Design Considerations for Visually-Aided Discussion Prompts:

Emphasizing Mathematical Reasoning in Teacher Education

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Abstract

The availability and familiarity of online discussion tools create new instructional options that teacher educators can use to foster prospective teachers' understanding of mathematics. In particular, online discussion blogs provide an avenue through which teacher educators can press prospective teachers to explore mathematical concepts and share their mathematical reasoning with peers. Furthermore, by incorporating visual stimulations as a design component of these discussion blogs, prospective teachers can make sense of and respond to others' ideas about mathematical concepts with greater clarity. This paper shares preliminary findings of a research study that examined the extent to which the design of a series of visually-aided online discussion prompts facilitated prospective elementary teachers' (PSTs) use of mathematical reasoning in a geometry and measurement course. Results suggest that (a) the wording of discussion prompts influences the nature of mathematical justifications that PSTs focus on in their responses and (b) social norms for communicating in online forums may influence the ways in which PSTs interact with peers in an online discussion blog about their mathematical reasoning.

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The National Council of Teachers of Mathematics ([NCTM], 2000; 2014) identifies technology as an essential resource for teaching and learning mathematics. Similarly, the Conference Board of the Mathematical Sciences (2012) promotes technology as a strategic tool that should be used in professional development and teacher preparation programs. Both organizations view technology as a tool for attaining particular mathematical goals or expanding ways to engage in mathematics learning. Among the many educational technologies available, online discussion blogs continue to be an accessible option for many post-secondary faculty through online Learning Management Systems platforms (e.g., Blackboard, D2L, and Moodle), which are exhibiting an annualized market penetration rate of 25 percent (Marketsandmarkets.com, 2013). As a mathematics professor, Offenholley (2006) used online "discussion[s] to: (a) encourage higher-order thinking, (b) monitor students' progress, and (c) encourage peer collaboration" (p. 8)—all skills that are important elements of teacher education programs. This paper shares preliminary findings of a research study that examined the extent to which the design of a series of visually-aided online discussion prompts facilitated prospective elementary teachers' (PSTs) use of mathematical reasoning in a geometry and measurement course.

Theoretical Perspective

The constructivist approaches to formative assessment espoused by both Popham (2011) and Wiliam (2011) informed our views on designing online discussion blogs as an evidence-based exploration of mathematical concepts. Through the lens of this theoretical perspective, we examined digital discussions as a venue for sharing mathematical evidence that was "elicited,

interpreted, and used by both [instructors] and learners" (p. 43). This perspective aligns with this study's purpose to examine the ways that an online discussion prompt – intended to provoke reflection and critique as it pertains to classifying quadrilaterals based on particular characteristics – fostered PSTs' usage of mathematical evidence to support their thinking in response to the prompt.

Context and Methods

The research team investigated 11 PSTs' experiences using an online discussion blog (DB) in a geometry and measurement course, the third in a series of required mathematics content courses for this population. The eight-week summer course met twice a week for two hours and thirty minutes and was partitioned into two content sections. The first section was devoted to geometry with an overarching mathematical goal for PSTs to learn characteristics of various shapes and then classify and group those shapes based on particular shared characteristics. The focus of the second section was on geometric measurement with an emphasis on understanding and distinguishing between measurement characteristics of perimeter and area of various shapes. The researchers designed a DB-Prompt (the second of four DB-Prompts) instructing PSTs to provide mathematical evidence to support their reasoning about classifying quadrilaterals based on possessing the characteristics of (1) one pair of parallel sides and/or (2) one line of symmetry.

Leading up to the second DB assignment, PSTs worked on several tasks (see Appendix) related to identifying different attributes of quadrilaterals and common characteristics across quadrilaterals. Initially, during an in-class activity, PSTs cut out paper shapes for different quadrilaterals and explored their geometric properties using right-angle testers, paper-folding, and tracing paper. PSTs noticed that certain quadrilaterals shared some common characteristics,

while others possessed characteristics that were unique. This difference led to a discussion of

finding a way to show relationships between trapezoids[1], parallelograms, and rhombuses.

Unexpectedly, all three groups of students chose to use the same incorrect Venn diagram with

three overlapping circles to show relationships between these quadrilaterals for an exit question.

The PSTs' exit question responses indicated a need for the instructor to discuss what different

areas of a Venn diagram represent and how to organize the shapes into those circles in

meaningful ways. Thus, in the subsequent class meeting, PSTs created the quadrilaterals using

AngLegs (ETA hand2mind) and worked in small groups to determine how to arrange the shapes

within physical circles to represent their Venn diagram placement. The second DB-Prompt

followed these discussions and asked PSTs to consider four different Venn diagrams and decide

which one would be the most appropriate for describing which of eight quadrilaterals had the

characteristics of a pair of parallel sides or a line of symmetry (see Appendix).

