Batching and sequencing in Bucket Brigade systems with heterogeneous items

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Objectives
- Analyze analytically no-blockage conditions for items with different work distributions
- Accordingly, suggest order batching & sequencing approach

Heterogeneous items/orders

Analytical model: description & assumptions

1. 2-worker Bucket Brigade line
2. Slowest to fastest order, \( v_i \), the deterministic speed of worker \( i \)
3. Multiple items to be produced /orders to be picked
4. Each item/order has heterogeneous work distribution

General conditions for no blockage

- \( W_j[x] \): cumulative work dist. of current item \( j \)
- \( W_j'[x] \): cumulative work dist. of new item \( j' \)
- \( w_j(x) = W_j[x] \): density work distribution
- \( x_0 \): hand-off point b/w \( j \) and \( j' \)

No-blockage condition: \( \frac{W_j(x)-W_j(x_0)}{W_j'[x_0]} \leq \frac{W_{j'}(x)-W_{j'}(x_0)}{W_{j'}'[x_0]} \quad \forall x \in [x_0, 1] \)

Equivalently: define \( g_{jj'}[x] = \frac{W_j(x)-W_{j'}(x)}{W_j'[x]} \). Then, \( \max_{x \in [x_0, 1]} g_{jj'}[x] \leq W_j'[x_0] \).

Sequence with no blockage when the total workload is identical

A single item type:
- Given a set of items, each with a work distribution \( W_j[x] \) and \( W_j'[1] = a_i \), no blockage occurs. The hand-off point converges to the steady state point, \( x^* \), which satisfies \( W_j[x^*] = a_i W_j'[x^*] \) (generalization of Bartholdi Eisenstein, 1996).

Multiple item types, identical workload \( a_i = a \): 
- The steady state hand-off point of each item \( j \), \( x_j^* \), satisfies \( W_j[x_j^*] = a \frac{W_j'[x_j^*]}{W_j'[x_0]} \).
- No blockage occurs when the items are ordered in a sequence with weakly decreasing \( x_j^* \).
- The process starts in \( \max x_j^* \) and each hand-off will occur in \( x_j^* \).

Sets of item types satisfying universal no-blockage

Definition: a set of item types satisfies universal no-blockage if no blockage occurs for any starting position, any pair of items from the set, and any order of that pair.

For some distribution function \( W_j[x] \), the set of all distribution functions \( W_j'[x] \), such that \( W_j[x] \in \bigcup_{x_0} W_j[x], W_j'[x] \) for all \( x \in [0,1] \), satisfies universal no-blockage.

Proof: \( \max_{x \in [0,1]} g_{jj'}[x] \leq \max_{x \in [0,1]} \left( W_j'[x] - \frac{a}{W_j'[x_0]} \right) \geq 0 \Rightarrow W_j[x_0] \).

Maximal set within a domain with universal no blockage

Definition: given a domain (a set of item types), a set of item types satisfies maximal universal no-blockage within the domain if:
(a) It satisfies universal no-blockage
(b) For any distribution within the domain and outside the set, there exists a starting position, an item with work distribution within the set, and an order of these two items that generate blockage.

Domain of general distributions

For any distribution function \( W_j \), the set of all distribution functions \( W_j' \in \bigcup_{x_0} W_j[x], W_j'[x] \) for all \( x \in [0,1] \), satisfies maximal universal no-blockage within the domain of general distributions.

Order batching

Constraint Programming for order batching

Minimize \( \sum_{k=1}^{n} \sum_{j=1}^{m} z_{kj} (z_{kj} - c_j^*)^2 \)
Subject to:
- \( z_{kj} = \sum_{k'=1}^{n} (x_i = k') \delta_{ij} \quad k = 1..UB^k, j = 1..m \)
- \( \sum_{k=1}^{n} z_{kj} \leq C \quad k = 1..UB^b \)
- \( z_k \leq \sum_{i=1}^{m} (x_i = k) \quad k = 1..UB^b \)
- \( z_k \geq \sum_{i=1}^{m} (x_i = k) \quad i = 1..n, k = 1..UB^b \)
- \( z_k \geq z_{k+1} \quad i = 1..n, k = 1..UB^b - 1 \)
- \( x_i = (0, UB^b) \quad z_{kj} = \text{int.} \quad z_k = (0,1) \)

CP batching solution (C=200)

Arbitrary batching solution (C=200)

Hamiltonian path for order sequencing

1. 2018 International Material Handling Research Colloquium
2. Savannah, Georgia USA, July 23-26, 2018