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Invited Expert on QR – Responder for QR Focus

Robert L. Mayes
Georgia Southern University, rmayes@georgiasouthern.edu

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Waterbury Summit
Response to Quantitative Reasoning Papers

Robert Mayes
Georgia Southern University
QR Papers

• Heather Johnson
  – QR in Mathematics Education: Directions in Research and Practice

• Richard Lehrer and Anthony Petrosino
  – A Learning Progression in Data Modeling Emerging in a Trading Zone of Professional Community and Identity

• Responses based on the work conducted through the Pathways Project, an NSF supported initiative to develop environmental literacy learning progressions.
  – Quantitative Reasoning (QR) Team on the project led efforts to incorporate QR into the science learning progressions, culminating in the formation of a QR learning progression.

This project is supported in part by a grant from the NSF: Culturally Relevant Ecology, Learning Progressions, and Environmental Literacy (DUE-0832173), which we refer to as Pathways.
Response to Johnson Paper

• Defining QR: broad use of QR in literature, differs both within math and between math & science areas
  – Johnson builds on Thompson’s definition of QR as a “process that could contribute to one’s construction of mathematical knowledge”
  – Cognitive development of multiplicative, algebraic, and covariational reasoning

• Act of Quantification (QA), as one of three progress variables in our proposed learning progression for QR
**QR Definition**

Quantitative reasoning is mathematics and statistics applied in real-life, authentic situations that impact an individual’s life as a constructive, concerned, and reflective citizen. QR problems are context dependent, interdisciplinary, open-ended tasks that require critical thinking and the capacity to communicate a course of action.
Contrasting QR Definitions

• Measurement issue in QR
  – Conceiving quantity as being possible to measure
  – Attribute has a unit measure
  – Moved beyond her notion of QR when we included identifying standard or non-standard unit of measure, since this implicates potential measurement
  – Measurement is so central to science that we felt compelled to include the act of measuring in our QA progress variable
  – Quantitative Literacy (QL): act of measurement, compare/contrast/combine variables using mostly arithmetic processes in an advanced way
Scale Analogy & Context

• Microscopic scale: determining the fundamental nature of quantification
• Landscape scale: determine overarching patterns of QR for literate citizens
• QA -> QL -> QI -> QM
• Role of Context: controlled QR tasks vs. authentic real-world tasks
Response to Lehrer & Petrosino Paper

• Defining modeling
  – set of practices including posing questions, generating measures and data representations, structuring and visualizing measures, and making inference in light of uncertainty
  – conceptual pathway from inquiry to inference; grounded in understanding measure, error in measure, and in structuring of the resulting quantities as data

• Procedural approach to learning math as barrier

• Interdisciplinary nature of QM and QI as barrier
Trajectory vs. Progression

• Learning progressions include prospective pathways of conceptual development and specific realizations of these we call learning trajectories.

• Learning trajectory crosses over our QR progression
  – difference among measures, multiple shapes of same data, and statistics-as-measure components fall within our QA and QL component
  – measure of uncertainty falls within our QI component
  – modeling measurement process with chance is QM
  – inference in light of uncertainty returns to QI, indicating the cyclic nature of the interplay between QI and QM
Trading Zones

• Learning progressions are trading zones in which different realms of educational practice intertwine
  – wrenching work of aligning disparate communities (learning researchers, teachers, psychometricians)
  – we found significant wrenching within the learning researchers in science & mathematics

• What mathematics is important for an environmentally literate citizen to possess: quantification, numeracy, measurement, calculation, representation, interpretation, or modeling?
QR in the Classroom

• Moving from progression to classroom implementation
  – revision of design documents into educative curricula that include teacher notes, thought-revealing questions, students’ ways of thinking, mathematical appendices supporting science contexts, and extensions
  – use of video annotated construct maps and formative assessment which move teachers’ practice away from initiate-respond-evaluate cycles are innovative strategies

• We are at LP and assessment development level
  – quantitative analysis of interviews
  – Closed-form assessments analyzed using Rasch Modeling
  – move to developing teaching experiments around LP and assessments
  – professional development workshops to move into classroom
Robert Mayes
Georgia Southern University
rmayes@georgiasouthern.edu

Franziska Peterson
University of Wyoming
fpeterso@uwyo.edu

Pathways QR Research Team:
Jennifer Harris Forrester – University of Wyoming
Jennifer Schuttlefield Christus – University of Wisconsin Oshkosh
Rachel Bonilla – Georgia Southern University
Nissa Yestness – Colorado State University
QR Pathways Research Overview

