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A ray model of sound focusing with a balloon lens: An experiment for high school students

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A weather balloon filled with carbon dioxide gas is used as a positive spherical acoustic lens. High frequency but audible sound from a circular loudspeaker ensonifies the balloon and produces increased sound pressure levels in a region along the principal axis according to a ray acoustics model. This enhancement was measured experimentally and was found to agree with theory. The possibility that interference from reflected sound off walls or the floor could mask or mimic the expected focusing was countered by calculating and measuring within a "shadow zone" in which only direct rays or rays refracted by the balloon exist by the method of Fresnel volumes. The experiment described in this paper would be a suitable learning experience for junior high and high school students showing how rays and Snell's law apply to sound as well as light and giving them a measurable predicted focal region for enhanced sound pressure levels.
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I. INTRODUCTION

Focusing sound, using a balloon filled with a gas in which sound travels slower than air, is a common demonstration in both classrooms and museums. The balloon focuses sound just as a glass sphere does light. The experiment described in this paper would be suitable for junior high and high school students. This paper itself is addressed at the college educated (presumably science major) science teacher who should be able to understand (and translate as needed) the somewhat advanced ideas beyond mere ray acoustics used here. These advanced ideas (such as the Fresnel volume) are used to warn the student experimenter away from geometrical configurations of the loudspeaker, balloon, and meter that would give rise to interference effects from the reflection of sound off laboratory room surfaces such as the floor and walls.

II. THEORY

Though wave acoustics is more exact than ray methods when solving acoustics problems, it also has many complications. For example, wave acoustics is mathematically demanding. It requires mastery of calculus and differential equations, and typically uses vector differential operators such as divergence, gradient, and curl. The typical approach to solve a wave equation requires expansion of the incident, scattered, and internal fields in orthogonal polynomials along with the use of proper boundary conditions as in Mie theory.[1] This particular approach to the current problem has been recently discussed in depth in a recent publication.[2] However, that study used the more complicated wave solution for distant sources (i.e., "Mie" theory[1]) and was aimed more toward college and graduate students learning scattering, computational, and analytical physics. With a proper choice of experimental parameters, this problem is solvable using the simpler ray acoustics method.[3] This would allow for a larger audience to more fully understand the experiment, including students in middle school and high school. Using the ray method to explain the physics of a balloon lens would also allow an easier consideration of near-field sound sources without the use of translational addition theorems.

Ray acoustics, on the other hand, is a simpler and easier conceptual approach. In order for ray acoustics to be accurate the size parameter needs to be relatively large. The size parameter is defined as the wave number multiplied by the radius of the scatterer. A size parameter of 100 is usually quoted as the minimum value for good quantitative comparison with wave solutions. However reasonably good qualitative results can be obtained for size parameters as low as 30 for typical scattering problems and in certain cases, such as scattering from bubbles in water for optical problems, good quantitative results have been obtained for size parameters as small as about 15.[4] Here, with a balloon radius of $r = 23.5$ cm and a wavelength of $\lambda = 3.445$ cm (corresponding to a temperature corrected speed of sound of $c = 344.5$ m/s and frequency $f = 10\,000$ Hz), the size parameter is $kr = 2\pi r/\lambda = 42.9$. This intermediate size parameter (between roughly 30 and 100) suggests that ray methods should give fair qualitative agreement but should show some quantitative disagreement. It should be noted that the point of this experiment is to predict and measure, using ray acoustics, a *geometrical position* where sound is focused. A quantitative prediction of the amount of enhancement (using, say, the van de Hulst localization principle[5] as used in, for example, Ref. 4) is not the purpose of the experiment proposed here. Such advanced hybrid methods are considered beyond the level of the usual high school student. The purpose of the proposed experiment is to predict a focal region where enhanced sound scattering due to focusing occurs. In

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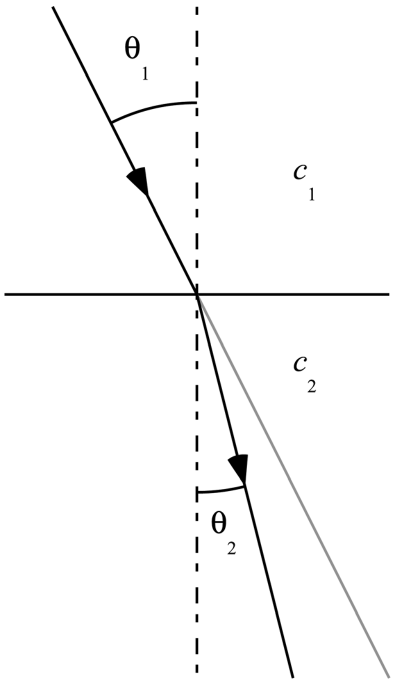

FIG. 1. An acoustical ray incident from a fluid medium with speed of sound c_1 is refracted at the interface with a second fluid medium with speed of sound c_2.

any case the predicted loci of ray focusing should still show significant sound pressure level increase. The latter is borne out by measurement in this experiment.

