19 Batching and sequencing in Bucket Brigade systems with heterogeneous items

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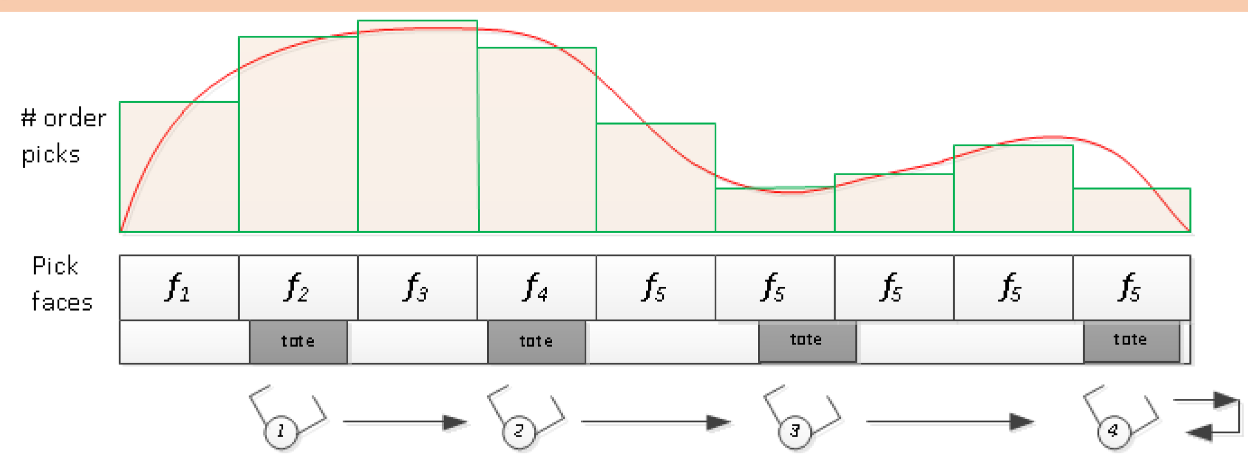
Objectives

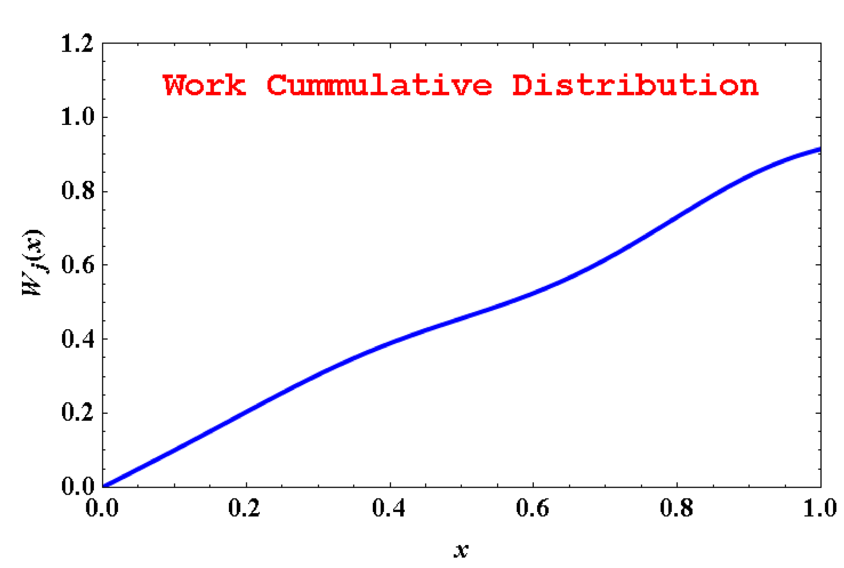
- Analyze analytically no-blockage conditions for items with different work distributions
- Accordingly, suggest order batching & sequencing approach