• The following slides provide a quick overview of the Pathways Team’s research on QR and development of a QR Progression. This will not be covered during the 8 minute response, but may be referred to during the follow up question and answer session.
Theoretical Framework

**Quantification act (QA):** mathematical process of conceptualizing an object and an attribute of it so that the attribute has a unit measure, and the attribute’s measure entails a proportional relationship (linear, bi-linear, or multi-linear) with its unit

**Quantitative literacy (QL):** use of fundamental mathematical concepts in sophisticated ways

**Quantitative interpretation (QI):** ability to use models to make predictions and discover trends, which is central to a person being a citizen scientist

**Quantitative modeling (QM):** ability to create representations to explain a phenomena
Theoretical Framework

• Learning progressions are central to the theoretical framework
  – Duschl (2007) learning and curriculum designs should be organized around learning progressions as a means of supporting learners’ development toward attaining the four proficiencies in science
  – (Corcoran, Mosher, & Rogat, 2009) identified learning progressions as a promising model that can advance effective adaptive instruction teaching techniques and thereby change the norms of practice in schools
Research Goals and Questions

• Purpose: establish a learning progression for QR within the context of environmental science for middle and high school students

• Research questions.
  – **Central research question**: How do students develop QR in the context of environmental science across 6th–12th grade?

• Procedural questions:
  – What are the QR progress variables (dimensions of understanding, application, and practice) that support the development of an environmentally literate citizen?
  – What level of QR within the context of environmental science do students bring to the discourse at the sixth grade level?
  – What are the key QR conceptual stepping stones to moving from a novice to environmentally literate citizen? How do these inform a QR learning progression?
  – What are the QR tasks students at a given learning progression level should be capable of performing?
Methods

• Iterative research design
  – Creating learning progressions involves
    • Grounding the lower anchor
    • Identifying intermediate levels of understanding
    • Upper anchor based on expert views of what QR a scientifically literate citizen should know
Methods

• First Iterative Cycle
  – 30-40 minute qualitative interviews (N=39 middle and high school students in STEM courses)
  – Grounded theory analysis, Nvivo as tool
  – Descriptive statistics to determine trends across grade levels (6-12), science scales (Macro,Micro,Landscape), QR progress variables (QA, QL, QI, QM) and QR process levels

• Revised LP and interviews, conducted second round of interviews (N=14) => Revision of LP and assessments
Methods

• Second Iterative Cycle
  – Closed-form QR assessment items, 5-point Likert scale, 96 items total (24 per scale/8 per element)
  – Data collected in 3 states (N > 500)
  – Rasch model analysis currently being conducted
Results

- Sample distribution across grades for overall QR, QA, QI, and QM

Table 1. QR by grade

<table>
<thead>
<tr>
<th>Grade</th>
<th>Number of students</th>
<th>Total QR mean</th>
<th>QA mean</th>
<th>QI mean</th>
<th>QM mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>3</td>
<td>1.75</td>
<td>1.78</td>
<td>1.78</td>
<td>1.50</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>1.77</td>
<td>1.90</td>
<td>1.67</td>
<td>1.00</td>
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<tr>
<td>8</td>
<td>12</td>
<td>2.01</td>
<td>2.08</td>
<td>2.06</td>
<td>1.33</td>
</tr>
<tr>
<td>9</td>
<td>4</td>
<td>2.22</td>
<td>2.25</td>
<td>2.17</td>
<td>2.33</td>
</tr>
<tr>
<td>10</td>
<td>3</td>
<td>2.08</td>
<td>2.11</td>
<td>2.00</td>
<td>–</td>
</tr>
<tr>
<td>11</td>
<td>7</td>
<td>1.96</td>
<td>1.90</td>
<td>2.05</td>
<td>1.75</td>
</tr>
<tr>
<td>12</td>
<td>3</td>
<td>1.67</td>
<td>1.78</td>
<td>1.78</td>
<td>1.00</td>
</tr>
<tr>
<td>Cor(^a)</td>
<td>0.076</td>
<td>0.012</td>
<td>0.294</td>
<td>0.026</td>
<td></td>
</tr>
</tbody>
</table>

\(^a\)Correlation between grade level and QR processes.
Results

• 39 students ranked from level 1 (lower anchor-novice) to level 4 (upper anchor-expert)
• ANOVA on QR x Science Scale: no significant difference

<table>
<thead>
<tr>
<th>QR level</th>
<th>Macro-scale</th>
<th>Landscape scale</th>
<th>Micro/atomic scale</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>QA</td>
<td>QI</td>
<td>QM</td>
<td>QA</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>9</td>
<td>7</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>28</td>
<td>27</td>
<td>6</td>
<td>28</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>3</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>NE/NA</td>
<td>0</td>
<td>0</td>
<td>25</td>
<td>0</td>
</tr>
</tbody>
</table>