Data from this study took on one of two forms: (1) PSTs' postings in response to the

discussion blog prompt (DB-Entries) and (2) PSTs' responses to two of their peers' postings

(DB-Responses). The pedagogical decision to include DB-Responses served as a feedback

mechanism, a core element of formative assessment protocol (Heritage, 2010). Data analysis

involved a two-cycle grounded-theory technique that began with open-coding the data for

emergent themes and segued into a second-cycle of pattern coding that further parsed the data

into one of two categories, strong or weak, reflecting PSTs' usage of mathematical evidence

(Saldana, 2009). Both researchers separately analyzed the data using this dichotomy and

identified most entries with the same code, that being weak – lacking mathematical reasoning or

supporting evidence. Any differences were reconciled through consensus (Harry, Sturgis, and

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[1] Note that an inclusive definition of trapezoids was used for this exploration. Thus, a trapezoid was considered to be a quadrilateral with at least one pair of parallel sides.

Klingner, 2005).

<div align="center">Results</div>

Analysis of the online discussion from the DB-Prompt illuminated two preliminary

findings. First, we realized our DB-Prompt wording directed PSTs to provide mathematical

evidence on which of four Venn diagrams best served quadrilateral classification without

specifically asking them to discuss how they knew the polygons' characteristics of parallelism

and symmetry were present. For example, PST DB-Responses included words such as noticed,

decided, and looked at to describe their Venn diagram placement of a particular polygon without

mentioning any mathematical properties such as equidistant or having a mirror image.

Consistent with the tenets of formative assessment, we responded to this lack of expressed

mathematical reasoning by making "subsequent instructional decisions on assessment-elicited

evidence" (W. J. Popham, 2011, p. 79). Specifically, this feedback provided us with insights on

how we could attend to our instructional language in a manner that was more precise with

regards to what PSTs should focus on when providing mathematical evidence to support their

thinking.

Second, our design of the second DB-Prompt was not successful in eliciting

mathematically critical responses from PSTs to their peers' thinking. In spite of very explicit

directions in the assignment rubric and prompt, PSTs offered superficial statements about their

peers' DB-Entry discussions. Many of the PST DB-Responses reflected weak evidence of

mathematical reasoning with statements such as "I agree with you on how you sorted the shapes.

This is what I did. Your presentation . . . was great" [PS03 DB-Response], and "I was confused

when I read [your DB-Entry]. I do see … 2 pairs of opposite sides are parallel, and it does not

seem to have parallel lines at all" [PS08 DB-Response]. In both cases, PSTs offered no

discussion detailing the nature of the relevant geometric properties. Thus, consistent with the findings of a PEW study on social networking usage (Hampton, Goulet, Rain, & Purcell, 2001), the PSTs' online discussions reflected communication relating to trust, tolerance, and social support. We believe these social norms may be incompatible with the DB assignments' learning objectives involving critique and analysis.

<div align="center">Discussion and Recommendations</div>

An overarching instructional goal of all the discussion blog assignments was to encourage PSTs to use mathematical evidence to improve their mathematical understanding and sense making. In reflecting on this study's preliminary findings, the majority of analyses identified most PSTs' DB-Entries and DB-Responses as weak. For example, some PSTs did not connect how they used paper-folding to the geometric properties of the shapes. In contrast, strong responses included details about their mathematical thinking pertaining to characteristics of the Venn diagram or geometric properties of particular polygons. For example, one PST explained, "I was able to identify the attributes of the quadrilateral by testing for lines of symmetry by folding or drawing the lines and seeing, if when folded, the edges fit perfectly as a mirror image" [PS04, DB-Entry]. We realized our DB-Prompt wording inadvertently directed PSTs to provide mathematical evidence on which Venn diagram best served the polygon classification without specifically asking them to discuss how they knew the attributes of parallelism and symmetry were present. Thus, one recommendation for teacher educators who want or need to use online discussion blogs as an instructional tool for assisting PSTs in thinking deeply about the mathematics is to carefully and strategically construct discussion blog prompts. Explicit instructions could elicit information from PSTs to direct them to reflect on and share their mathematical reasoning with probes such as: How did your thinking process evolve? or

How was your approach useful in exploring the mathematical relationships? Furthermore, we see additional value in modeling a sample discussion thread (including DB-Entry and multiple DB-Responses) to reflect the instructor's expectations for how to participate in discussion blogs involving sharing mathematical reasoning and critiquing peers' mathematical ideas.