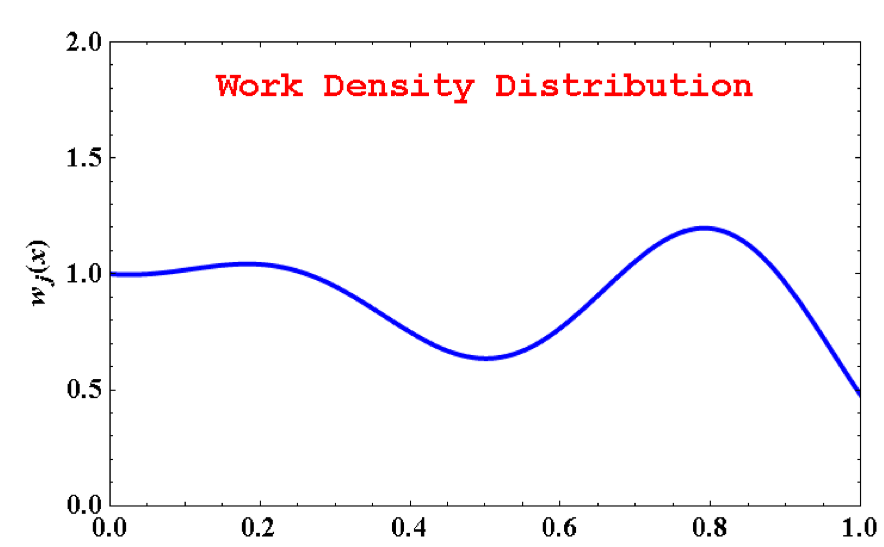
Heterogeneous items/orders



Analytical model: description & assumptions

1. 2-worker Bucket Brigade line
2. Slowest to fastest order, v_i the deterministic speed of worker i
3. Multiple items to be produced /orders to be picked
4. Each item/order has heterogeneous work distribution



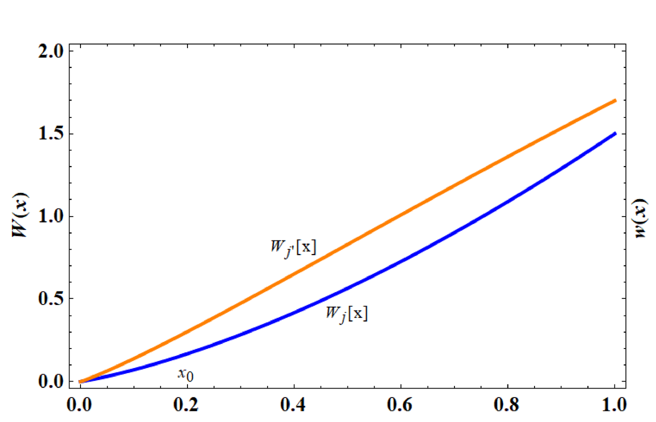


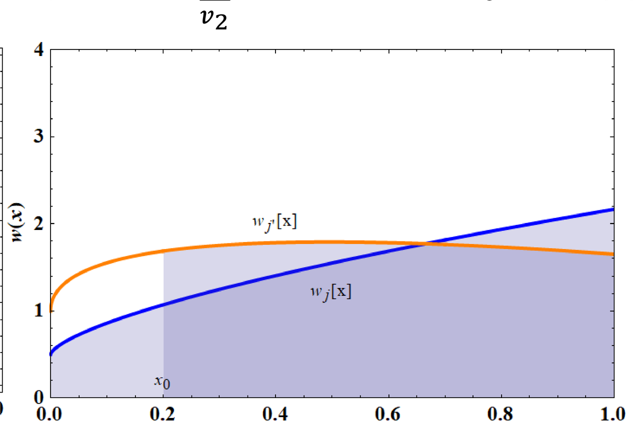
General conditions for no blockage

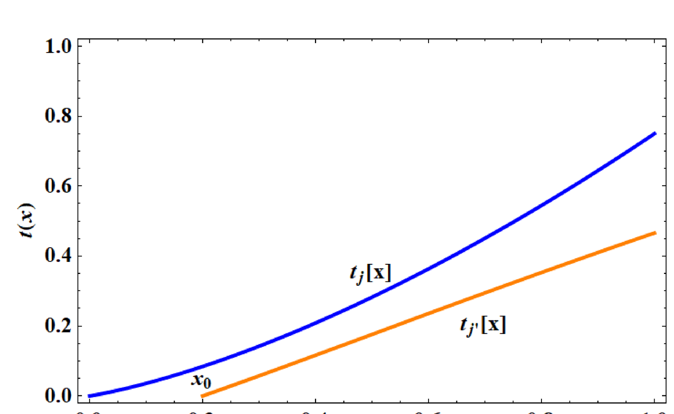
$W_j[x]$	cumulative work dist. of <u>current</u> item j
$W_{j'}[x]$	cumulative work dist. of <u>new</u> item j'
$w_j[x] = W_j'[x]$	density work distribution
x_0	hand-off point b/w j and j'

No-blockage condition: $\frac{W_j[x]-W_j[x_0]}{v_2} \le \frac{W_{j'}[x]}{v_1} \quad \forall x \in [x_0, 1]$

Equivalently: define $g_{j'j}[x] := W_j[x] - \frac{W_{j'}[x]}{\frac{v_1}{v_2}}$. Then, $\max_{x_0 < x \le 1} g_{j'j}[x] \le W_j[x_0]$.