Pedagogically speaking, ray methods handle far-field and near-field problems easily, using the same method commonly known as Snell's law. Consider an acoustical ray incident from a fluid medium with sound speed c_1 that is bent or refracted when it crosses into a second fluid medium with sound speed c_2 as in Fig. 1. The angle of the incident ray to the dashed normal line is θ_1. The angle of refraction of the ray in the second medium with respect to the normal is θ_2. Then Snell's law describing this bending of the ray can be written as

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{c_1}{c_2} = n, \tag{1}$$

where n is the relative index of refraction. This method is appropriate for middle school and high school students and can be quickly mastered.

Because a spherical carbon dioxide filled balloon is used as a positive acoustic lens, spherical aberration will occur. This aberration causes different rays to focus at different points. In the experimental situation described here the sound does not converge to a focal point but rather to a locus of focal points called a caustic region (see Fig. 2). The temperature corrected speeds of sound in air and carbon dioxide ($T = 22.5\ ^\circ$C) are 344.5 and 268 m/s, respectively, for a relative index of refraction $n = 1.2854$. Using that value the ray diagram for the experiment is shown to scale in Fig. 2. The reader should note that in Fig. 2, by eye, the strongest focusing appears to occur at about 2.4r where r is the radius of the balloon. There might be some concern that the loudspeaker has some lateral extent yet is modeled in Fig. 2 as a point source. However for a 10.5 cm diameter loudspeaker positioned 4.00 m away (the distance from the center of the balloon to the loudspeaker in this experiment), the loudspeaker subtends only a 1.5° angle. In any case, experiment trumps theory, and the predictions of this somewhat simplistic ray model are borne out by measurement.

In this experiment special attention must be paid to interference due to reflections off the walls and the floor. It was reasonable to ignore reflection from the ceiling because it had a sound absorbent tile treatment. The problem is that interference could mask or mimic the enhancement in the sound pressure level due to focusing. In order to successfully model the dominant contributions to the reflected sound field, the sound emerging from the circular aperture of the loudspeaker is modeled as that of the so-called Fresnel volume of the axial acoustic ray.[3,6–9] Sound coming out of the circular loudspeaker is treated as if it were a plane wave diffracted by a circular aperture of the same size. The axial ray Fresnel volume includes all those paths from emitter to receiver that differ in phase from the direct path (the axial ray) by no more than π radians or 180°. For diffraction from a circular aperture this axial Fresnel volume contains over 83% of the sound energy and consists of a cone that has a half angle equal to the Rayleigh minimum angle (the angle of the first minimum of the circular diffraction pattern).[10,11] There are additional side lobes, but they contain relatively little energy. The secondary and tertiary side lobes hold only about 5 and 2 %, respectively, of the sound energy, so they will be ignored in this model.

This approach corresponds to treating the emitted sound as being substantially contained within a Fresnel volume associated with the paraxial ray, meaning the sound within the Rayleigh minimum angle cone is the only significant source for reflection. The Rayleigh minimum angle occurs at an angle θ_R given by

$$\theta_R = \sin^{-1}\left(1.22\frac{\lambda}{D}\right), \tag{2}$$

where D is the diameter of the loudspeaker and λ is the wavelength of the emitted sound. Here $D = 10.5$ cm and $\lambda = 3.445$ cm (corresponding to a frequency of 10 000 Hz

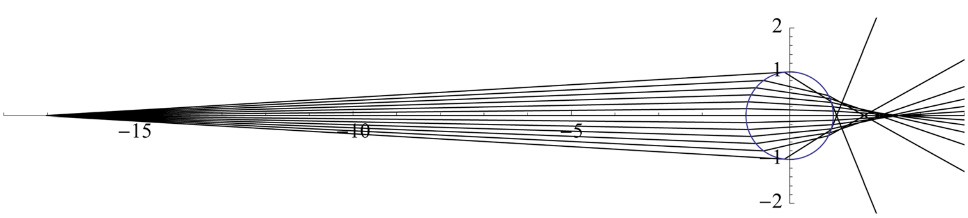

FIG. 2. (Color online) To-scale experimental ray diagram where the radius of the balloon is taken to be 1 m. Note that focusing is very strong at about 2.4 times the radius on the acoustical axis.