QA, quantification act; QI, quantitative interpretation; QM, quantitative modeling.
Results

• Qualitative Interview Analysis
  – Trajectory issue: no consistent increase in learning progression levels across grade levels
  – Scaling issue: no consistent differences in QR use on the micro/atomic, macro and landscape-scale
  – Tool implementation issue: failed to select the appropriate mathematical or statistical tool from their toolbox, and even when the correct tool was selected they failed to use QR to apply the tool within the science context
Results

• Rasch Analysis of Assessment
  – Analysis of items indicates which items were problematic (difficult item, poorly constructed item, miscoded item)
  – Analysis of persons allows removal of students not completing the assessment as intended (student answers are too predictable)
  – Analysis of persons x items allows improvement of instrument as well as results of students by scale (macro, micro, landscape scale), by strand (biodiversity, carbon, water) and by QI element (trend, translation, revise, predict)
Discussion

• Implications for teaching
  – engage in real-world problem-based learning
  – require students to provide quantitative as well as qualitative support for their arguments
  – provide multiple quantitative representations (tables, graphs, equations, science models) within a science context and use QR to provide data-based informed decisions about critical issues that impact their place
  – engage in building their own models representing these issues, then test and refine those models
Discussion

• QR Learning Progression
  – QA progress variable is the trigger for QR: student first quantifies an object within a context, allowing them to operate on that quantity using the arithmetic processes within quantitative literacy
    • Elements of QA: Variation, QL, Context, Variable
  – QI progress variable is the ability to interpret a model provided to the student
    • Elements of QI: Trends, Predictions, Translation, Revision
  – QM progress variable is creation of the model by the student
    • Elements of QM: Create model, Refine Model, Model reasoning, Statistical
<table>
<thead>
<tr>
<th>Achievement Level</th>
<th>Quantification Act</th>
<th>QR Progress Variable</th>
<th>Quantitative Modeling</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level 4 (Upper Anchor)</td>
<td>4a Variation: reasons about covariation of 2 or more variables; comparing, contrasting, relating variables in the context of problem</td>
<td>4a Trends: determine multiple types of trends including linear, power, and exponential trends; recognize and provide quantitative explanations of trends in model representation within context of problem</td>
<td>4a Create Model: ability to create a model representing a context and apply it within context; use variety of quantitative methods to construct model including least squares, linearization, normal distribution, logarithmic, logistic growth, multivariate, simulation models</td>
</tr>
<tr>
<td></td>
<td>4b Quantitative Literacy: reasons with quantities to explain relationships between variables; proportional reasoning, numerical reasoning; extend to algebraic and higher math reasoning (MAA)</td>
<td>4b Predictions: makes predictions using covariation and provides a quantitative account which is applied within context of problem</td>
<td>4b Refine Model: extend model to new situation; test and refine a model for internal consistency and coherence to evaluate scientific evidence, explanations, and results; (Duschl)</td>
</tr>
<tr>
<td></td>
<td>4c Context: situative view of QR within a community of practice (Shavelson); solves ill-defined problems in socio-political contexts using ad-hoc methods; informal reasoning within science context (Steen &amp; Madison; Sadler &amp; Zeidler)</td>
<td>4c Translation: translates between models; challenges quantitative variation between models as estimates or due to measurement error; identifies best model representing a context</td>
<td>4c Model Reasoning: construct and use models spontaneously to assist own thinking, predict behavior in real-world, generate new questions about phenomena (Schwarz)</td>
</tr>
<tr>
<td></td>
<td>4d Variable: mental construct for object within context including both attributes and measure (Thompson); capacity to communicate quantitative account of solution, decision, course of action within context</td>
<td>4d Revision: revise models theoretically without data, evaluate competing models for possible combination (Schwarz)</td>
<td>4d Statistical: conduct statistical inference to test hypothesis (Duschl)</td>
</tr>
<tr>
<td>Achievement Level</td>
<td>Quantification Act</td>
<td>Quantitative Interpretation</td>
<td>Quantitative Modeling</td>
</tr>
<tr>
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<tr>
<td>Level 3</td>
<td>3a Variation: recognizes correlation between two variables without assuming causation, but provides a qualitative or isolated case account; lacks covariation</td>
<td>3a Trends: recognize difference between linear vs. curvilinear growth; discuss both variables, providing a quantitative account</td>
<td>3a Create Model: create models for covariation situations that lack quantitative accounts; struggle to apply model within context or provide quantitative account</td>
</tr>
<tr>
<td></td>