Sequence with no blockage when the total workload is identical

<u>A single item type:</u>

given a set of items, each with a work distribution $W[x]$, and $W[1] = a$, no blockage occurs. The hand-off point converges to the steady state point, x^*, which satisfies $\frac{W[x]}{v_1} = \frac{a - W[x]}{v_2}$.

$x^* = W^{-1}\left[\frac{a v_1}{v_1+v_2}\right]$ (generalization of Bartholdi Eisenstein, 1996).

<u>Multiple item types, identical workload $a_j = a \ \forall j$:</u>

The steady state hand-off point of each item j, x_j^*, satisfies $\frac{W_j[x_j]}{v_1} = \frac{a - W_j[x_j]}{v_2}$.

$x_j^* = W_j^{-1}\left[\frac{a v_1}{v_1+v_2}\right]$.

no blockage occurs when the items are ordered in a sequence with weakly decreasing x_j^*.

The process starts in $\max_j x_j^*$, and each hand-off will occur in x_j^*.

Sets of item types satisfying universal no-blockage

<u>Definition</u>: a set of item types satisfies ***universal no-blockage*** if no blockage occurs for any starting position, any pair of items from the set, and any order of that pair.

For some distribution function $W^0[x]$, the set of all distribution functions $W[x]$, such that $W[x] \in \left[\frac{v_1}{v_2} W^0[x], W^0[x]\right]$ for all $x \in [0,1]$ satisfies universal no-blockage.

<u>Proof</u>: $\max_{x_0 \le x \le 1} g_{j'j}[x] \le \max_{x_0 \le x \le 1} \left(W^0[x] - \frac{\frac{v_1}{v_2} W^0[x]}{\frac{v_1}{v_2}} \right) = 0 \le W_j[x_0]$.

Maximal set within a domain with universal no blockage

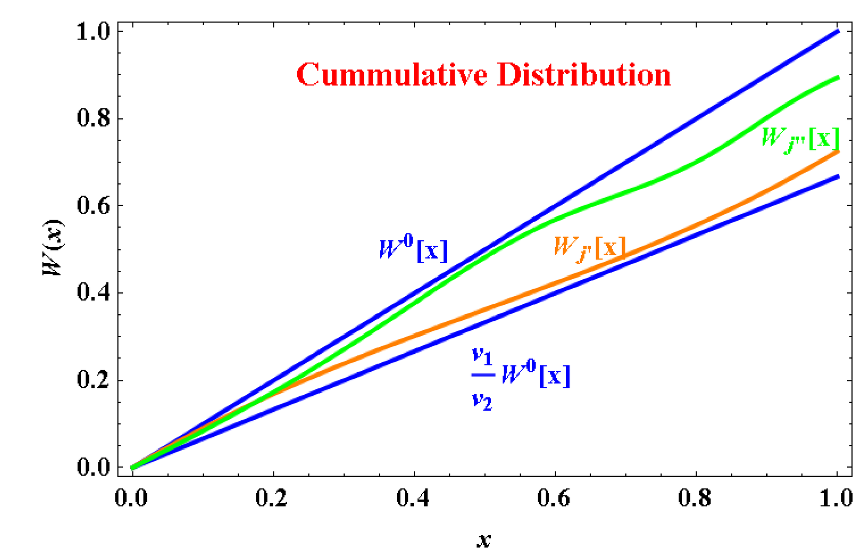
<u>Definition</u>: given a domain (a set of item types), a set of item types satisfies *maximal universal no-blockage within the domain* if:

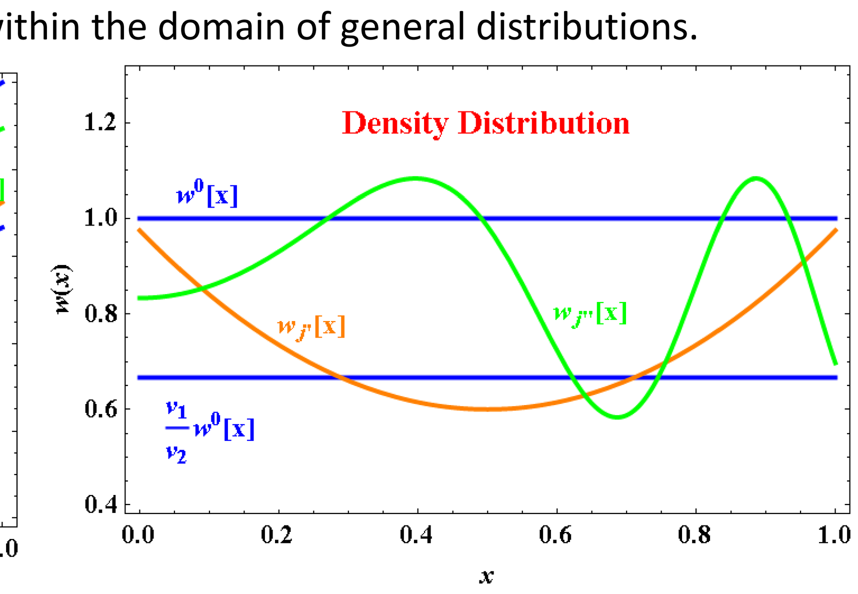
(a) It satisfies universal no-blockage

(b) For any distribution within the domain and outside the set, there exists a starting position, an item with work distribution within the set, and an order of these two items that generate blockage.

