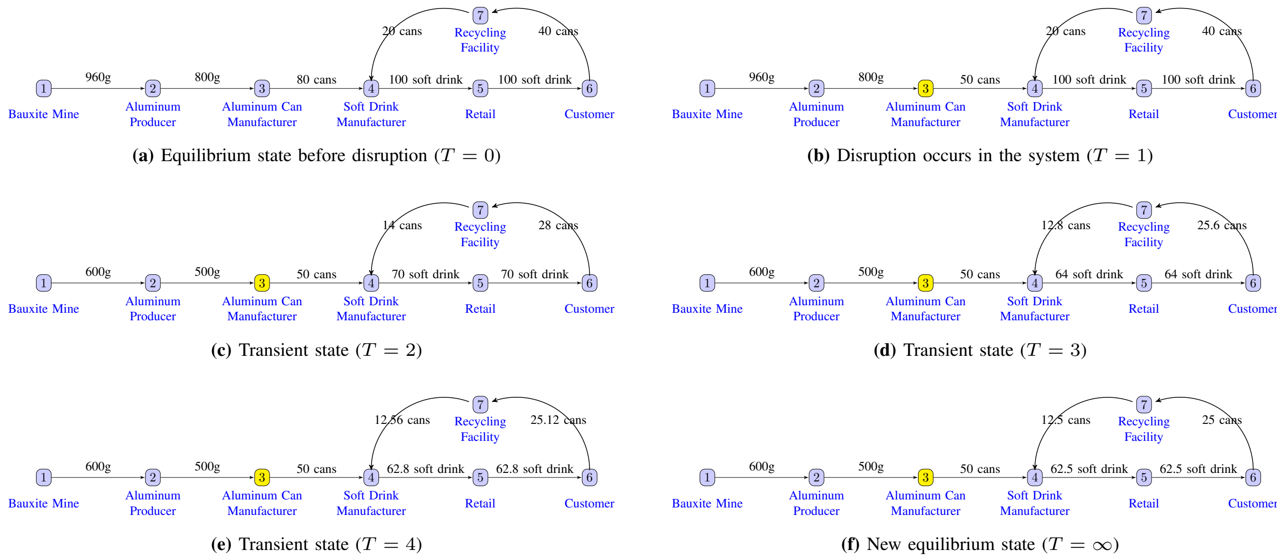
$$\max_{\{x_i\}} \sum_d \bar{p}_d Z_d + \left(1 - \sum_d \bar{p}_d\right) \tag{11}$$

$$\text{s.t. } \bar{p}_d = f_d(p_d, \{x_i |\ i \in \mathcal{I}_d\}) \qquad \forall d \in \mathcal{D} \tag{12}$$

$$\sum_i C_i x_i \leq B \tag{13}$$

$$x_i \in \{0, 1\} \qquad \forall i \in \mathcal{I} \tag{14}$$

In equation (12), $f_d(\cdot, \{\cdot\})$ denotes a function that takes in two arguments: the previous probability of disruption d occurring, and a set of investment decisions that affect the disruption d. The output of the function is the updated probability



(a) Equilibrium state before disruption ($T = 0$)

(b) Disruption occurs in the system ($T = 1$)

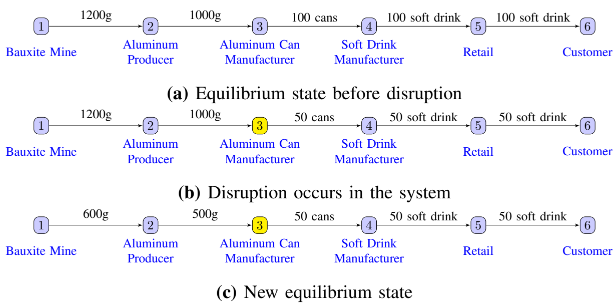
(c) Transient state ($T = 2$)

(d) Transient state ($T = 3$)

(e) Transient state ($T = 4$)

(f) New equilibrium state ($T = \infty$)

Fig. 3: Temporal evolution of flows over a circular supply chain after a disruption at node 3 (maximum production is reduced to 50).



(a) Equilibrium state before disruption

(b) Disruption occurs in the system

(c) New equilibrium state

Fig. 4: Evolution of flows over a linear supply chain after a disruption at node 3 (maximum production is reduced to 50).

of disruption d occurring and is denoted by $\bar{p}_d$. Equation (13) signifies that the total cost used for all investments must be less than the available budget. The objective is maximizing the updated resilience metric. Note that $\bar{p}_{\text{base}} = 1 - \sum_d \bar{p}_d$. In solving the above optimization problem, apart from the computational complexity arising from having to solve an integer programming, a major difficulty arises from having to accurately characterize the function $f_d(\cdot, \{\cdot\})$.

V. Resilience Measure Considering Transient States

So far, we have defined the resilience of a supply chain based on how much demand it can satisfy at new equilibrium states after disruptions. Here, we introduce a second measure of resilience that is based on the transient states. Due to the circular nature (e.g. reuse, recapture) of CSCs, it takes some time for the effect of a disruption to settle down and drive the network to a new equilibrium. This phenomena is illustrated through a series of figures. The evolution of operational states for the circular supply chain and the linear supply chain (both originally supplying 100 soft drink cans to end-customers) are shown in Figures 3 and 4. The results are summarized in Figure 5, where one can observe that the disruption on the linear supply chain results in an immediate decrease in production, whereas the exact same disruption on the circular supply chain results in a buffered decrease in production that flattens out over time, reaching a new equilibrium point. Furthermore, the new equilibrium state of CSC delivers more products than that of the linear supply chain. The benefit of CSC over linear supply chain can therefore be measured by integrating the difference between two curves from the time when the disruption starts to the time when the disruption is fixed. In practice, we consider discrete time periods, so a Riemann sum would be appropriate.

VI. Findings and Conclusion

In this paper, we proposed a framework for characterizing the resilience of circular supply chains. Compared to conventional linear supply chains, computing the equilibrium state of a circular supply chain after a disruption is nontrivial due to cycles in the network. We develop a methodology for computing the new equilibrium states after disruptions and use this to measure the resilience of circular supply chains. Numerical simulations illustrating our concept were performed on a synthetic toy-example. More simulations on real-world data will be performed as they become available. In addition, we introduced a secondary measure of resilience that encapsulates the transient stages of a supply chain after disruption. This measure is able to highlight the potential strength of CSCs over LSCs resulting from the circulation of reused materials. Material handling is an important component of supply chains and is also subject to disturbances. The framework that we propose is general enough to incorporate different types of disruptions including those that happen in the material handling operations.

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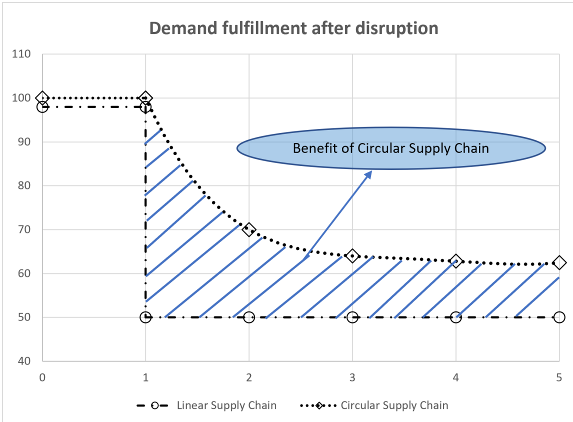

Fig. 5: Post-disturbance demand fulfillment on a circular supply chain versus a linear supply chain.

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