<td>3b Quantitative Literacy: manipulates quantities to discover relationships; applies measure, numeracy, proportions, descriptive statistics</td>
<td>3b Predictions: makes predictions based on two variables, but relies on qualitative account; uses correlation but not covariation.</td>
<td>3b Refine Model: extend model based on supposition about data; do not fully verify fit to new situation</td>
</tr>
<tr>
<td></td>
<td>3c Context: display confidence with and cultural appreciation of mathematics within context; practical computation skills within context (Steen); lacks situative view</td>
<td>3c Translation: attempts to translate between models but struggles with comparison of quantitative elements; questions quantitative differences between models but provides erroneous qualitative accounts for differences</td>
<td>3c Model Reasoning: construct and use multiple models to explain phenomena, view models as tools supporting thinking, consider alternatives in constructing models (Schwarz)</td>
</tr>
<tr>
<td></td>
<td>3d Variable: object within context is conceptualized so that the object has attributes, but weak measure (Thompson); capacity to communicate qualitative account of solution, decision, course of action within context, but weak quantitative account</td>
<td>3d Revision: revise model to better fit evidence and improve explanatory power (Schwarz)</td>
<td>3d Statistical: use descriptive statistics for central tendency and variation; make informal comparisons to address hypothesis</td>
</tr>
<tr>
<td>Achievement Level</td>
<td>Quantification Act</td>
<td>Quantitative Interpretation</td>
<td>Quantitative Modeling</td>
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</tbody>
</table>
| Level 2           | **2a** Variation: sees dependence in relationship between two variables, provides only a qualitative account; lacks correlation, erroneously assumes causation  
|                   | **2b** Quantitative Literacy: poor arithmetic ability interferes with manipulation of variables; struggle to compare or operate with variables  
|                   | **2c** Context: lack confidence with or cultural appreciation of math within context; practical computation skills are not related to context  
|                   | **2d** Variable: object within context is identified, but not fully conceptualized with attributes that are measurable; fails to communicate solution, decision, course of action within context; qualitative account without quantitative elements (Thompson) | **2a** Trends: identify and explain single case in model; recognize increasing/decreasing trends but rely on qualitative account or change in only one variable  
|                   | **2b** Predictions: makes predictions for models based on only one variable, provides only qualitative arguments supporting prediction  
|                   | **2c** Translation: indicate preference for one model over another but do not translate between models; acknowledge quantitative differences in models but do not compare  
|                   | **2d** Revision: revise model based on authority rather than evidence, modify to improve clarity not explanatory power (Schwarz)  | **2a** Create Model: constructs a table or data plot to organize two dimensional data; create visual models to represent single variable data, such as statistical displays (pie charts, histograms)  
|                   | **2b** Refine Model: extends a given model to account for dynamic change in model parameters; provides only a qualitative account  
|                   | **2c** Model Reasoning: construct and use model to explain phenomena, means of communication rather than support for own thinking (Schwarz)  
<p>|                   | <strong>2d</strong> Statistical: calculates descriptive statistics for central tendency and variation but does not use to make informal comparisons to address hypothesis |</p>
<table>
<thead>
<tr>
<th>Achievement Level</th>
<th>Quantification Act</th>
<th>Quantitative Interpretation</th>
<th>Quantitative Modeling</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Level 1 (Lower Anchor)</strong></td>
<td>1a Variation: does not compare variables; works with only one variable when discussing trends, 1b Quantitative Literacy: fails to manipulate and calculate with variables to answer questions of change, discover patterns, and draw conclusions; 1c Context: does not relate quantities to context or exhibit computational skills 1d Variable, fail to relate model to context by identifying objects no attempt to conceptualize attributes that are measurable; discourse is force-dynamic; avoids quantitative account, provides weak qualitative account</td>
<td>1a Trends: do not identify trends in models 1b Predictions: avoids making predictions from models 1c Translation: fail to acknowledge two models can represent the same context 1d Revision: view models as fixed, test to see if good or bad replicas of phenomena (Schwarz)</td>
<td>1a Create Model: does not view science as model building and refining so does not attempt to construct models 1b Refine Model: accepts authority of model, does not see as needing refinement 1c Model Reasoning: construct and use models that are literal illustrations, model demonstrates for others not tool to generate new knowledge (Schwarz) 1d Statistical: does not use statistics; no calculation of even descriptive statistics</td>
</tr>
</tbody>
</table>