<u>Domain of general distributions</u>

For any distribution function W^0, the set of all distribution functions $W[x] \in \left[\frac{v_1}{v_2} W^0[x], W^0[x]\right]$ for all $x \in [0,1]$ satisfies maximal universal no-blockage within the domain of general distributions.





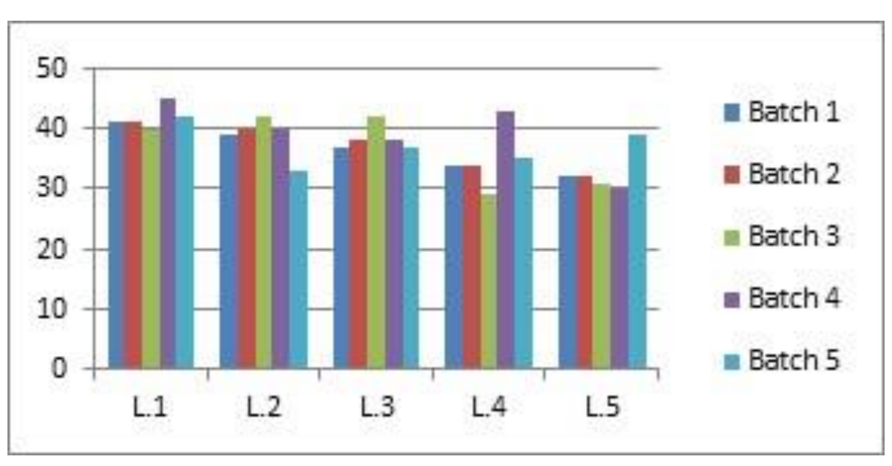
Order batching

Constraint Programming for order <u>batching</u>

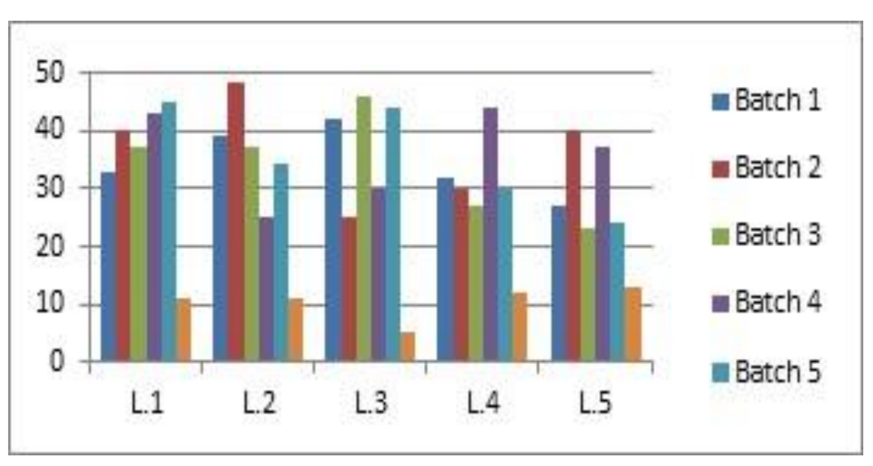
$$minimize \sqrt{\sum_{k=1}^{UB^b} \sum_{j=1}^{m} z_k (z_{kj} - c_j^b)^2}$$

s.t.

$$z_{kj} = \sum_{i=1}^{n} (x_i = k) c_{ij} \qquad k = 1..UB^b, j = 1..m$$
$$\sum_{j=1}^{m} z_{kj} \le C \qquad k = 1..UB^b$$
$$z_k \le \sum_{i=1}^{n} (x_i = k) \qquad k = 1..UB^b$$
$$z_k \ge (x_i = k) \qquad i = 1..n, k = 1..UB^b$$
$$z_k \ge z_{k+1} \qquad k = 1..UB^b - 1$$
$$x_i = (0, UB^b)$$
$$z_{kj} = int.$$
$$z_k = (0,1)$$



CP batching solution (C=200)



Arbitrary batching solution (C=200)

Hamiltonian path for order <u>sequencing</u>

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