Another instructional expectation for the discussion blog assignment was for PSTs to attend to each other's mathematical ideas, to analyze them, and then to respond to those ideas by using mathematical reasoning and providing mathematical evidence. One of the challenges associated with using online discussion blogs that require analyzing peer's work is related to PSTs needing to overcome their desire to avoid criticizing others' thinking. A possible consideration is that PSTs may have established norms for how they converse in virtual discussions, such as social networking, that do not foster critical analysis of peers' ideas. Hence, teacher educators will need to consider this type of reluctance when designing online discussion assignments. Teacher educators may find it helpful to (a) specify that DB-Responses include an excerpt from the original post to facilitate evidence-based analysis, (b) offer a word bank of mathematical terminology to encourage the use of mathematical language, (c) provide PSTs with a hypothetical DB-Entry that may reduce their hesitation to critically analyze others' mathematical ideas in an online forum, and (d) share a sample discussion thread in which the DB-Responses include specific, evidence-based justifications rather than superficial critiques of the mathematical thinking.

Overall, our analysis of the study's data has enlightened our perspectives on how to design a more effective online discussion prompt for PSTs. We found it helpful to see how different participants responded to the task of analyzing specific mathematical concepts and how they interacted with their peers in this mode of online communal discussion.

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In-class task: exploration of attributes of quadrilaterals using paper folding, testing whether different quadrilaterals have particular attributes, grouping quadrilaterals based on shared attributes	In-class exit question: using a Venn Diagram to show relationships between quadrilaterals based on their shared and unique attributes	In-class discussion: what different areas of a Venn Diagram represent; different arrangements of Venn Diagrams In-class task: using physical circles to create a Venn Diagram and anglegs to represent the quadrilaterals to show relationships between quadrilaterals based on their shared and unique attributes
Week 2, Tuesday	Week 2, Thursday	Week 3, Tuesday

In class task – Week 2, Tuesday:

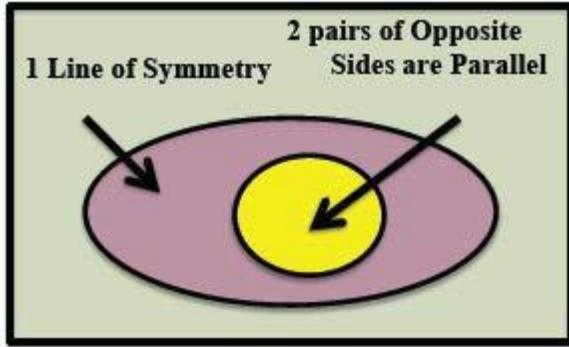## Quadrilateral Classifications

Shape Group 1: Trapezoids

Shape Group 2: Parallelograms

Shape Group 3: Rhombuses (rhombi)

In-class exit question – Week 2, Thursday:

Small-Group Work: Grouping Shapes Based on Their Attributes

Consider the attributes of the shapes in shape groups 1, 2, and 3.

1. Draw a Venn Diagram showing how the shape groups are related based on their attributes. Use quadrilaterals as your universe. Explain what each part of the Venn Diagram represents. Color-coding might be helpful.

2. Sketch examples of shapes that belong in each part of your Venn Diagram, and justify why each of your examples fits into that part of the Venn Diagram. Folding, using a straight edge, and/or tracing paper might be helpful to use in your justification.
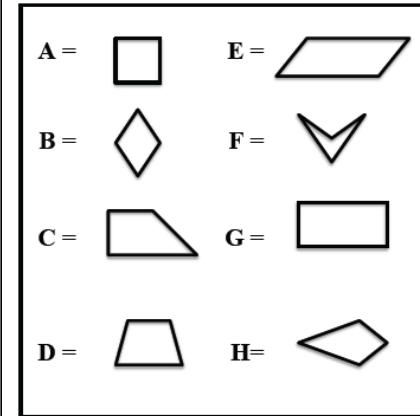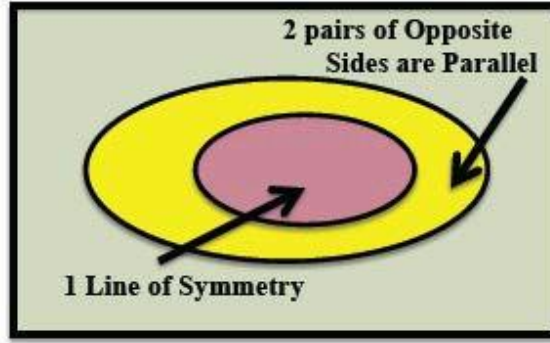
In-class discussion – Week 3, Tuesday:

EQ2: Grouping Shapes Based on Their Attributes

Consider the attributes of the shapes in shape groups 1, 2, and 3 and the Venn Diagram that we developed as a class.