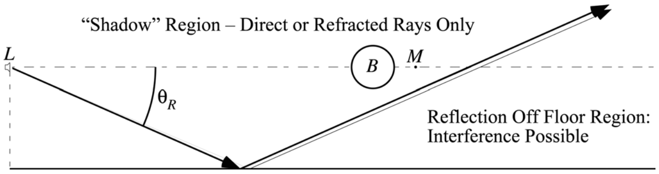

FIG. 3. Side view: reflection off the floor. The diagram is to scale. The position of the loudspeaker, balloon, and sound meter are denoted by L, B, and M, respectively. The center of the balloon is 4.00 m from the loudspeaker. The radius of the balloon is $r = 23.5$ cm, while the wavelength used is $\lambda = 3.445$ cm. Also see Fig. 5.

and a speed of sound in air of 344.5 m/s) such that $\theta_R = 23.6°$.

Figures 3 and 4 show how sound reflects off the floor and walls. The experiment was designed such that all needed measurements could be taken in the so-called "shadow region" such that only direct or doubly refracted acoustic rays (i.e., rays passing through the balloon) originating in the paraxial Fresnel volume occur.

All measurements were performed in the "shadow" region, that is, without interference effects. All rays inside the cone reflect off the wall or floor further on causing the reflected sound to miss the balloon and the various points of measurement.

This paper shows just one possible way to measure enhanced sound pressure levels due to sound focusing by a spherical carbon dioxide lens. For a more quantitative analysis, using the wave solution, i.e., Mie theory,[1] the interested experimenter is strongly recommended to see Ref. 2 wherein sufficient information is available to the motivated inquirer to deduce the pitfalls/challenges and educational applications of alternative setups.

III. EXPERIMENT

The sound pressure levels were measured using a Brüel & Kjær Type 2230 Precision Integrating Sound Level Meter mounted on a tripod. The heights of the meter and balloon center were made level with the center of the sound source

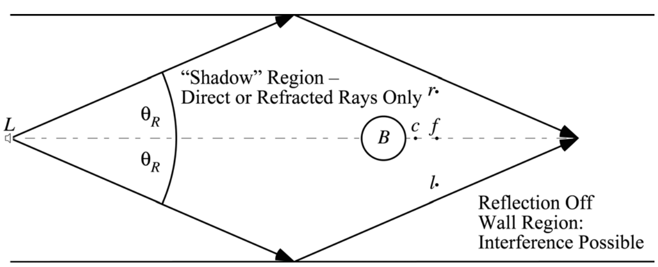

FIG. 4. Top view: reflections off the walls. The diagram is to scale. The position of the loudspeaker, balloon, and the various meter positions are denoted by L, B, c, f, r, l, respectively, where c stands for close position, f stands for focal region, r stands for right, and l stands for left. The loudspeaker, balloon, close, and focal region positions are all centered between the walls. The balloon center is placed 4.00 m from the loudspeaker. The close position is placed 10 cm plus the radius of the balloon ($r = 23.5$ cm, also wavelength is $\lambda = 3.445$ cm) from the center of the balloon. The focal region is placed $2.4r$ from the center of the balloon where r is the radius of the balloon. The right and left positions are located 50 cm to the right and left perpendicular to the acoustical axis of the focal region position in the horizontal plane. Also see Fig. 5.

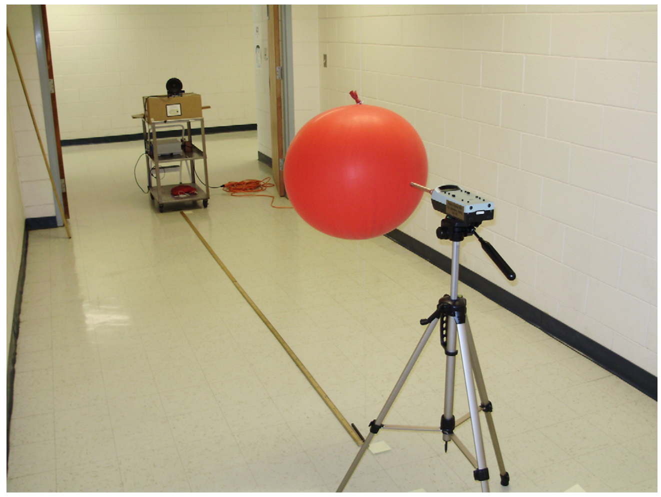

FIG. 5. (Color online) Experimental setup showing Brüel & Kjær Type 2230 Precision Integrating Sound Level Meter in "close" position.