1. Place characteristics into different parts of the Venn Diagram based on the descriptions that we came up with about what each section represents.

2. Sketch examples of shapes that belong in the different part of your Venn Diagram, and explain why each of your examples fits into that part of the Venn Diagram.

3. If you have parts that do not have characteristics or shapes, explain why based on the characteristics of the groups.

In-class task – Week 3, Tuesday:

EQ2 (revision): Grouping Shapes Based on Their Attributes

Revisit your responses to EQ2 and the associated comments. Construct shapes using the anglegs based on the characteristics of each shape group. Then place those shapes into different parts of a Venn Diagram based on the criteria for being included in each group. It might be helpful to consider the different ways that you can draw a Venn diagram (particularly if you find that there are empty spaces in your Venn). Turn in a revised version of EQ2 with the following information:

1. Place characteristics into different parts of the Venn Diagram based on the descriptions that we came up with about what each section represents.

2. Sketch examples of shapes that belong in the different part of your Venn Diagram, and explain why each of your examples fits into that part of the Venn Diagram.

3. If you have parts that do not have characteristics or shapes, explain why based on the characteristics of the groups.
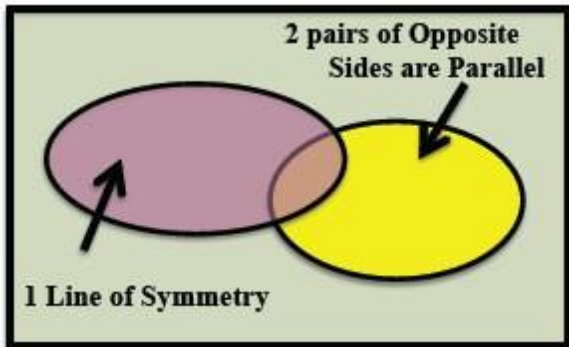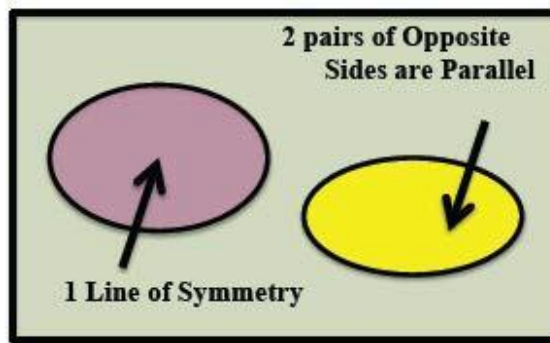
Venn Diagram #1	Venn Diagram #2	
2 pairs of Opposite Sides are Parallel	2 pairs of Opposite Sides are Parallel	A = E =
1 Line of Symmetry		B = F =
	1 Line of Symmetry	C = G =
		D = H =
Venn Diagram #3	Venn Diagram #4	
2 pairs of Opposite Sides are Parallel	2 pairs of Opposite Sides are Parallel	
1 Line of Symmetry	1 Line of Symmetry	

Discussion and Prompt:

Recall that a Venn diagram is a visual display representing mathematical or logical sets as circles of closed curves (cells) within an enclosed rectangle (the universal set), in which the "common" elements of the sets are placed in the "overlap" areas of these cells.

- For this activity the universal set is the set of all Quadrilaterals.
- Next consider how different quadrilaterals can be classified in terms of 2 attributes: (a) 1 Line of Symmetry, and (b) 2 Pairs of Opposite sides that are
                             Parallel.
- Choose 1 of the 4 Venn diagrams that provides an efficient visual presentation of how the 8 given quadrilaterals (A through H) can be classified using (a) and (b).

1) Which Venn diagram did you choose? Discuss how you decided which Venn diagram to use. Be specific about the mathematical thinking that you used.
2) Which quadrilaterals are placed in which part of your chosen Venn diagram? Did any of the quadrilaterals fit into more than one circular cell areas? Explain why or why not. Be specific about the mathematical thinking that you used.
3) Did any of the quadrilaterals not fit into any of the circular cell areas? Explain why or why not. Be specific about the mathematical thinking that you used.

Example: I chose Venn diagram #3 because …. I placed Quadrilaterals B, C, and H in the Pink area because .... I placed Quadrilaterals in the intersection/overlap area because … I think there is a connection between overlapping cell areas and ... because … Venn diagrams are connected to
           …. because …