which was a Pasco 30 W rms, 60 W peak, open circular loudspeaker, model WA-9900, driven by a BK Precision model 4011A 5 MHz function generator. The function generator was set to emit a harmonic continuous wave. The frequency of the wave was maintained at 10 000 Hz. The balloon was suspended from above by a string and stabilized against drafts by a weighted string attached directly below the suspension point with tape. The position of the sound source, the radius of the balloon, and the positions of various resonances were measured on the floor using a plumb line and single and double meter sticks.

The sound pressure levels were recorded in four places with and without the balloon (see Fig. 4). The sound meter was set to measure peak values with slow time weighting, frontal incidence, no filter, and linear frequency weighting. Peak values were chosen since by swaying slightly in a draft, the balloon's shifting could cause the meter's microphone to go "off-focus." Measurements were taken in a suitably featureless building corridor (see Fig. 5).

The experiment was done in a relatively noisy area. The decibel reading of the background sound pressure level was 71.2 decibels. Background sound sources included the air conditioning, the compressor on a nearby water fountain, and the intermittent noise of people walking, talking, slamming doors, etc. Instantaneous sound pressure levels varied considerably when using fast averaging. In one such run the

TABLE I. Sound pressure levels with and without the balloon: Close position is on the acoustical axis 10.0 cm from the balloon surface; focal region is on axis $2.4r$ from the balloon center (where r is the radius of the balloon [$r = 23.5$ cm], and the balloon center is 4.00 m from the loudspeaker); right and left are in the focal plane but displaced horizontally 50 cm to the right and left, respectively (see Fig. 4).

Sound pressure levels (dB)				
Positions	Close position	Focal region	Right	Left
With balloon	86.4	87.3	85.9	85.8
Without balloon	85.6	85.6	85.7	85.6

sound pressure level varied between 82 and 85 dB in a time interval of 2 s. The experimental criterion for acceptance of a measurement was that the meter had to maintain a certain maximum reading for at least two seconds using slow averaging. Measurements that were obviously contaminated by background noise (slamming doors and so forth) were rejected. The measurements are listed in Table I. Note that one would expect some enhancement at the close position but more at the focal position or focal region, which from the ray diagram of Fig. 2, is at about 2.4 times the radius of the balloon (= 23.5 cm). The right and left positions are at the same height and distance (measured parallel to the axial direction) from the loudspeaker as the focal measurement, but are displaced 50 cm to the right and left, respectively. The right and left positions according to the ray diagram of Fig. 2 should have little or no enhancement in sound pressure level, which is confirmed by the experimental results.

Finally the meter was moved by hand through the air along the acoustical axis in the exterior region where interference from sound reflections off of the walls and floor was predicted. The instantaneous sound pressure level readings varied up and down in a periodic fashion with a spatial wavelength of about 20 cm. This result cannot be taken by itself as definite proof of the predicted interference due to the time variation in the background noise, which might have mimicked the expected spatial variation. Further experimentation is required to resolve this issue.

IV. CONCLUSIONS

The balloon had a significant impact on the measurements. The focal region measurement was almost a full decibel louder than the close-in measurement when the balloon was present. Both measurements were higher with the balloon than without showing that there was some focusing at both positions with the balloon present but more so at the predicted focal region position (see Fig. 2). The sound pressure level measurements at the right and left positions were essentially unchanged with or without the balloon. As can be seen by comparing Figs. 2 and 4 (both to scale, but not the same scale) no significant number of rays is predicted to be deflected to these positions by the balloon. In consequence the only effective way for sound to get to the left and right positions is by a direct ray path. So the presence of the balloon should make no substantial difference in the measured sound levels at these positions, which is confirmed by the experimental data. The increase in sound at positions "c" and "f" from Fig. 5 is noticeable to the naked ear but only to the attentive listener. Normal background noise makes this observation somewhat difficult.

ACKNOWLEDGMENTS

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